Finding the Blind Spots Before It's Too Late: A (Reverse) Stress Testing Approach for Asset Liability Management.*

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Abstract

The rapid increase of interest rates across global economies and the failure of Silicon Valley Bank, in large part driven by interest rate risk, have increased public and regulatory focus on diligent interest rate risk management. This paper introduces a new quantitative toolkit for (reverse) stress testing of Net Interest Income (NII) and Economic Value of Equity (EVE) – the key metrics of Interest Rate Risk in the Banking Book (IRRBB) – as required by various regulations. Our toolkit combines classic yield curve modelling and valuation tools with Machine Learning (ML)/ Artificial Intelligence (AI) clustering techniques to systematically identify blind spots in a bank's balance sheet.

We illustrate the model's use by applying it to realistic balance sheets of two hypothetical banks and draw several risk management insights and policy implications from this exercise:
1. Supervisory stress scenarios for IRRBB, as defined in BCBS 368, may fail to identify blind spots;
2. For banks operating in multiple currencies, it is cross currency correlations that can give rise to scenarios which adversely affect NII and EVE *simultaneously*;
3. We identify a strong interdependency between EVE and NII suggesting that banks should set the risk appetite for these metrics jointly rather than independently;
4. We outline how a macroprudential regulator could extend the proposed framework to a system-wide (reverse) stress test aimed at identifying whether the banking system as a whole is exposed to interest rate risk concentrations that could pose systemic risk.

Because a safe and robust financial system is vital for a well-functioning economy and society, we aspire to make a timely contribution to ongoing academic, regulatory and practitioner debates on how to make banks more resilient when facing interest rate risk and asset liability management challenges.

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1 Executive Summary

It ain't what you don't know that gets you into trouble;
It's what you know for sure, that just ain't so.

Often attributed to Mark Twain.

1.1 Why one should get the Asset Liability Management (ALM) basics right

The unprecedented fiscal and monetary emergency measures in support of global economies through the severe shock induced by the COVID-19 pandemic has led to a sharp increase in inflation across core economies since 2022, followed by monetary tightening and higher interest rates. On Friday, March 10 2023, US authorities stepped in to save Silicon Valley Bank (SVB), a medium-sized US lender of approximately USD 210 bn total assets, from failing after a bank run started to materialize due to large unrealized losses on SVB's bond portfolio following interest rate increases.

Interest rates have remained quite stable at very low, and even negative, levels since the Global Financial Crisis. This may have contributed to some complacency in the management of interest rate risk. However, the materialisation of interest rate risk, which was one of the key elements of SVB's failure, is not a historical novelty. Indeed, the Savings and Loan (S&L) Crisis in the United States during the 1980s is a prominent example of the catastrophic consequences of interest rate risk inadequately managed. The S&L crisis was largely a result of the mismatch in maturities of the assets and liabilities of many Savings and Loans associations (the S&Ls). They had long-term, fixed-rate mortgage assets funded by short-term liabilities. When interest rates rose dramatically in the late 1970s and early 1980s, the cost of their short-term liabilities exceeded the income from their long-term assets, leading to insolvency for many S&Ls. Because the maturity transformation banks undertake in order to intermediate between the different temporal needs of borrowers and lenders, robust management of Interest Rate Risk in the Banking Book (IRRBB), or Asset Liability Management more generally, remains a timeless focus area for banks[1] and forms a key pillar for a well-functioning economy.

1.2 Why existing regulation would have helped (if applied)

International standards, such as the Basel Committee on Banking Supervisions' Standards Paper 368 ("BCBS 368" now integrated into the "Basel Framework") set regulatory expectations around the treatment of IRRBB. The regulatory framework around IRRBB is focused on the Supervisory Review Process including internal stress testing (Pillar 2) and public disclosures (Pillar 3).

First, under Pillar 2, supervisors set out expectations how banks should measure, monitor and manage these risks, including e.g. the expectation that the Risk Appetite be owned by and decided upon at the most senior level by the Board of Directors of a bank. Moreover, under Pillar 2, banks are required to conduct so-called "Supervisory Outlier Tests", which quantify interest rate risk both in terms of EVE and NII, and put the interest rate risk taken in relation to Tier 1 capital. If a bank fails these tests, it is identified as an "outlier bank" taking excessive interest rate risk, and may be subject to supervisory action. The "EVE Supervisory Outlier Test" (EVE SOT) threshold is set at 15% of Tier 1 capital. However in the US, with less than USD 250bn in total assets, Silicon Valley Bank was not bound by the Supervisory Outlier Test, and in effect running an EVE of 27.7% according to its last public disclosures; essentially double the amount of interest rate risk a bank would be allowed to take before being designated as taking "excessive interest rate risk" under the standard EVE SOT.

Second, Pillar 3, sets out requirements for banks to disclose certain key metrics around IRRBB, thus seeking to foster market discipline. However, in contrast to Credit Risk, Market Risk or Operational Risk there are no Pillar 1 miminum capital requirements for IRRBB. While the former Chairman of the Basel Committee on Banking Supervision, Stefan Ingves, recently remarked that the post financial crisis "lack of appetite" to include IRRBB in the Pillar 1 minimum requirements may have left a hole

[1] The Swiss National Bank's "Financial Stability Report" of June 2023 states *"Interest rate risks are limited at the globally active Swiss banks and were not the cause of Credit Suisse's problems."* (See Section 4.2 p.32 Swiss National Bank (2023))

in the system, a consistent application of BCBS 368 would have reduced the interest rate risk that Silicon Valley Bank was able to take.[2]

The regulatory framework also sets expectations around "Reverse Stress Testing" of interest rate risk. While the Basel Standards set general expectations around reverse stress testing of IRRBB, the EBA are more prescriptive in their regulatory expectations of reverse stress testing. Indeed, Paragraph 98 of the recently updated EBA IRRBB Guidelines (EBA/GL/2022/14) stipulates: *Institutions should perform reverse stress tests in order to (i) identify interest rate scenarios that could severely threaten an institution's capital, economic value and net interest income measures plus market value changes; and (ii) reveal vulnerabilities arising from its hedging strategies and the potential behavioural reactions of its customers. Reverse Stress Testing (RST) methodology and associated processes are required.*[3] Moreover, in June '23, the Federal Reserve Vice Chair Michael Barr has called for "Reverse Stress Tests" as a part of the solution to avoid future failures such as the one of Silicon Valley Bank (Tett, 2023). However, to the best of our knowledge, no best practice for reverse stress testing of interest rate risk or ALM more generally exists to date.

1.3 Where are your blind spots? A "best practice" proposal for (Reverse) Stress Testing of ALM and IRRBB.

This paper sets the ambitious goal of proposing a first step towards building an operational framework for systematic vulnerability identification and reverse stress testing of IRRBB and thereby answering the below pressing questions for a bank:

What type of interest rate shocks will compress our balance sheet's NII or EVE respectively? Does our bank's current suite of stress test scenarios fully cover this range or are there blind spots? Is the interest rate position the bank takes a conscious positioning or are there risks that have gone unnoticed? Do scenarios exists that could adversely impact NII and EVE simultaneously? Is it actually "enough" to evaluate NII and EVE risks via a dozen scenarios (leveraging the prescribed supervisory scenarios and a few additional internal ones)?

A (Reverse) Stress Testing Approach for ALM and IRRBB in a nutshell. We take the above mentioned questions and regulatory guidelines as the starting point for our proposed approach on how to conduct a systematic vulnerability assessment of IRRBB risks. Fig.1 gives a high-level overview of the framework:

1. **Scenario generation:** We introduce a modified Principal Component Analysis (PCA) scenario generator which allows us to efficiently simulate thousands of historical and hypothetical scenarios. Our scenario generator takes FX dependencies across currencies into account and covers a very wide range of interest rate shocks, covering almost the entire space of possible yield curves.

2. **NII and EVE computation:** We compute Net Interest Income (NII) and Economic Value of Equity (EVE), the two core metrics of IRRBB for every scenario generated under step 1. In our simulations, we leverage validated methodologies and models to calculate EVE and NII of two mock balance sheets. However, as the framework is modular, any tool to estimate NII and EVE such as a bank's internal models, the EBA standardised methodologies, or vendor software can be used.[4]

3. **Clustering:** We develop a modified spectral clustering algorithm to systematically identify patterns of similar yield curve shocks and summarize the information contained in thousands

[2] See: `https://www.centralbanking.com/central-banks/financial-stability/micro-prudential/7959049/rate-risk-under-pillar-2-left-hole-in-the-system-ingves`

[3] The equivalent requirement in the FINMA circular 2019/2 on IRRBB states:"Banks shall consider interest rate risk as part of qualitative and quantitative stress tests (reverse stress tests) as part of their overall stress test framework. In such stress tests, banks shall assume a severe worsening of their capital or earnings in order to reveal vulnerabilities in view of their hedging strategies and the potential behavioural reactions of their customers."'

[4] In general, we recommend using "dynamic balance sheet models" to account for shifts in size and composition of the bank's balance sheet under extreme rate moves. For instance, it is well known that when interest rates increase, balances on non-maturing deposits (e.g. current accounts) decrease as customers move funds to higher yielding product classes.

of simulations into actionable information. In this step, a systematic link is established to understand which type of interest rate move is going to affect the NII and/or EVE of a balance sheet in what particular way.

4. **Vulnerability identification and integration into the existing risk management framework:** In the final step, insights drawn from the three previous steps are integrated into the existing risk management framework of a bank. If heightened risk sensitivities to certain yield curve moves have been identified as part of the previous steps, a deep dive may be required to assess whether this positioning of the balance sheet is reflective of a conscious position the bank takes or whether a blind spot has been identified. On the one hand if a blind spot has been identified, mitigating actions via hedging for instance can be undertaken. On the other hand, if a position is consciously taken as a view on the market, the information may help in quantifying a risk-adjusted PnL.

Fig. 1: Our proposed (Reverse) Stress Testing Approach for ALM Risk Management: 1. Comprehensive scenario generation, covering all possible yield curve moves; 2. Systematic evaluation of Net Interest Income and the Economic Value of Equity, inclusive of dynamics that depend on the scenario; 3. Use of clustering techniques to identify patterns of adverse scenarios; 4. Integration into regular risk management processes such as risk identification, risk appetite setting or ALM strategy setting.

1.4 Main Results

We seek to contribute to the ongoing regulatory, academic and practitioner debates around the sound management of interest rate risks (particularly in the new normal) through the development of our operational framework and the insights we are able to draw from it:

- **Result 1: A first step towards a "best practice" approach for ALM (reverse) stress testing**: We propose a novel quantitative toolkit for joint NII and EVE (Reverse) Stress Testing and Vulnerability Identification: To the best of our knowledge, so far no "best practice" has emerged regarding how regulatory required reverse stress testing of IRRBB should or could be conducted. Our approach is modular in the sense that it can be integrated into any bank's existing framework for calculating NII and EVE. The framework helps to systematically identify interest rate vulnerabilities in a bank's balance sheet through a comprehensive exploration of possible interest rate shocks and then using AI/ML tools to identify patterns (i.e. "clusters") of vulnerabilities. Beyond proposing the framework outlined in Fig.1, our methodological contributions consist of i) extending well-known PCA decompositions of yield curves to generate hypothetical scenarios that comprehensively cover a very wide range of "yield curve space" and ii) developing a modified spectral clustering algorithm, which has shown a high performance in pattern recognition of interest rate shocks. – (See Section 4)

- **Result 2: Supervisory stress tests may fail to identify blind spots**: Applying our framework to two hypothetical yet realistic bank balance sheets, we find that supervisory scenarios for stress testing EVE and NII as defined by the BCBS 368 scenarios may fail to identify significant IRRBB risks. In particular, our simulations reveal that both of our hypothetical banks would *pass* their EVE Supervisory Outlier Test with ΔEVE losses of approx. 4% of Tier 1 capital (well below the 15% outlier threshold), despite having very different interest rate risk profiles. While the different interest rate risk profile is not picked up by the BCBS scenarios, our comprehensive scenario analysis does identify this risk: Across 1000 simulated scenarios, the well-hedged bank suffers ΔEVE losses of approx 5% of Tier 1 capital, close to the BCBS outcome, while the other bank suffers a *median* loss of more than 50% of its Tier 1 capital across the *same* 1000 scenarios. Importantly, this result is *not* a reflection of our PCA generated scenarios being extreme as both banks are tested against the same PCA scenarios. Rather, it is a reflection of one bank being well-hedged while the other one is taking on material interest rate risk, which goes unnoticed by the BCBS scenarios. – (See Section 5.1)

- **Result 3: The correlation of cross-currency shocks may lead to *simultaneously* adverse ΔNII and ΔEVE impacts**: We show that for (well-hedged) banks with material operations in more than one currency, cross-currency moves become the key driver to adversely affect ΔNII and ΔEVE *simultaneously* despite the well-known trade-off often present between these two IRRBB metrics. – (See Section 5.2). While we recognise that interest rate moves in currencies are often correlated; firstly, such correlations are far from perfect, and secondly, there are macroeconomic situations in which some of these correlations can be dramatically broken, hence to entirely neglect considering such decoupling events would be a major risk management oversight.

- **Implications for ALM Risk Management:** Our results indicate that evaluating IRRBB risks across a handful of scenarios may miss blind spots and lead to an illusion of precision. Instead, a robust evaluation of IRRBB requires the evaluation of *distributions* of interest rate scenarios akin to how banks assess Market Risk. Moreover, because cross currency correlations become the key driver in determining scenarios that are simultaneously bad for NII and EVE, banks may miss adverse scenarios if ALM decisions are taken on a currency-by-currency basis rather than being looked at holistically in a multi-currency setting. Consequently, the risk identification process and risk appetite framework that banks are required to have should be implemented by considering ΔEVE and ΔNII jointly rather than independently. (See Section 6.1)

- **Implications for Policy Makers**: We find that instantaneous ΔEVE impacts and 1-year ΔNII impacts (e.g. as defined in the EBA's Regulatory Technical Standards) are of the same order of magnitude for well-hedged banks. This may suggest reviewing the different Supervisory Outlier Thresholds for ΔEVE and ΔNII of 15% and 5% of Tier 1 capital respectively. In general, ΔNII is quite sensitive to assumptions such as non-maturing deposit replication, investment of equity and the time evolution of the scenario, which are all aspects that could be explored in further depth. Finally, we believe our model may also form a first step towards a "macroprudential reverse stress test": Instead of evaluating how a system of banks fares in a single adverse scenario, which is common regulatory practice, we believe systemic risk analyses could benefit from evaluating how the financial system fares in a wide range of scenarios in order to then understand whether systemic concentration risk is present (because many banks are exposed to the same scenario) or whether a "Tolstoy'ian diversification" in the sense of "all "unhappy" scenarios are different" exists. – (See Section 6.2)

More generally, we also hope to trigger an interdisciplinary discussion between the academic fields of banking, mathematical finance, machine learning / data science, macroprudential policy making, as well as among practitioners in asset liability management / interest rate risk management.

2 Literature Review

We aspire to make a timely and interdisciplinary contribution to improve the identification, management and reverse stress testing of interest rate risk in the banking book. First, we build on standard PCA decompositions of the yield curve and extend these to create a comprehensive yield curve simulator. Second, we modify spectral clustering algorithms to identify patterns in yield curve. Leveraging these innovations, we run a systematic analysis on two mock balance sheets to draw practical implications for risk management and policy making. Our results thereby contribute to the literature on (reverse) stress testing, macroprudential stress testing and practical ALM risk management. We attempt to survey some key contributions in these different fields and our relation to these works and disciplines below.

The stress of 2023, banking crises and (macroprudential) stress testing. Cecchetti and Schoenholtz (2023) take a look at the banking stress of 2023, going through the macroeconomic and banking drivers of the failures of several US banks. At the heart of the crisis, a loss in confidence of uninsured depositors, who doubted the solvency of their banking institutions after increasing rates created significant unrealized losses across bond portfolios, is identified. The book draws comparisons to earlier crises such as the Global Financial Crisis, the Savings and Loan Crisis or Continental Illinois in 1984 and seeks to draw lessons for policy reform. The authors discuss revisiting the pros and cons of expanding mark-to-market accounting for banks' bond portfolios, how to strengthen supervisory and resolution frameworks as well as ideas to redesign deposit insurance schemes.

More fundamentally, Duffie (2010) discusses in detail the failure mechanics of large banks, highlighting the interplay between (perceived) solvency on the one hand and liquidity on the other hand and how to address this nexus. Cont et al. (2020) develop a quantitative model of joint liquidity and solvency stress testing and show how *exogenous* shocks to solvency can lead to *endogenous* shocks to liquidity, which eventually lead to amplification of equity losses. The authors define the concept of "Liquidity at Risk", which quantifies the liquidity resources that a financial institution requires to withstand a severe stress scenario. Coelho et al. (2023) also highlight the significance of the interconnectivity between interest rates, solvency and liquidity positions of Banks, and hence the need for an holistic approach to risk management. In Budnik et al. (2020), authors from the European Central Bank introduce the "Banking Euro Area Stress Test (BEAST)", a semi-structural model designed to assess the resilience of the euro area banking system from a macroprudential perspective. Calibrating the model to the EU-banking system, the authors find higher system-wide capital depletions for a given stress scenario compared to a similar constant balance sheet exercise without dynamics. Aikman et al. (2023) provide a comprehensive overview of the current state of the literature on macroprudential stress testing reviewing more than 150 academic papers and classifying their contributions into the various contagion mechanisms and the policy insights drawn.

Reverse stress testing. The academic literature on "reverse" stress testing is emerging and has not yet reached the level of maturity of the more general ("forward") stress testing literature. Eichhorn et al. (2021) provides a broad overview of the current status of reverse stress testing in banking, both from an individual bank perspective, as well as from the macroprudential perspective. Eichhorn and Mangold (2021) propose a theoretical framework for integrated reverse stress testing considering both solvency and liquidity effects. Albanese et al. (2023) propose a stylized model for quantitative reverse stress testing leveraging Monte Carlo simulations to assess capital losses with particular attention to credit and funding valuation adjustments as well as considering cost of capital. Ojea-Ferreiro (2021) builds on the concept of "Conditional Expected Shortfall" to design a market-based reverse stress test and applies the model to the Spanish Fund sector.

Reverse stress testing has also been a focus of system-wide and macroprudential analyses: Grigat and Caccioli (2017) develop a model of financial contagion and apply it to identify the smallest exogenous shock that would lead to system-wide repercussions should it occur. Grundke and Pliszka (2018) develop a theoretical framework for macroprudential reverse stress testing, highlighting the necessary steps for such a framework including principal component analysis for dimension reduction and the need for numerous robustness checks given general data limitations and model risk. Baes and Schaanning (2023) develop an operational model of fire sales contagion and apply it to the EU-banking system to identify worst-case shocks under the assumption that every bank responds optimally to a given exogenous shock. The authors find that the EBA bank stress test, being a microprudential stress test misses some key system-wide vulnerabilities and identify a small subset of banks which are core to loss amplification during stress periods.

Agent based models. Many of the above-mentioned models build on insights from the "Agent Based Model" (ABM) literature. Bookstaber et al. (2018) was among the first to propose the use of ABMs to study system-wide contagion mechanisms between dealer banks, hedge-funds and cash providers. Bookstaber (2017) takes a high-level perspective on the application of ABMs in the future of modelling economic and financial phenomena. Taking a broad macro-perspective beyond stress testing but related to interest rate modelling, Knicker et al. (2023) provide a significant contribution to the ongoing debate of central bank models and the role of ABMs in central bank policy decision making in the post COVID inflation areas. By calibrating the well-known Mark-0 ABM to the 2020-2023 period, with the ability to generate a veriety of plausible scenarios, the authors study the influence of regulatory policies on inflation dynamics and draw important policy lessons such as the success of policy actions depending on expectation anchoring rather than direct interest rate hikes.

Yield curve modelling and decompositions. Redfern and McLean (2014) provides a detailed introduction to the modelling of yield curves using principal components, which we leverage to build our scenario generator in order to simulate interest rate shocks. Decompositions of yield curves into (often three) principal components is closely related to the well-known Nelson Siegel model, introduced in the seminal paper Nelson and Siegel (1987). We benchmark our PCA generator, which (by design) creates severe instantaneous shocks to interest rates, to the classic HJM model from Heath et al. (1992). While exploring a smaller subspace of interest rate shocks, the HJM-generated scenarios are based on current market conditions and thus provide a market-based view of "likely" scenarios as opposed to more extreme "what-if?" scenarios. Harms et al. (2018) introduce a new class of HJM models, called consistent recalibration (CRC) models, which fit the dynamics of real market data and remain tractable, resolving the issue of loss of analytical tractability of affine short rate models due to time-dependent parameters that arise when seeking to properly account for market dynamics. More generally, Filipovic (2009) and Brigo and Mercurio (2006) are considered the standard references for interest rate and yield curve modelling and provide rigorous introductions to the topic.

Clustering algorithms. We leverage the work of Von Luxburg (2007) to develop a modified spectral clustering algorithm, which we found to be particularly effective at clustering yield curves. A comprehensive overview of the latest research on spectral clustering is provided in the survey paper Jia et al. (2014). While to our knowledge the clustering of yield curves has not received particular attention in the academic literature yet, Cheam and Fredette (2020) provides an overview of similarity characteristics for curve clustering. Hastie et al. (2009) is a standard text for a comprehensive introduction into various clustering techniques and statistical learning.

Comprehensive ALM overviews. We would like to highlight the works of Farahvash (2020) and Bohn and Elkenbracht-Huizing (2018), which provide, in the authors' personal view the most comprehensive and hands-on introductions to key IRRBB modelling topics. In the authors' view, the importance of these monographs has not yet been recognized by the academic literature, despite their concrete and quantitative treatment of core banking topics such as the modelling and "replication" of non-maturing client deposits (ie deposits without contractual maturity). Lastly, Choudhry (2022) is a standard providing an overview of bank operations, with a particular focus on ALM and treasury operations and Hull (2012) provides a broad introduction into a variety of financial products.

The IRRBB Regulatory Framework. The original Basel Committee on Banking Supervision's (BCBS) Standard 368 (see Basel Committee on Banking Supervision (2016)), has now been fully integrated into the Basel III framework. Local regulators such as the Swiss Financial Market Supervisory Authority (FINMA) or the European Banking Authority (EBA) have implemented these Standards in their respective jurisdictions, see Swiss Financial Markets Supervisory Authority (FINMA) (2019) and European Banking Authority (2022a). The EBA has issued Regulatory Technical Standards that provide clear guidance on the implementation of the NII and EVE Supervisory Outlier Tests in European Banking Authority (2022c). Lastly, standardized and simplified standardized models for IRRBB are laid out in European Banking Authority (2022b) for EBA-regulated entities. These standardized approaches apply to smaller banks, which may choose to leverage these models, as well as for more sophisticated banks whose internal models have been deemed "inadequate" by supervisors. For a historical account of the development of IRRBB regulations, see Chapter 12 in Newson (2021) or Chapters 8 and 9 in Farahvash (2020).

3 A Primer on Asset Liability Management and Interest Rate Risk in the Banking Book

In this section we introduce two hypothetical banks with realistic but imaginary balance sheets: Reversius Bank and YOLO Bank. We will illustrate key IRRBB concepts by concretely discussing these two banks and shall use these realistic but mock balance sheets to conduct simulations in Section 5. Seasoned ALM practitioners may wish to skip to Section 4 directly. An introduction to the key IRRBB risk categories (gap-, basis- and optionality risk), the IRRBB regulatory framework and the key positions that determine IRRBB is provided in the annex. Our analysis of Reversius Bank and YOLO Bank will focus primarily on gap- and basis risk.[5]

3.1 The balance sheets of Reversius Bank and YOLO Bank

We assume that Reversius and YOLO Bank are two banks with USD 100bn in total assets, consisting of assets and liabilities in two currencies only (USD and CHF).[6] Reversius Bank and YOLO Bank will be identical in size and composition of assets; they will only differ in the interest rate risk profile of their assets and liabilities. Fig.2 shows the balance sheet of Reversius (and YOLO), who hold 60bn in loans, split equally between fixed rate and floating rate loans, 20bn in cash and HQLA (e.g. bond portfolios), and 20bn in secured lending and other assets. On the liability side, both banks hold 10bn equity, and fund themselves via 60bn customer deposits, split equally between term and non-maturing deposits, 20bn long-term debt and 5bn in structured notes and CDCPs. We randomly generate assets and liabilities of the bank and obtain a rough 50/50 split of USD and CHF positions. For the sake of illustration, we assume that both banks hold equity in CHF and in USD, but they have decided to stabilize only their USD NII.[7] Both banks also use interest rate swaps to hedge their interest rate risk. We have intentionally constructed the balance sheets of Reversius Bank to be more prudent as the one of YOLO Bank, see Section 3.2 and Fig.4 and Fig.5.

[5]Our framework can also be used to assess optionality risk (such as deposit redemptions by clients), however this would require a detailed discussion of the modelling of non-maturing deposits, which is beyond the scope of this paper.

[6]Our model can easily be extended to further currencies. However, as all interesting phenomena already arise in the two currency setting, we limit ourselves to this setting for ease of exposition.

[7]"NII stabilisation", "investment of equity", "hedging of equity" or "equity term out" refers to the process of holding a receiver swap portfolio against the shareholder equity. The basic idea is that equity is a fixed rate (at 0%) liability in perpetuity, which is funding assets that (post hedging) earn an overnight rate, which results in an interest rate mismatch. By holding a fixed receiver portfolio of swaps against the equity, the stream of volatile overnight cashflows is swapped into a fixed rate receiver stream, which leads to less volatile NII. For a deeper discussion, we refer to Newson (2021).

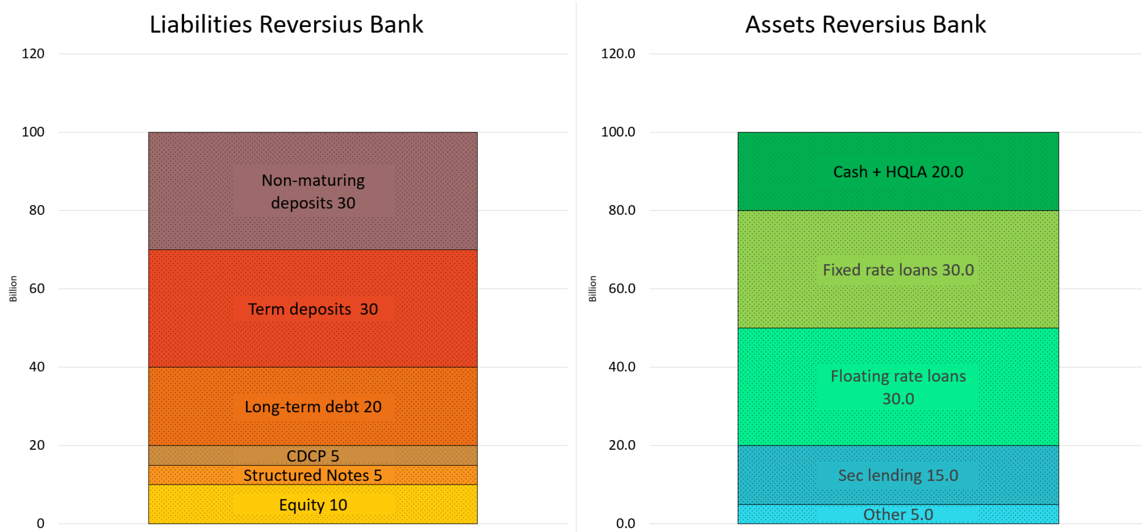

FIG. 2: We consider two 100bn hypothetical balance sheets of banks called Reversius and YOLO. Reversius and YOLO are indistinguishable from the size and composition of their assets and liabilities, but will differ markedly in the interest rate risk that these products generate as discussed below in Fig.4 and Fig.5.

Next, we will look at the interest rate risk that the positioning of Reversius and YOLO creates and how this can be captured by the two standard IRRBB metrics.

3.2 The two key metrics of IRRBB: Net Interest Income and Economic Value of Equity

3.2.1 NII and the repricing cashflow profile.

NII is the difference between interest earned on interest bearing assets and the interest paid on interest paying liabilities, and is often the most important source of revenues for banks.[8] For the purpose of risk management (and supervision), NII is often projected over a horizon of 1-3 years (sometimes up to 5). A critical aspect for the risk assessment of NII is the so-called "repricing gap profile", shown in Fig.3. The repricing gap profile shows when the interest rates on assets and liabilities is reset, which is called "repricing".[9] Reversius Bank is slightly liability sensitive in the overnight to 1-month bucket (i.e. more liabilities than assets reprice), while it asset sensitive in the 1 to 3 month bucket (i.e. more assets than liabilities reprice). Hence, if rates were to move up in the next month, due to more liabilities repricing than assets repricing, the NII of both banks would decrease because there are more liabilities requiring a higher rate to be paid on than assets earning a higher rate. Conversely, if rates fell, the bank would make a profit. Due to the bank being asset sensitive instead of liability sensitive in the 1 to 3 months bucket, the situation is reversed if rates were to move in that bucket. The same holds true in fact for YOLO Bank, which is essentially indistinguishable from Reversius on the repricing gap profile. Indeed, we have constructed the balance sheets of both banks such that the amount of assets and liabilities repricing over a 12 month horizon, which will be our focus for the NII simulations, is identical for both banks.

[8]When asked, ChatGPT told us that smaller retail banks in the US and Europe derive 50-70% of their total income from NII, while larger international banks have more diversified income sources and may derive 40-60% of their revenues from NII.

[9]Notably, there is a critical difference between the liquidity duration and the interest rate duration here: a 10-year floating rate loan, which resets every quarter to SOFR, say, has a 10 year duration from a liquidity perspective, but only a 3 month duration from an interest rate perspective.

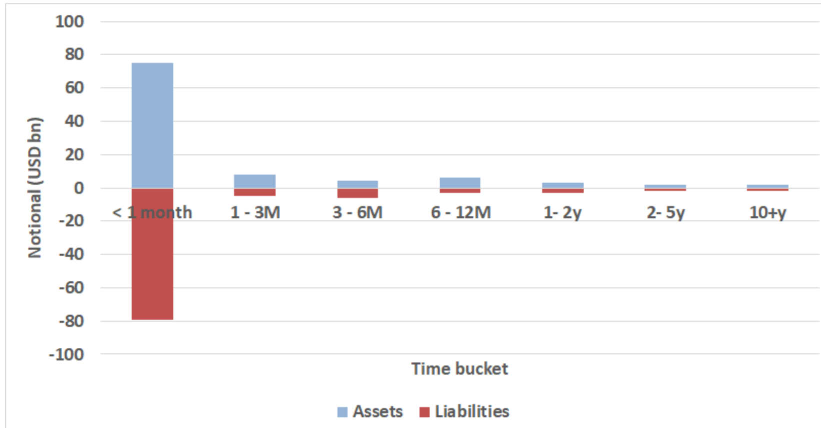

Fig. 3: The repricing gap profile of Reversius Bank shows that the majority of its balance sheet reprices within one year. The profile is typical for a well-hedged institution (by construction of the authors). As NII is simulated over a 1-year horizon, only the first four buckets (up to 12 months) in the plot are of relevance, as all other positions only reprice outside of the 1-year simulation horizon.

Overall, NII takes a rather short-term (and going concern) perspective on IRRBB and helps answering the question whether a bank will manage to remain profitable if interest rates change.

ΔNII. ΔNII is the difference of NII under a baseline scenario (usually the current spot yield curve) and a stress scenario, which assumes a shock to interest rates:

$$\Delta\text{NII}(s; b) := \text{NII}(s) - \text{NII}(b),$$

where b denotes the baseline scenario (ie current spot yield curve) and s denotes a stress scenario defined as a shocked yield curve.

3.2.2 Economic Value of Equity (EVE) and the key rate duration sensitivity profile.

The Economic Value of Equity of a bank is the difference of the present value of all (interest bearing) assets minus the present value of all (interest paying) liabilities. In order to compute EVE, a bank calculates all cash flows paid or received, under the assumption of a run-off balance sheet, and sums these across all time buckets j while discounting these cash flows to obtain the present value:

$$\text{EVE}(s) := \sum_{j=1}^{P} \sum_{t=1}^{T_j} \text{CF}_{j,t}(s)\text{DF}_t(s)$$

where s is a scenario, which may refer to the current spot yield curve, or a shocked version thereof, $\text{CF}_{j,t}(s)$ is a cash flow (positive or negative) for product j at time t, and $\text{DF}_t(s)$ is the discount factor for time t. We note that discount factors do depend on s (if rates go up, discount factors decrease reflecting a decrease in the time value of money), while cash flows may or may not depend on s (e.g. they do not in the case of fixed rate products, but they do for floating rate products).

ΔEVE. ΔEVE is the difference of EVE under a baseline scenario (usually the current spot yield curve) and a stress scenario, which assume as shock to interest rates:

$$\Delta\text{EVE}(s; b) := \text{EVE}(s) - \text{EVE}(b), \tag{1}$$

where b denotes the baseline scenario (ie current spot yield curve) and s denotes a stress scenario defined as a shocked yield curve.

Rather than looking at the full impact of a scenario, one may also ask the question how sensitive the value of a position is to a 1 basis point shift of the yield curve at a specific tenor point. These sensitivities are generally called "Key Rate Durations", and measure the change in Net Present Value of a product (or entire portfolio) under the assumption that the interest rate at a specific tenor is increased by 1 basis point,

$$\mathrm{KR01}(k) := \mathrm{EVE}(b_k^+) - \mathrm{EVE}(b),$$

where b_k^+ designates the current spot (baseline) yield curve, where the interest rate at tenor k is increased by 1 basis point. This can be defined equivalently in terms of cash flows:

$$\mathrm{KR01}(k) := \mathrm{CF}_k^{+1bp}\mathrm{DF}_k^{+1bp} - \mathrm{CF}_k\mathrm{DF}_k,$$

where CF_k^{+1bp} and DF_k^{+1bp} are the cash flows at time k under the assumption that the market rate in bucket k has increased by 1 basis point. While cash flows may or may not change following the shift (depending on whether or not they reprice in that bucket), the discount factor will always change. Fig.4 and 5 show the key rate duration profiles of Reversius and YOLO Bank respectively. The key rate duration profile shows by how much the economic value of a bank changes if interest rates at a specific tenor were to increase by 1 basis point. For Reversius Bank in Fig. 4, the profile tells us that if e.g. interest rates in the 4 year bucket were to increase by 1 bp, the liabilities of Reversius Bank would increase in value by USD ∼10 mn. The economic value of liabilities increases as rates move up, because all else equal, the bank should be paying 1bp more on these liabilities but it does not yet (as they will only reprice in 4 years from now), which thus corresponds to an economic gain. Similarly, a payer swap in which which the counterparty pays the fixed rate and receives the floating leg, will also increase in value as the fixed leg remains fixed but the amount received on the floating leg increases. However, Reversius Bank also stands to lose about USD 10mn in economic value from assets and receiver swaps losing value if rates increased by 1bp in the 4 year bucket. The logic is the same for assets: if rates increase by 1 basis point, then all else equal, the bank should earn 1 basis point more. But because the assets only reprice in the future, it will not earn this additional basis point, which corresponds to an economic loss. Overall, Reversius is thus well hedged and would see only little change in its economic value if rates moved.[10] Indeed, the main driver of it's EVE sensitivity is the receiver swap portfolio that Reversius holds to stabilize its NII, see Section 3.3.2.

While the repricing gap from Fig.3 shows that the majority of the NII risk lies in the shorter term buckets below 1 year, the key rate duration profiles show that from an economic value perspective, the main risks lie in fact in the time buckets between 1 and 10 years. This is quite natural as profile: a rule of thumb to compute the KR01 of a position is to multiply the notional of the position with the time to maturity (expressed in years) and divide this by 10'000.[11] This is why the large positions in the bucket below 1 year of the repricing gap profile are squeezed to a small sensitivity on the key rate duration profile and vice versa the smaller amounts repricing in later years on the repricing gap profile are the key drivers of the economic sensitivity shown on the key rate duration profile.

[10] In the repricing gap profile, assets with a positive sign are placed above the x-axis, while liabilities with a negative sign are placed below the x-axis. Assets and liabilities are flipped in the key rate duration profile relative to the repricing gap profile because the y-axis capture the change in economic value for a 1-bp increase in rates, which is negative for assets and positive for liabilities.

[11] For instance, a 10mn loan repricing in one week will have a USD KR01 $\approx 10'000'000\frac{5}{365\times 10'000} \approx 14$ USD, while the same loan maturing in 2 years has a KR01$\approx 10'000'000\frac{2}{10'000} \approx 2000$ USD.

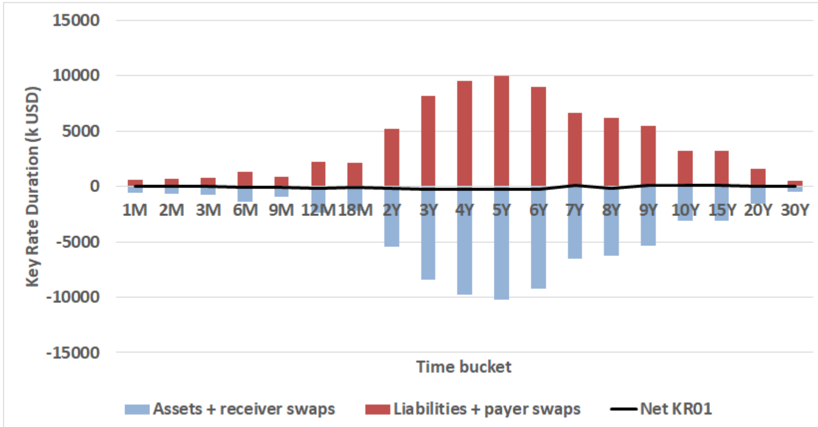

Fig. 4: The Key Rate Duration profile of Reversius Bank shows that - in contrast to the repricing gap profile - the majority of the economic value risk of Reversius Bank lies in between the 1 and 10 year tenors. Such a profile is again typical of a well-hedged institution.

We have constructed the balance sheet of YOLO Bank, by leaving all short-term positions (repricing in less than 12 months) unchanged. As such the gap profile (below 1 year) and the NII which YOLO Bank will earn over the next 12 months is identical to Reversius Bank. However, YOLO Bank, as illustrated in Fig.5, has taken on substantial longer-term interest rate risk. Specifically, YOLO Bank has an excess of long-term assets in the 6 year to 10 year buckets, which are funded by an excess of short-term liabilities in the 1 year to 5 year buckets. This gives rise to the hump-shaped KR01 profile of YOLO: For instance, per 1 basis point increase of interest rates in the 3y bucket, YOLO stands to gain net USD 2.5 mn, while per 1 basis point increase in the 6 year bucket YOLO stands to lose net USD 2.5 mn. We have intentionally constructed YOLO Bank's balance sheet to be net flat. Therefore any parallel move of the yield curve leads to a near zero change in the economic value of the bank as the losses in the longer term buckets are exactly offset by the gains in the shorter term buckets . However, YOLO would suffer a double whammy if the yield curve were to flatten by rates increasing in the 6 - 10 year tenors, while falling in the 1 - 5 year tenors, and this at a rate of approx. USD $\sim$ 25mn per "unit of flattening". As a result, a (minor) move of rates falling by 20bps in the short term buckets, while increasing by 20bps in the longer term buckets would lead to a 0.5bn loss in economic value for YOLO. Or put differently, given YOLO's 10bn Tier 1 capital, a 60 basis point flattening of the yield curve would lead YOLO Bank to lose approx. 15% of its Tier 1 capital, equivalent to the level set by the supervisory outlier threshold.

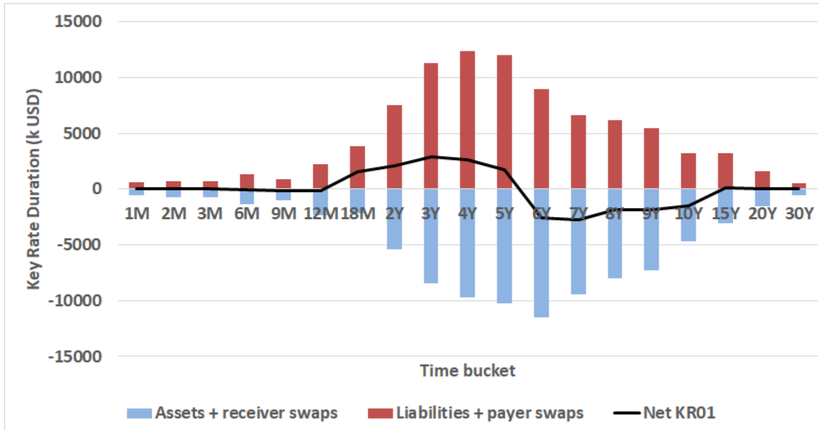

Fig. 5: The Key Rate Duration profile of YOLO Bank shows that YOLO Bank is performing aggressive maturity transformation by funding a significant part of its long-term assets with shorter term liabilities. We have constructed the balance sheet of YOLO Bank such that this excessive maturity transformation only occurs in the time-buckets post the 1-year tenor. As a result, the distribution of NII across our scenarios for YOLO Bank will be identical to the more prudent Reversius Bank; however, when considering EVE results will differ quite drastically as shown below.

3.2.3 How do NII and EVE behave when rates move? Two intuitive examples.

Let us consider two simple examples of a fixed and a floating rate loan in order to illustrate how NII and EVE behave when interest rates move up or down. For simplicity we assume a spot base interest rate curve for discounting that is flat at 2% and use a 30/360 day count convention. Consider the following product characteristics:

- **Fixed-rate loan**: Notional: 1000USD; Interest rate: 3.5% (annualized); Payment frequency: semi-annual payments; Time to maturity: 2 years
- **Floating-rate loan**: Notional 1000USD; Interest rate: SOFR (at 2%) + 50bps; Payment frequency: semi-annual payments; Time to maturity: 2 years

Moreover, we assume that the products have been initiated just today such that no interest has been paid yet, no interest has been accrued either, and with the first payment due in 6 months. Table 1 shows the cash flows (in the NII rows) and the present value (in the EVE rows) of the two products. The payment in 6months on the fixed rate loan will be $1000 \times 0.035 \times \frac{180}{360} = 17.5$ USD, which will be the same for all other periods given the fixed rate nature of the loan irrespective of what interest rates do. Let us now consider the present of the cash flow in 1 year: The cash flow is 17.5, and the corresponding discount factor is $\text{CF}_1 = \frac{1}{\left(1+0.02\times\frac{360}{360}\right)^{\frac{360}{360}}} \approx 0.98039$, whence the PV is $17.5 \times 0.98039 \approx 17.157$. The rest of the table is populated by completing the equivalent computations for the other time periods.

Clearly, the fixed rate loan is independent of any interest rate moves after the constant rate has been agreed at issuing. From an EVE perspective however this fixed rate loan does carry risk. Any increase/decrease in the market interest rate, will make the fixed rate offered become less/more attractive in the current market, hence the net present value of the loan will increase if market rates decrease and decrease if rates increase. Indeed, all else equal, if rates have increased, the bank could have issued that fixed rate loan at a higher rate than it effectively has, and as such the bank has incurred a loss in economic value. The reverse applies if rates decrease. For the floating rate loan the situation is reversed: from an NII perspective the loan is directly sensitive to interest rate moves impacting the interest income as the cash flows generated by the loan will vary in tandem with market rates. However, due to cash flows moving in grid lock with market rates, from an EVE perspective the floating rate loan's net present value is essentially constant as cash flows increase when discounting increases (with higher rates) and decrease when discounting decreases (with lower rates).

Metric	Product	t	Baseline (2%)	+100bps shock	-100bps shock
NII	Fixed rate loan	6m	17.5	17.5	17.5
		12m	17.5	17.5	17.5
		18m	17.5	17.5	17.5
		24m	17.5	17.5	17.5
		Total cash flows	**70**	**70**	**70**
		ΔNII	N/A	**0**	**0**
	Floating rate loan	6m	12.5	17.5	7.5
		12m	12.5	17.5	7.5
		18m	12.5	17.5	7.5
		24m	12.5	17.5	7.5
		Total cash flows	**50**	**70**	**30**
		ΔNII	N/A	**+20**	**-20**
EVE	Fixed rate loan	6m	17.33	17.24	17.41
		12m	17.16	16.99	17.33
		18m	16.99	16.74	17.24
		24m	16.82	16.50	17.15
		Principal	**961.17**	**942.60**	**980.30**
		Present Value	**1029.46**	**1010.07**	**1049.43**
		ΔEVE	N/A	**-19.39**	**+19.97**
	Floating rate loan	6m	12.38	17.24	7.46
		12m	12.25	16.99	7.42
		18m	12.13	16.74	7.39
		24m	12.01	16.50	7.35
		Principal	**961.17**	**942.60**	**980.30**
		Present Value	**1009.95**	**1010.07**	**1009.93**
		ΔEVE	N/A	**+0.12**	**-0.02**

Tab. 1: The complementary nature of NII and EVE as rates move: While a fixed rate loan has predictable cash flows, it's economic value will fluctuate wildly as rates move by increasing with falling rates and falling with increasing rates. Conversely, the economic value of a floating rate loan is stable as its cash flows move in tandem with interest rate and discount factor changes, though this leads to more volatile cash flows.

3.2.4 Why do we need two metrics to monitor IRRBB?

As illustrated in the example of the previous section, when trying to manage interest rate risk (of a position e.g. single loan or more structural positions such as non-maturing deposit replications or NII stabilisation), banks face a dilemma as it needs to choose whether it wishes to reduce its net present value risk (at the expense of more volatile cash flows) or whether it wishes to achieve a stable and predictable stream of cash flows (thus increasing its economic value risk). While the repricing gap profile in Fig.3 shows that the majority of NII risk lies in the short term buckets below 1 year, the key rate duration profile in Fig.4 shows that from an economic perspective the main risk of the bank lies in the 1 to 10 year buckets. These two sensitivity graphs thus capture the different time scales of IRRBB, which are important to obtain a holistic picture of a bank's IRRBB risks. While in principle, a bank could fully mitigate its ΔNII and ΔEVE risks, e.g. by hedging all interest rate risk and only earning a fixed margin over the entire duration of its balance sheet. In practice, this is not feasible however, and a bank will end up taking short-term interest rate risk positions (often driven by client activity) and longer-term interest rate risk positions (often driven by strategic positioning such as deposit replication or NII stabilisation / investment of equity).

3.2.5 The Supervisory Outlier Tests of ΔEVE and ΔNII.

While in contrast to Credit, Market or Operational risk for instance there is no Minimum Capital Requirement (Pillar 1), the regulatory framework for IRRBB focuses on the Supervisory Review Process (Pillar 2) and market discipline through public disclosures (Pillar 3). In particular, banks are required to evaluate the change in their Economic Value of Equity (1) under the six regulatory scenarios shown in Fig.6. The left panel in the figure shows the shocks, while the right panel shows the current yield curve in black and how the yield curve changes through application of each of the six shock scenarios.

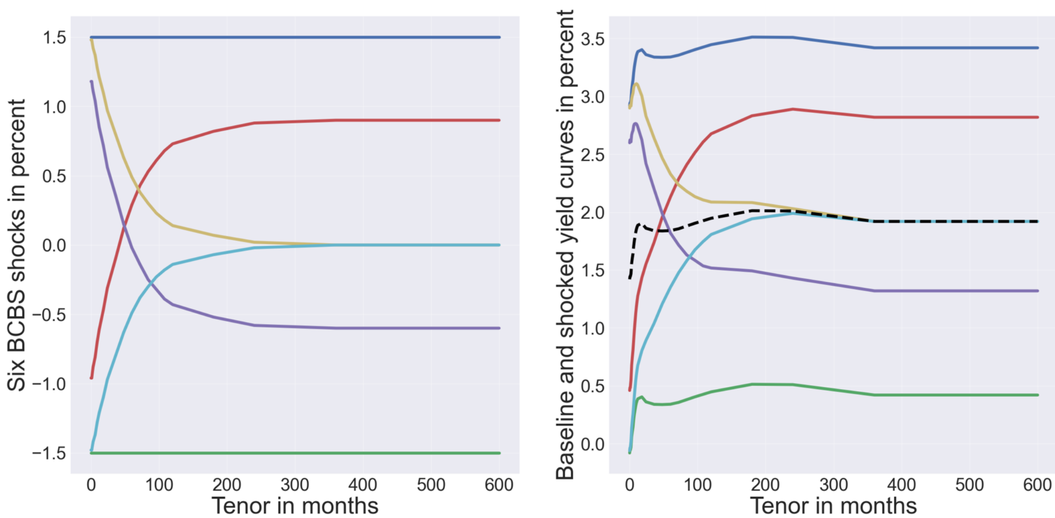

Fig. 6: Left: The six regulatory EVE shocks for CHF: Parallel Up (dark blue), Parallel Down (green), Steepener (red), Flattener (violet), Short Rate Up (yellow), and Short Rate Down (light blue). Right: The six regulatory EVE scenarios as resulting from the application of the six shocks to the current yield curve (dashed). The green yield curve in the right-hand plot, for instance, is obtained by adding the shock of the green parallel down scenario in the left-hand plot to the current spot yield curve. The corresponding plot for USD is relegated to the annex as Fig.23

The Supervisory Outlier Test then stipulates that the biggest decrease[12] in EVE should be no more

[12] The definition of ΔEVE in (1) coincides with the regulatory one and a loss then corresponds to a negative number, which is also the convention used in our scatter plots below. An alternative way is to consider a loss to be a positive number (and a gain a negative number), in which case the definition of the ΔEVE SOT might become a bit more intuitive, stipulating "the maximum (ie largest) loss in ΔEVE should be at most as large as 15% of Tier capital", ie:

$$\max_{s \in \mathcal{S}_{\mathrm{EVE}}} \Delta\mathrm{EVE}(s; b) \leq 0.15 \times \text{Tier 1 Capital}.$$

than 15% of Tier 1 capital, i.e.

$$\min_{s \in \mathcal{S}_{\text{EVE}}} \Delta\text{EVE}(s; b) \geq -0.15 \times \text{Tier 1 Capital},$$

where $\mathcal{S}_{\text{EVE}}$ is the set of six regulatory EVE scenarios. Any bank breaching this requirement is identified as an "outlier bank" which is taking "excessive" interest rate risk.

In October 2022, the EBA has published revised set of Guidelines and Regulatory Technical Standards on IRRBB, which have among others also introduced a Supervisory Outlier Test for Net Interest Income, stipulating that the maximum change in NII under the two adverse NII scenarios should be less than 5% of Tier 1 capital, i.e.

$$\min_{s \in \mathcal{S}_{\text{NII}}} \Delta\text{NII}(s; b) \geq -0.05 \times \text{Tier 1 Capital},$$

where $\mathcal{S}_{\text{NII}}$ consists of the Parallel Up and Parallel Down scenarios only.

3.3 Key ALM risk management questions and the hedging of IRRBB with derivatives

While banks do indeed engage in maturity transformation to match the different inter-temporal needs of borrowers and lenders, it is a common misconception[13] that banks fully assume the entire interest rate (and liquidity) risk arising from this process. The management (and regulation) of IRRBB put limits on how much risk banks may take on though term transformation. The next two sections describe three key ALM concepts: i) the hedging of interest rate risk via derivatives, ii) the behavioural modelling of non-maturing deposits and iii) the stabilisation of Net Interest Income. These are core issues at the heart of a bank's ALM strategy. We refer the interested reader to Bohn and Elkenbracht-Huizing (2018) and Farahvash (2020) and references therein for an in-depth discussion of these topics.

3.3.1 Hedging the banking book and non-maturing deposit replication

The left panel of Fig.7 shows a toy example of a variant of YOLO Bank, funding a 10 year fixed rate loan with an overnight funding. If rates move up, this will be a painful experience. The right panel of Fig.7 shows that this interest rate risk can be mitigated by entering a 10 year payer swap: the fixed rate received on the loan is passed on to the swap counterparty, while the floating leg which is received is paid to the customer.[14] If rates move up, nothing will change on the fixed legs, but we will receive a higher amount on the floating leg, which can be passed on to the customer.[15]

[13] Another related misconception is that banks "intermediate" between borrowers and lenders, i.e. they first receive money from saving households and lend proceed to them out to borrowers. The reality is rather that new money is created when banks grant customers a loan. Banks then need to manage the liquidity (and other) risks that arise from this process, in part by ensuring they have well diversified funding sources, which do include deposits. We refer the interested reader to the Bank of England's Quarterly Bulleting for a detailed exposition of this subtle though important issue (McLeay et al., 2014).

[14] Note that while interest rate risk has been mitigated in this example, liquidity risk is still glaring.

[15] We note that interest rate risk has only been hedged from an economic value perspective, but not from an accounting perspective: As rates move, the swap will need to be marked to market and its change in value will hit the PnL statement of the bank, while there will be no corresponding offset in mark to market change for the loan. In order to align the "accounting reality" with the "economic reality", the bank will need to put the loan and the swap into a so-called "hedge accounting" relationship, which will result in only the *net* change in market value of both swap (hedge) and loan (hedged item) to flow to PnL. We refer the reader to Chapter 12 of Bohn and Elkenbracht-Huizing (2018) for an overview of hedge accounting, which is beyond the scope of this paper.

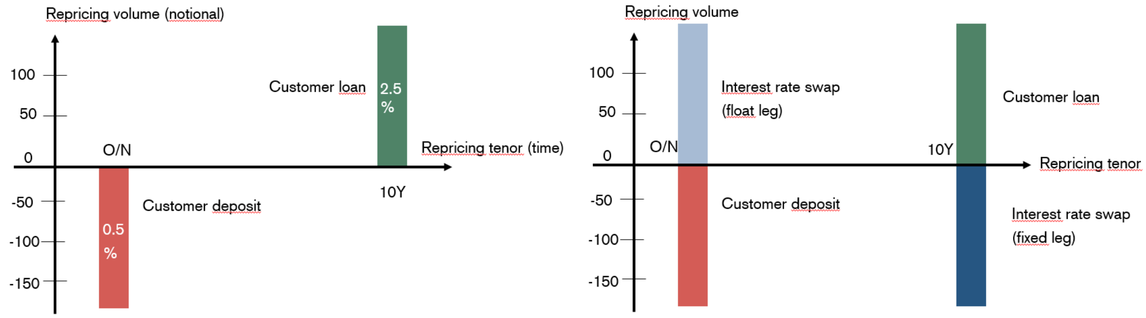

Fig. 7: To mitigate the (blatant) interest rate risk of funding a 10 year fixed rate loan by an overnight deposit (left panel), a bank can choose to enter a payer swap, which fully hedges the interest rate risk.

In practice, banks will not hedge the interest rate risk arising from lending 1:1 with interest rate swaps, but will use behavioural modelling in order to determine "behavioural durations" of deposits and thus use deposits as "natural hedges" for a portion of their loan book. The behaviour of deposits is analysed using statistical methods, and based on this assessment the bank, generally via its Asset Liability Management Committee (ALCO) will assign a duration to its deposits. Fig.8 illustrates simplistically how (contractual) overnight deposits are turned into a series of (modelled) term deposits, which in turn function as natural hedges for the loan book of the bank, often called "deposit term-out" or "deposit replication". In practice, the term-out of deposits can be viewed as identifying the fair Funds Transfer Pricing (FTP) rate at which deposits should be remunerated given their behavioural. This process is critical to correctly allocating profit and loss generation between the front office of a bank (attracting customer deposits and granting loans) and the treasury function of a bank, which centrally manages the resulting interest rate and funding risks. For a more detailed discussion of the FTP process and the split between a bank's Treasury and customer-facing functions, we refer to Cadamagnani et al. (2015), Bohn and Elkenbracht-Huizing (2018) and Farahvash (2020).

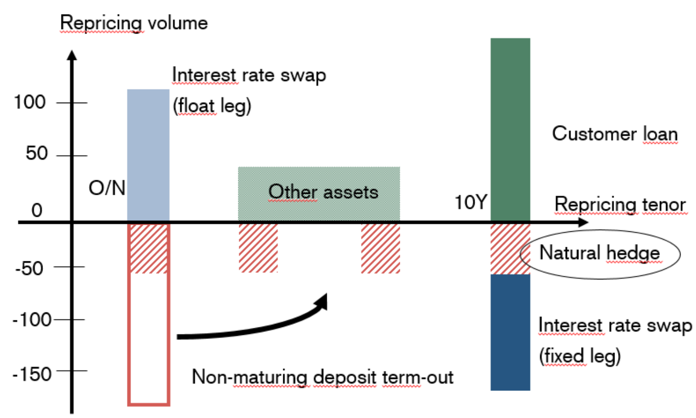

Fig. 8: In the non-maturing deposit replication process, the bank analyses the behaviour of customers and replaces the contractual duration of deposits with a modelled duration. In this process, overnight deposits (red outlined box) are transformed into a series of term deposits (red diagonally striped boxes) which in turn can function as natural hedges for customer loans and thus reduce the need for payer swaps to hedge the interest rate risk.

3.3.2 NII stabilization and investment of equity

In principle, a bank could decide not to perform any maturity transformation at all, and solely focus on earning a fee (which is relatively interest-rate independent) for providing loans and deposit taking services. The balance sheet in Fig.9 assumes that indeed, no interest rate position is taken on neither assets or liabilities and that all fixed rates earned on assets and paid on deposits have been hedged down to overnight using interest rate swaps. In this situation, the bank will earn a stable stream of margins, independent of where spot rates are, because similar to a boat floating in the water, the level of rates does not affect the bank as an increase in interest paid on liabilities is offset by an increase in interest earned on assets. However, the non-interest bearing net[16] equity is funding interest bearing assets, which creates an interest rate mismatch, and exposes the bank to a fall in rates. In order to hedge this risk, banks often choose to stabilize their net interest income and swap the more volatile and generally lower overnight interest rate earned on assets funded by the net equity into a less volatile and generally higher[17] NII. To do so, banks commonly hold an receiver swap portfolio, which is put into a cash flow hedge accounting relationship. Over the interest rate cycle, the swap portfolio maintains a higher NII when rates fall (because the longer fixed rates fall slower than the overnight rate), at the expense of a more sluggish uptake when rates increase. For a more detailed discussion we refer to Newson (2021). The key trade-off, and thus a key ALM strategy decision, is i) whether or not a bank wishes to stabilize its NII, and ii) if so at what duration the equity should be invested.

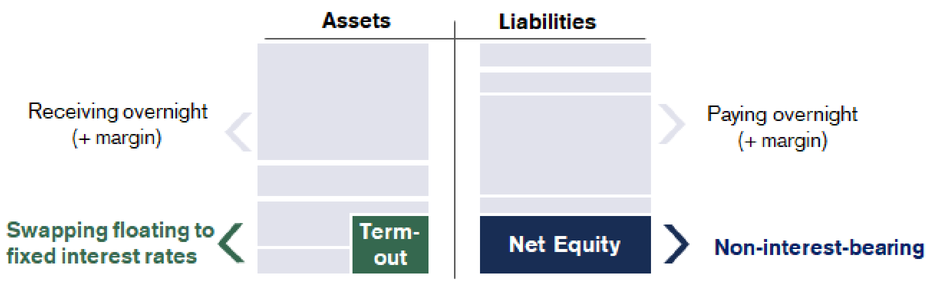

Fig. 9: Post (full) hedging, a bank receives the overnight rate on assets and earns a margin; similarly, it pays the overnight rate and earns a margin on liabilities. However, an interest rate mismatch arises as equity, which is non-interest bearing (or is paid a "fixed rate" of 0%) is funding assets that are earning an overnight rate. In order to stabilize the stream of net interest income, some banks then choose to hedge their net equity. This is most commonly done by holding a portfolio of (receiver) swaps against the net equity, thus converting a more volatile (and generally lower level) of overnight income into a less volatile (and generally higher level) of fixed rate income.

4 A (Reverse) Stress Testing Approach for ALM and IRRBB

This section describes our proposed (Reverse) Stress Testing Approach and presupposes a standard level of ALM-jargon and expertise, which are introduced in Section 3. As mentioned in the Executive Summary, the key steps of the ALM (Reverse) Stress Testing Approach are:

1. **Scenario generation**: Generate a comprehensive set of yield curve scenarios, which essentially cover the entire space of how rates could possibly move.
2. **NII and EVE estimation**: Calculate the NII and EVE of each scenario using models that include behavioural reactions of customers.
3. **Clustering**: Using clustering techniques, identify patterns of "common" scenarios and group these.
4. **Vulnerability identification and integration into the existing risk management framework**: Depending on what scenarios XYZ are identified as critical in step 3, the bank should engage in a discussion regarding i) what would need to happen in the macroeconomy for rates to behave as captured in scenarios XYZ, and ii) whether as a result additional hedging activity should take place and / or the risk appetite reviewed.

The next four subsections will lay out these steps in greater detail with further technical details in the annex.

[16] Net equity in this context refers generally to the gross shareholders equity minus i) goodwill, ii) premises and equipments and iii) deferred tax assets, all of which are considered to be "non interest bearing assets" and are to be deducted, as these non-interest bearing assets can be viewed as being funded by non-interest bearing equity and thus no interest rate risk arising in the process.

[17] In an upward sloping yield curve environment, the receiver swap portfolio will lead to a positive carry trade.

4.1 Scenario generation

For the purpose of Reverse Stress testing, or indeed any assessment of the largest vulnerabilities of the balance sheet from an interest rate perspective, one needs to model interest rate scenarios not in a way attempting to predict the most probable interest rate moves, but rather by seeking to holistically cover the "whole space" of all possible rate moves. It is with this goal in mind that we have expanded a well-known PCA yield curve decomposition to generate a comprehensive set of scenarios for reverse stress testing purposes. We perform a comparison of our approach to the well-known Heath-Jarrow-Morton (HJM) model. We limit the scenario generator description in this Section to the PCA generator, and provide an additional description of a modified HJM model in the Annex, including user-defined volatility functions, that also allow the user to scale the scenario severity.[18]

Yield Curve sampling. We describe a yield curve at time t and in currency c by the vector $y_c(t) = (y_1^c(t), \ldots, y_M^c(t))$, where M is the number of discrete tenors that are being modelled. In our empirical applications in Section 5 we will focus on USD and CHF yield curves, modelled with $M = 8$ tenors each. and interpolating between the tenors.[19]

In order to reduce the dimensionality of modelling and simulating yield curves in this space, we first perform a standard PCA decomposition on the historical interest rate datasets in CHF and USD. We refer the reader to any standard statistics text for an introduction to PCA. Conducting this PCA on interest rate data in every currency leads to the identification of three principal components, displayed in Fig.10 for USD, with the CHF decomposition relegated to Fig.33.

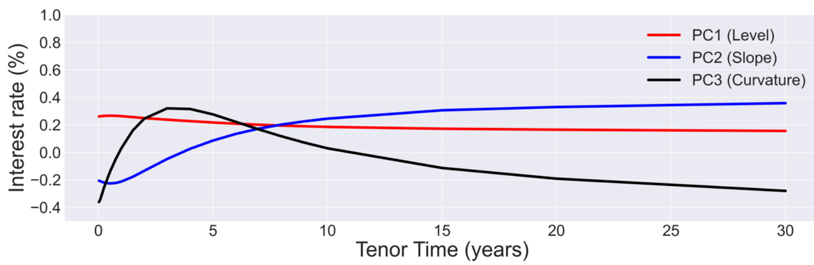

Fig. 10: The first three principal components of USD yield curves, with unity coefficients.

These three principal of yield curves are quite well known as corresponding to parallel, steepening, and curvature moves of the yield curve and are close to the representation of the 'Nelsen-Siegel' interest rate model (Nelson and Siegel, 1987).

Having calculated the three principal components which account for more than 99% of the variance in yield curve moves, we know that a linear combination of these principal components can accurately represent yield curves and hence serve as a useful and parsimonious mathematical basis for the efficient sampling of new yield curves with a minimum number of degrees of freedom and a high degree of accuracy. That is, we can write the yield curve

$$y^c(t) = \Sigma_{i=1}^3 \alpha_i^c(t) PC_i^c,$$

where PC_i^c corresponds to the ith principal component of currency c, and $\alpha_i^c(t)$ are coefficients. Fig.34 in the Annex shows the distribution of $\alpha_1, \alpha_2, \alpha_3$ respectively for $c =$USD. If we sample the coefficients from the joint distribution of $(\alpha_1, \alpha_2, \alpha_3)$, we retrieve historical yield curves (with a precision of ∼99%, as we limit ourselves to the first three principal components. With the basis and the range of coefficients now established, what remains is to sample within this space. Rather than sampling from the joint distribution of $(\alpha_1, \alpha_2, \alpha_3)$, we determine the minimum and maximum coefficients for each $\alpha_i^c(t)$, then

[18] While HJM-generated scenarios are useful to assess "likely moves" of the yield curve given current rates, we found the method generally less well suited to explore a wider range of "what-if?" scenarios.

[19] To calibrate our scenario generator model, we start with USD and CHF yield curve data from 01/01-2006 – 01/06/2023.

add a margin[20] and sample uniformly from the resulting hypercube. Fig.11 shows an example of 1000 such scenarios sampled for USD and CHF. The tweaking of the coefficients, which is arguably "expert judgement", will influence the range and shapes of yield curves generated. For instance, if one would like to see USD yield curves up to 10% or 12% instead of the 8%, one simply needs to increase the upper bound α_1^{USD} from 0.29 to 0.38 or 0.48 respectively.

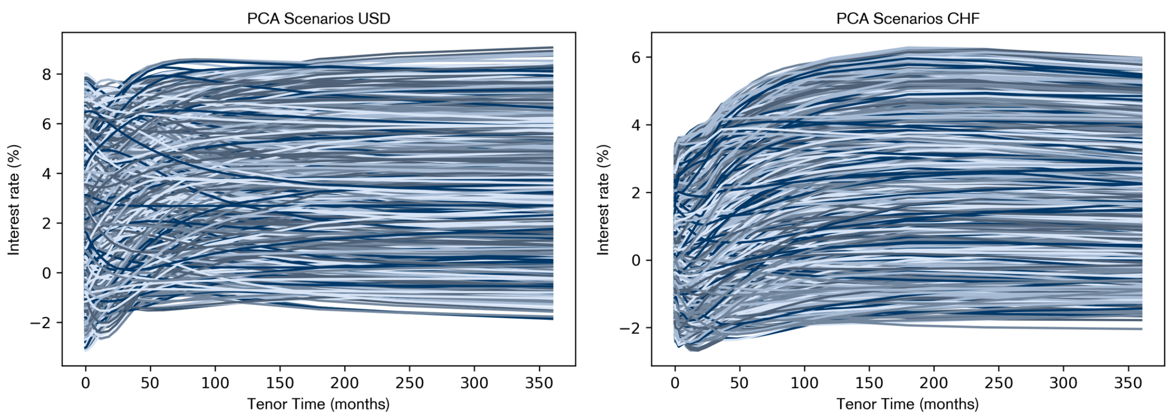

Fig. 11: 1000 yield curves (scenarios) generated for USD (left) and CHF.

Cross currency dependencies and covered interest rate parity. We begin by sampling the yield curves in each currency separately when generating the above scenarios, each based on the associated historical data. This results in a differing range of plausibility in each case, where for instance in Fig.11 the USD short rate moves above 8%, while the CHF short rate remains below 4%. The question then is how to combine these scenario in order to form meaningful multi-currency shocks. Covered interest rate parity dictates that interest rates and FX rates for different currencies may not move arbitrarily, but are intimately linked. For example, if interest rates in USD increase while those in CHF remain unchanged, then all else equal the USD will weaken in the future relative to the CHF. In order to avoid USD and CHF interest rates to move in very implausible ways, we bound the distribution of (implied) forward FX rates to its historical distribution plus a margin, and discard combinations of USD and CHF rate moves that would result in "implausible" forward FX rates. This explicit bound on the forward FX rate will generate an implicit bound on how USD and CHF interest rates may move relative to each other in our scenarios. For a more detailed discussion of Covered Interest Rate Parity, and deviations thereof under certain market conditions, we refer the reader to Rime et al. (2022).

Evolution of yield curves through time. Thus far we have dealt purely with spot yield curves. However, in order to simulate a time horizon (e.g. 12 months) and calculate NII impacts, we require forward curves. While we obtain these curves "for free" in the context of the HJM model, we need a way to generate forward curves for our PCA generator. To do so, we simply assume that implied forward rates of the spot yield curve will materialize. The reader may consult Farahvash (2020) for a detailed discussion on implied forward rates.

4.2 NII and EVE estimation

Valuation and cash flow projection models. The next step in our framework requires the systematic computation of the impact of each interest rate scenario on NII and EVE. For NII, this step requires a full computation of all cash flows, which do depend on the scenario in question, as shown in our examples in Table 1. For EVE, this step requires a full valuation model in order to present value

[20] We reduce the minimum coefficient by defining $\underline{\alpha_i^c} < \min_t \alpha_i^c(t)$ and increase the maximum coefficient by defining $\overline{\alpha_i^c} > \max_t \alpha_i^c(t)$ and then sample uniformly from $\left[\underline{\alpha_i^c}, \overline{\alpha_i^c}\right]$.

the cash flows arising from loans, interest rate swaps, non-maturing deposits, bonds held and debt issued etc. A description of the valuation technique for each product is significantly beyond the scope of this paper, even for our mock balance sheets. The EBA has introduced Regulatory Technical Standards for a standardised methodology and a simplified standardized methodology in order to compute NII and EVE European Banking Authority (2022b). A simplified framework could use these methodologies although we believe that for the purpose of reverse stress testing it is important to capture expected (and unexpected) customer behaviour and balance sheet dynamics. The regulatory required sensitivity analyses may therefore be meaningfully embedded in a broader RST framework. For a comprehensive overview of valuation techniques, including securitisations and more exotic products one may encounter in the banking book, we refer the reader to Farahvash (2020) and Hull (2012).

Balance sheet dynamics and behavioural modelling. Compared to NII, EVE is "simpler" to calculate in the sense that it considers a snapshot of the balance sheet today and calculates the net present value of all cash flows assuming a run-off balance sheet. Under a run-off balance sheet maturing positions are not replaced. In contrast, when calculating NII, a run-off balance sheet is generally **not** assumed, but assumptions are made regarding how the balance sheet will evolve through time and as a function of different interest rate moves. The most simple assumption is a "Constant Balance Sheet", which means that maturing positions are replaced by identical products, thereby keeping the size and composition of the balance sheet constant.[21] The third, and in the authors' opinion most relevant, option for (reverse) stress testing and vulnerability identification is the so-called "Dynamic Balance Sheet" approach. The dynamics of the balance sheet can be separated into essentially three categories:

1. **Expected changes in size or composition of the balance sheet as a result of interest rate moves**: When rates increase customers generally withdraw funds from non-maturing deposits and place them into either higher yielding fixed-term deposits, or withdraw them altogether e.g. by investing into fixed income or other assets. Conversely, if rates fall, banks can generally expect inflows into non-maturing deposit categories. The uptake of Lombard loans and/or prepayment frequencies fall under the same category of dynamic modelling. The importance of dynamic balance sheet modelling is particular relevant to capture optionality risks such as customer redemptions for instance for non-maturing deposits. The margin on a loan is calculated as difference between the rate paid by the client and the funding rate (usually the bank's "Funds Transfer Pricing" (FTP) rate). Similarly, the margin earned on a deposit is calculated as the difference between the rate paid to the client and the internal reference rate. The corresponding NII is obtained by multiplying margin with volume, i.e.

$$\mathrm{NII}(s) = \mathrm{volume}(s) \times (\mathrm{ref.rate}(s) - \mathrm{client\ rate}(s)),$$

   where the reference rate is directly linked to the scenario and the volume and client rate may or may not depend on the scenario, depending on the bank's modelling choices (see Bohn and Elkenbracht-Huizing (2018)).

2. **Financial plan modelling**: A second part of dynamic modelling comprises the inclusion of the financial plan and the bank's goals regarding how it may want to evolve its business. This may for instance include divestments from certain business segments or the growth of new client segments and thus entail a change in the total size and composition of the balance sheet. Most often such forecasts are only available for the "baseline" scenario.

3. **Idiosyncratic stress events and management actions**: This category of dynamics is particularly relevant for stress testing and may include the modelling of extreme deposit withdrawals, drawing of committed credit lines (especially if commitments have been made at fixed rates and rates increase) for large interest rate moves, or management actions that the management intends to take if the bank got into a difficult situation. These actions tend to be scenario-specific and require ad-hoc modelling.

[21] By way of example, if a loan matures in 6 months from today, then the loan will generate NII for the next 6 months, and for months 7-12 it is assumed that a new loan is issued with the same characteristics (e.g. client rate, margin etc) than the matured loan.

The authors have not gone to the length of developing a full-blown business plan or management actions for Reversius and YOLO bank and restrict the dynamics of the NII calculations to the first category above. To do so, we leverage existing models to calculate EVE and NII, including dynamics on how volumes of customer deposits and the client rate paid on these are expected to behave under various interest rate moves (see Bohn and Elkenbracht-Huizing (2018)).

ΔNII and ΔEVE scatter plots. Fig.12 illustrates the computation of ΔNII and ΔEVE for Reversius Bank across the 1000 scenarios generated with our PCA methodology as illustrated in Fig. 11 as well as for 1000 scenarios generated using a modified HJM methodology. The difference (Δ) for NII or EVE is always calculated against the baseline scenario, which was the spot yield curve of USD and CHF respectively as of April 2023. The plot shows both the scatter plot of Δ EVE (x-axis) against Δ NII (y-axis) as well as their marginal distributions, where the origin (0,0) denotes the baseline scenario. While the HJM generated scenarios cluster closely around the baseline scenario, the PCA generated scenarios (by design) cover a significantly wider range of yield curve moves. Hence, HJM scenarios can be used to provide a market based view on what is likely to happen, while PCA generated scenarios are better suited to ask "what if?" questions.[22]

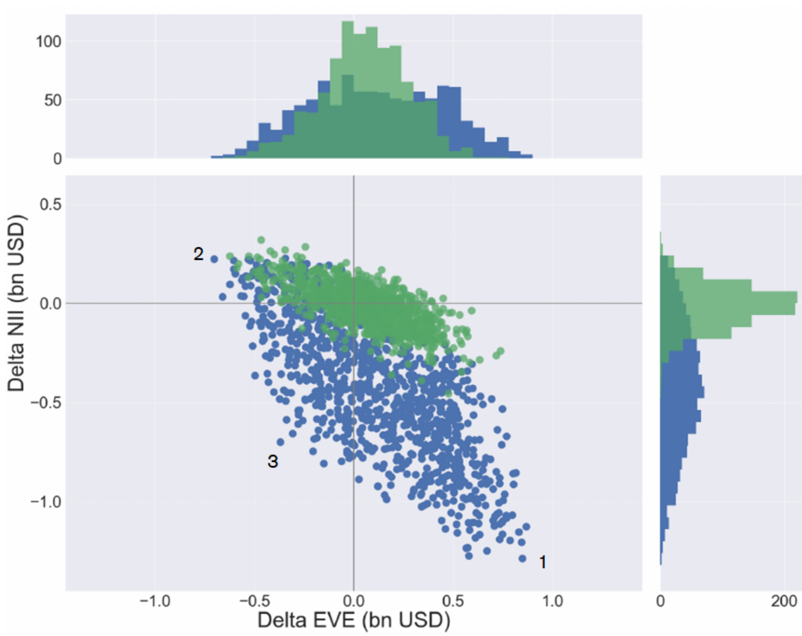

Fig. 12: Comparison of market-based HJM scenarios (green dots and marginal distributions) to our PCA-generated scenarios (blue dots and distributions). Notably, the HJM scenarios are centred around the baseline scenario (at 0,0). HJM scenarios thus provide a good indication of "what is likely to happen" given current market implied covariance matrices, while PCA generated scenarios take a broader "what if?" perspective. While the worst case ΔNII is approx. 1.25bn (scenario 1) loss and the worst case ΔEVE loss is approx. 0.5bn (scenario 2), the worst *joint* loss in both is approx. 0.7bn and 0.4bn respectively (scenario 3). We will analyse the structure behind these scenarios in more detail in Section 5. Moreover, while a bank's balance sheet intimately links NII and EVE behaviour, and a trade-off often exists between these two measures, this is not true in general - as the more detailed analysis of YOLO bank in Section 5 shall reveal.

[22]The HJM distribution is wider in EVE space compared to NII space because the HJM scenarios develop "over time" taking as starting point the current baseline, which leads to NII only gradually deviating from the baseline (which assumes that implied forward rates are realized). In contrast, in order to compute EVE, we just require a single curve. To do so, we have taken the 12-month yield curves as one-off shocks. By taking e.g. the 6-month yield curves instead the EVE distribution for HJM would be significantly more narrow compared to the PCA distribution for EVE.

4.3 Clustering

While the scatter plots and marginal distributions in Fig.12 provide all information on the impact on NII and EVE, it is not at all clear what the underlying structure of the yield curves and interest rate shocks are that lead to this outcome. Are the yield curves that lead to a bad outcome in terms of EVE or NII similar or very different? In particular are there different yield curve shocks that can get the bank to the joint EVE and NII loss as highlighted by scenario 3 in the plot or is there just "a single" combination of yield curve shocks that will achieve this outcome?

In the last quantitative step of our framework, we modify well-established spectral clustering techniques to identify patterns of similar yield curve moves that support us in wading through a large amount of interest rate shocks and thus answer these questions

Amongst a variety of clustering approaches tried, we found the best performance was achieved when conducting Normalized Laplacian Spectral clustering on a specially constructed similarity matrix. Meta Algorithm 1 lays out the steps implemented by our approach in Python. The key transformation occurs in steps 2 and 3: First, we compute all dot products of yield curves. Next, rather than normalizing the dot products by the respective standard deviations, which would correspond to calculating cosine similarity between two yield curves, we find that normalizing the dot by the maximum entry of G yields better results. A similar approach is taken in Baes and Schaanning (2023) to compute "liquidity-weighted overlaps" and thus identify similar portfolios in a banking network.

Algorithm 1 Normalized Laplacian Spectral Clustering with Similarity and Affinity Matrix Construction for Pattern Identification in Yield Curve Samples.

1: **Inputs**: 1) Yield curves data matrix $Y \in \mathbb{R}^{N \times M}$ with N-many yield curves of M tenor points, 2) number of clusters k, 3) affinity calculation method (e.g., nearest neighbors or radial basis function (Von Luxburg, 2007))
2: Compute the Gram matrix $G := Y \cdot Y^T$
3: Compute the matrix H, with elements $H_{ij} := \exp(1 - (G_{ij}/\max(|G|)))$
4: Compute the affinity matrix S based of H using the specified affinity calculation method in 3) above
5: Compute the degree matrix D with diagonal elements $D_{ii} := \sum_j S_{ij}$
6: Compute the Laplacian matrix $L := D - S$
7: Compute the normalized Laplacian matrix $L_{\text{norm}} := D^{-\frac{1}{2}} L D^{-\frac{1}{2}}$
8: Perform the eigenvalue decomposition of $L_{\text{norm}} = V \Lambda V^{-1}$
9: Perform k-means clustering on the first k eigenvectors of V to get clusters $\{C_1, C_2, \ldots, C_k\}$
10: **Output** clusters $\{C_1, C_2, \ldots, C_k\}$

Fig.14 and Fig.13 illustrates the results of our machine learning algorithm when applied to the 1000 generated scenarios and segregating the data into 9 clusters for CHF and 16 clusters for USD. Common methodologies for identifying the optimal number of clusters are the "elbow-plot", the Silhouette score or analysing gap statistics (see Hastie et al. (2009)), which we have supplemented with a "you know it when you see it" approach. We acknowledge the "as much art as science" touch to this approach and encourage interested readers to develop improved clustering techniques for yield curves, which we are confident do exist.

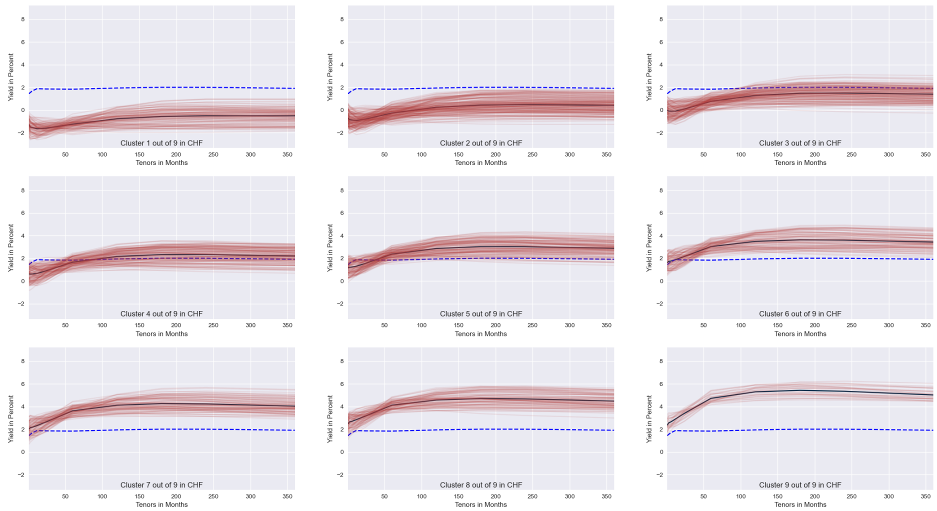

Fig. 13: 1000 generated yield curves for CHF are clustered into 9 families. The blue dashed curve corresponds to the current baseline yield curve. Every red curve corresponds to a scenario. The black curve corresponds to the average curve within a cluster.

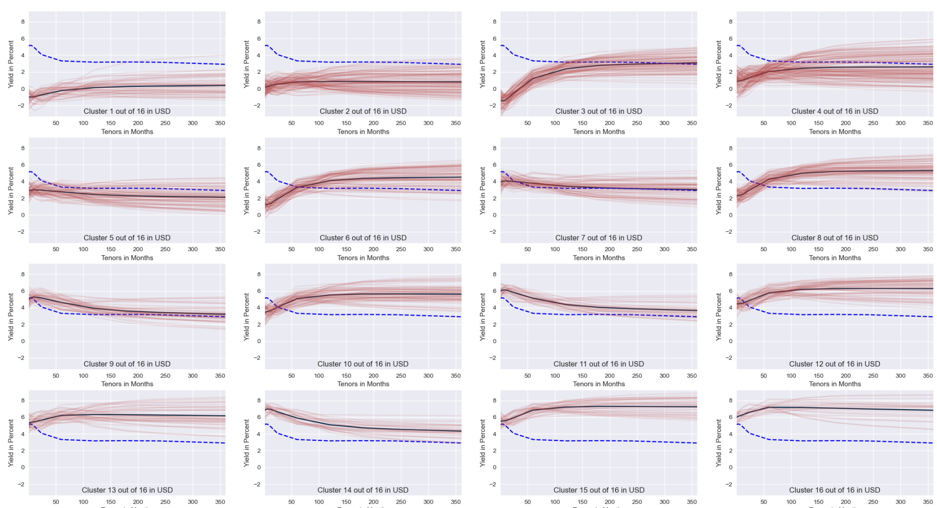

Fig. 14: 1000 generated yield curves for USD are clustered into 16 families. The blue dashed curve corresponds to the current baseline yield curve. Every red curve corresponds to a scenario. The black curve corresponds to the average curve within a cluster.

4.4 Vulnerability identification and integration into the existing risk management framework

The last quantitative step of our (reverse) stress testing approach consists in evaluating and interpreting all outputs of the scenario generation, ΔNII / ΔEVE computation and clustering. One may say that these steps in fact are "preliminary" and that the "actual" risk management work only begins thereafter. In the next section we will outline a couple of use cases for such analyses with this framework and draw some important - we believe - implications for ALM risk management.

5 Results

We now proceed to applying the full framework from Section 4 to the balance sheets of Reversius Bank and YOLO Bank, introduced in Section 3.1. The key rate duration profiles of Reversius Bank and YOLO bank are depicted in Fig.4 and Fig.5 respectively in Section 3.2.2. Fig.5 highlights in particular how YOLO bank is taking considerable - or dare one say "excessive" - economic value risk, relative to the more prudent and well-hedged Reversius Bank. The NII repricing cashflows of Reversius Bank are shown in Fig.3 in Section 3.2.1. We have not reproduced the gap profile of YOLO Bank, which is identical to the repricing cashflows of Reversius Bank below the 12-month bucket (intentionally by-design). Both will thus produce identical 12-month ΔNII outputs, while their ΔEVE results will differ markedly.[23]

While some of our conclusions hold in a general context, it is important to note that no inference can be drawn with respect to what the "the *general* worst case scenario" is: This depends intimately on the balance sheet of the bank in question, and thus varies from bank to bank.[24]

5.1 Supervisory IRRBB stress test scenarios may fail to identify blind spots in banks' balance sheets, in particular when risks are pronounced

ΔNII, ΔEVE and Supervisory Outlier Tests for (the well-hedged) Reversius Bank.

Similarly to Fig.12, Fig.15 shows a scatter plot of ΔNII and ΔEVE across 1000 PCA-generated scenarios along with the marginal distributions of ΔNII and ΔEVE respectively. The (0,0) point identifies the baseline scenario on the chart. Positive numbers indicate an increase of NII or EVE relative to the baseline scenario, while negative numbers indicate a loss relative to the baseline. The graph shows that the six BCBS scenarios provide a good coverage in terms of ΔEVE risks (for Reversius Bank) but that the scenarios appear insufficient to cover the potential range of ΔNII impacts (for Reversius Bank), because non-linear effects appear for larger shock sizes for ΔNII. We discuss the structure of which scenarios generate these losses in more detail in Section 5.2. More importantly, none of the BCBS scenarios generate a simultaneous adverse ΔNII and ΔEVE loss for Reversius Bank, whereas such scenarios do exist, creating a joint loss of approx. 0.7bn in ΔNII and 0.4bn in ΔEVE. The graph also reveals that Reversius Bank would pass the EVE Supervisory Outlier Test with a ΔEVE loss of approx. 0.4bn corresponding to 4% of its Tier 1 capital, well below the 15% supervisory outlier threshold.

Furthermore, the graph reveals that ΔEVE and ΔNII have roughly similar distributions: These range from approx. -0.5bn to +0.75bn for ΔEVE and from -1bn to +0.25bn for ΔNII. The similar order of magnitude of these two distributions (for a well-hedged bank) may call into question the threshold calibration of Supervisory Outlier Tests. For instance, from an EVE perspective, "outlier banks" taking excessive IRRBB risks are identified by the EBA Guidelines as those banks whose worst loss across the supervisory scenarios would consume 15% of Tier 1 capital, while on the NII side losing more than 5% in NII would lead to the identification as an outlier bank. This needs to be viewed in the context of i) our hypothetical bank, which is well-hedged from an IRRBB perspective by design and shows a larger sensitivity towards NII than towards EVE, which will be different for YOLO Bank in the subsequent paragraph, and ii) NII generally being sensitive to assumptions on the replication of deposits, non-linearities such as prepayments or other optionality risk among others, which are important to realistically reflect the behaviour of ΔNII[25].

[23] Underlining why it's important to consider both the NII and EVE view of an institution to assess IRRBB, as discussed in Section 3.2.4

[24] It bears repeating at this point that the balance sheets of Reversius and YOLO bank have been constructed as realistic but mock balance sheets and that no inference on any entities that the authors are affiliated with can be made.

[25] Albeit the NII SOT currently considers a constant balance sheet approach for comparability reasons.

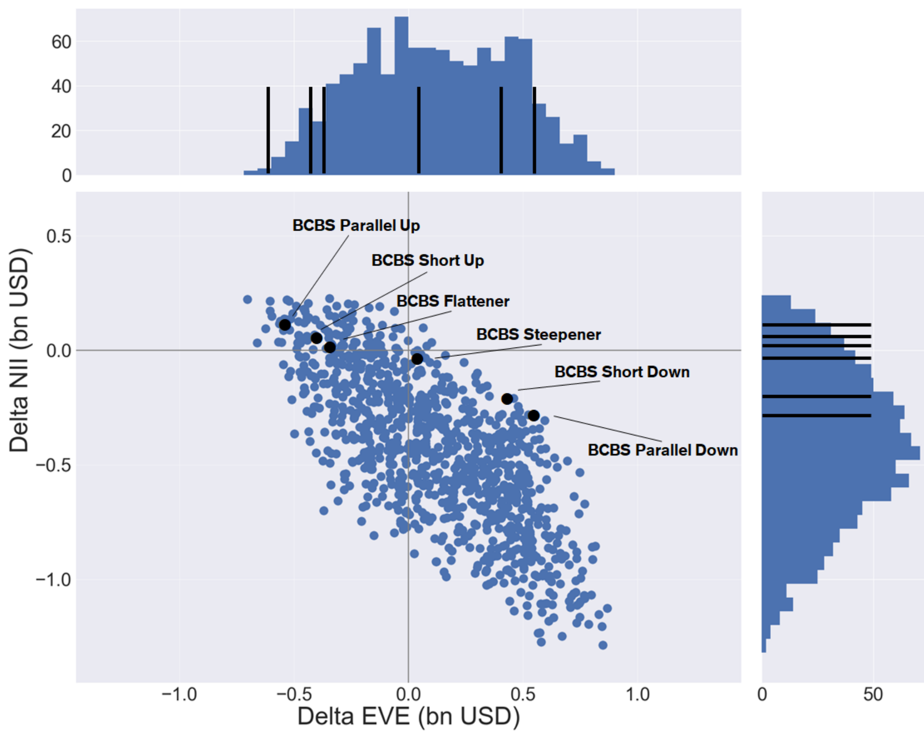

Fig. 15: Distribution of ΔEVE and ΔNII across 1000 PCA scenarios for (the well-hedged) Reversius Bank: The six regulatory BCBS scenarios provide good coverage of ΔEVE risks, but miss some of the non-linearities that appear for larger ΔNII shocks (discussed in more detail in Section 5.2). Notably, the BCBS scenarios do not create jointly adverse ΔEVE and ΔNII impacts for Reversius, while such scenarios do exists (also explored in more detail in Section 5.2).

ΔNII, ΔEVE and Supervisory Outlier Tests for YOLO Bank.

We now turn to YOLO Bank, which is conducting more aggressive maturity transformation by funding longer term assets with shorter term liabilities, as revealed by its key rate duration profile in Fig.5.

Fig.16 shows the ΔNII and ΔEVE impacts for YOLO Bank across the *same* 1000 scenarios compared to Reversius Bank. As by construction the balance sheets of YOLO Bank and Reversius bank are identical below the 1-year time bucket, the ΔNII impacts are exactly the same for both banks, even though the different scale of the axes may hide this at first sight.

However, the picture is drastically different for ΔEVE: Similar to Reversius Bank, YOLO Bank also passes its EVE Supervisory Outlier Test by suffering a maximum loss of approx. 0.4bn in the BCBS Steepener[26] scenario (corresponding to 4% of its Tier 1 capital, again well below the 15% supervisory threshold). However, the graph reveals that YOLO Bank suffers a *median* ΔEVE loss of approx. 50% of its Tier 1 capital in the PCA scenarios. Moreover, in several of the scenarios, YOLO even stands to suffer a ΔEVE loss of more than its entire capital - similar to the magnitude of losses Silicon Valley Bank suffered on its investment portfolio relative to its capital. YOLO would thus experience more than a four-fold excess of the EVE SOT under our PCA-generated scenarios compared to the six BCBS scenarios. It is important to note that this is not a reflection of the PCA-generated scenarios being extreme: Reversius Bank lost a maximum of 5% of its Tier 1 across the exact same set of 1000 scenarios. Hence, this is rather a reflection of the (excessive) interest rate risk position that YOLO Bank has taken, but which has gone undetected by the standard BCBS supervisory scenarios.

[26] Given YOLO's close to zero net sensitivity (obtained when summing key rate durations across all time buckets), it is clear that the Parallel Up and Down scenarios will have close to no impact on YOLO's EVE SOT.

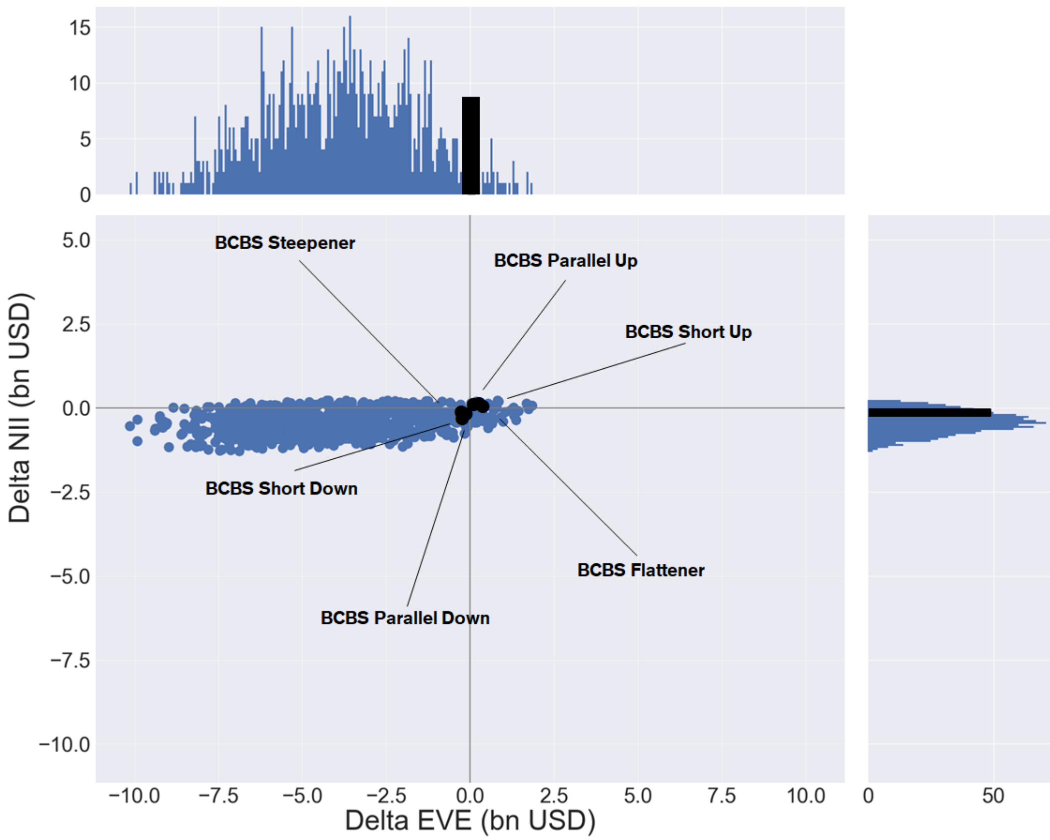

Fig. 16: Distribution of ΔEVE and ΔNII across the same 1000 PCA-generated scenarios for YOLO Bank. While the distribution of ΔNII losses is identical (by construction) for Reversius and YOLO, YOLO Bank's material maturity transformation (see Fig.5) generates considerably larger losses for ΔEVE, which goes unnoticed by the BCBS scenarios. Indeed, YOLO stands to suffer ΔEVE losses of approx. 50% of its Tier 1 capital in the median PCA scenario and up to the entire Tier 1 capital in several scenarios. Meanwhile, YOLO Bank also passes its EVE Supervisory Outlier Test with a maximum of approx. 4% losses in terms of Tier 1 capital, which is well below the supervisory threshold and similar to Reversius' BCBS scenario outcomes (though for a different scenario). Importantly, this is not a reflection of PCA scenario severity, but YOLO's – dare one say "excessive" – interest rate risk position. The better hedged Reversius Bank was suffering a maximum loss of 5% of Tier 1 capital across the same set of scenarios in Fig.15.

ΔNII and ΔEVE for Reversius Bank 2.0.

While Fig.15 might suggest that the BCBS scenarios lying on a line roughly along the $y = -x$ diagonal, this is not true in general. The balance sheet of Reversius Bank was set up such that we assumed Reversius was stabilizing its NII (in USD only) through executing an Investment of Equity programme (cf. Sections 3.1 and 3.3.2). If we remove the swap portfolio and consider how "Reversius without NII stabilisation portfolio" fares, we observe that the ordering of the regulatory scenarios flips: While with an NII stabilisation programme, the parallel up scenario will tend to be the worst (because the receiver swap portfolio will lose in net present value if rates increase), banks without such a programme tend, in general, to be more vulnerable to a parallel down scenario. Consequently, as shown in Fig.17, with this "change" in strategy, the worst case BCBS scenario changes from "Parallel Up" to "Parallel Down" and thus reinforces our earlier point that no general statement can be made about the worst case scenario for a bank, rather this depends on the ALM strategy of the bank and how this influences its interest rate risk position.

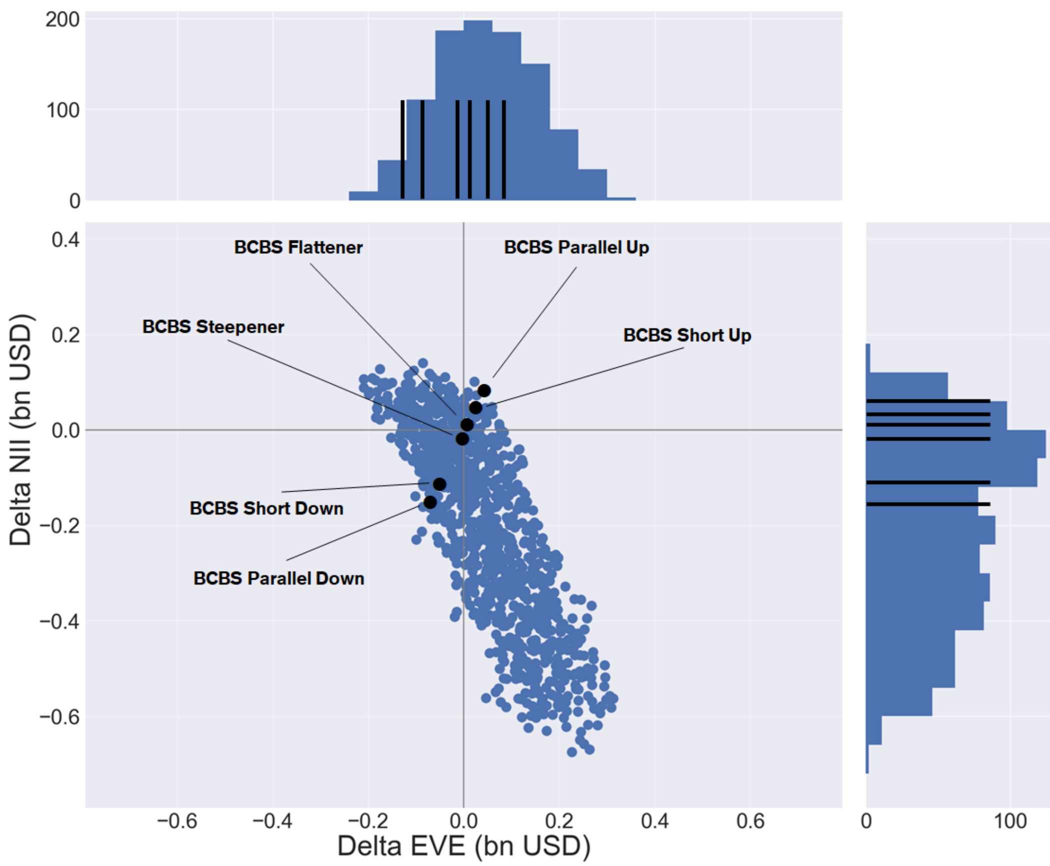

Fig. 17: The distribution of ΔEVE and ΔNII losses for the modified balance sheet of Reversius Bank without NII stabilisation portfolio. While previously the worst NII scenario was Parallel Up, this has (intuitively) flipped to a Parallel Down scenario. Nonetheless, the BCBS still fail to cover non-linear ΔNII impacts and the coverage in terms of ΔEVE has also worsened relative to the "original" balance sheet of Reversius including the NII Stabilisation portfolio.

5.2 IRRBB Black Swans? The correlation structure of cross-currencies shocks can generate scenarios that hit ΔNII and ΔEVE simultaneously

We will now investigate 1) how the aggregate ΔNII and ΔEVE impacts disaggregate into CHF and USD respectively, 2) apply our clustering algorithm on USD and CHF yield curves to identify families of interest rate shocks and their respective impact on ΔNII and ΔEVE, and 3) combine these pieces of information in order to understand how yield curve shocks in different currencies combine to create joint ΔEVE and ΔNII losses.

1. How do ΔNII and ΔEVE impacts disaggregate into CHF and USD?

Fig.18 shows the ΔNII and ΔEVE impacts of the USD (blue) and CHF (red) positions respectively for Reversius. First, we note that the trade-off of ΔEVE and ΔNII impacts is much "sharper" in each individual currency when compared to the aggregate impact across both currencies in Fig.15. Second, we note that the stabilization of NII that Reversius Bank performs through a USD equity hedge "reduces the slope of the cloud" - this is intuitive: the stabilization of NII reduces ΔNII risk and thus reduces the "height" of the cloud, but this comes at the expense of ΔEVE risk, which in turn "widens" the cloud. As a result, the more NII is stabilized (at the expense of ΔEVE risk), the more the cloud "rotates" in a counter-clockwise sense. We also note that contrary to Fig.15, which displays scenarios with joint negative ΔNII and ΔEVE impacts, this does not happen at the individual currency level (for Reversius bank). We will now explore why.

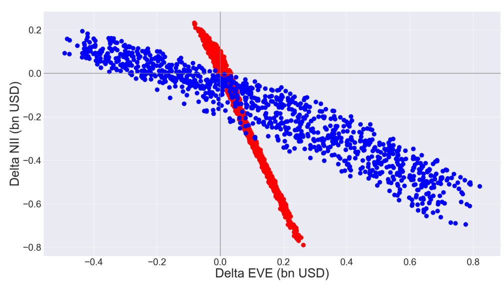

FIG. 18: Superimposed ΔEVE and ΔNII clouds for USD (blue) and CHF (red) respectively for Reversius Bank. Due to the NII stabilisation of Reversius' Investment of Equity programme (in USD only), the "slope" of the USD scatter is less steep than the "slope" of the CHF scatter: ΔNII risk has been traded against ΔEVE risk. Notably, in each individual currency, not many scenarios create joint ΔNII and ΔEVE losses, whereas Fig.15 shows that joint negative ΔNII and ΔEVE scenarios exist when aggregating the two currencies.

2.a CHF Interest rate clusters and their impact on ΔNII and ΔEVE.

We deploy the clustering technique introduced in Section 4.3 to better understand how interest rate shocks across USD and CHF combine to create bad ΔNII and ΔEVE outcomes for Reversius Bank in the individual currencies.

The left-hand chart of Fig.19 shows the average yield curve in each of the nine identified clusters for all CHF yield curves; the right-hand chart shows the ΔNII and ΔEVE impacts, coloured by cluster. Every dot on the right-hand panel of Fig.19 thus corresponds to one yield curve (in red) of Fig.13. The graph clearly reveals that, **given Reversius' balance sheet positioning**, the bank would stand to profit from a ΔEVE perspective if CHF rates were to decrease (while losing at the same time in terms of ΔNII), and it would stand to lose from an ΔEVE perspective if rates were to increase (while at the same time gaining in terms of ΔNII). However, the ΔNII distribution is quite skewed for Reversius, with the bank standing to lose more in our considered scenarios than possibly gaining in upside scenarios.

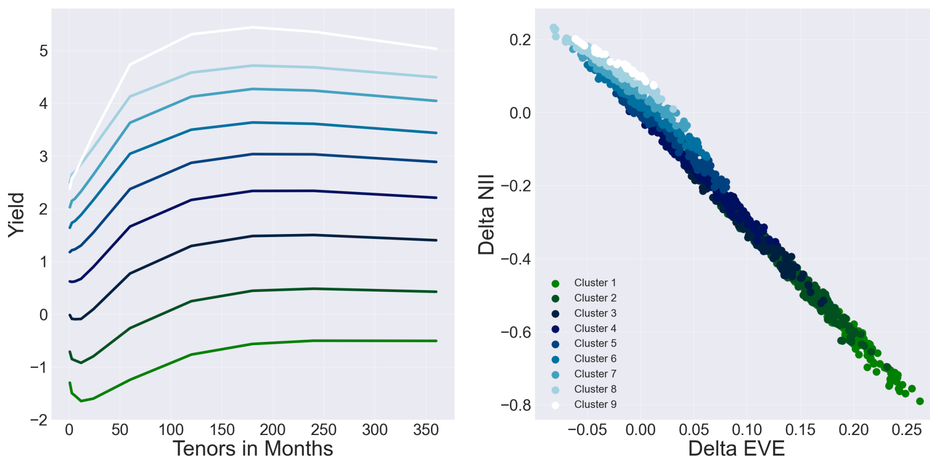

Fig. 19: Left: The average yield curve for each of the 9 clusters in CHF. Right: ΔNII and ΔEVE impacts in CHF, coloured by cluster. If CHF interest rates increased, Reversius Bank - given its balance sheet - stands to lose in terms of ΔEVE but gain in terms of ΔNII and vice versa if rates decreased. Our clustering algorithm helps us to shed light on how every scenario with its particular ΔNII and ΔEVE impact in CHF (in red) of Fig.18 relates to the CHF yield curves simulated in Fig.11.

2.b USD Interest rate clusters and their impact on ΔNII and ΔEVE.

Performing the same clustering analysis on USD rates, we see in the left-hand chart of Fig.20 the average yield curve in each of the 16 identified clusters for all USD yield curves. The right-hand chart shows the ΔNII and ΔEVE impacts, coloured by cluster, for USD products.

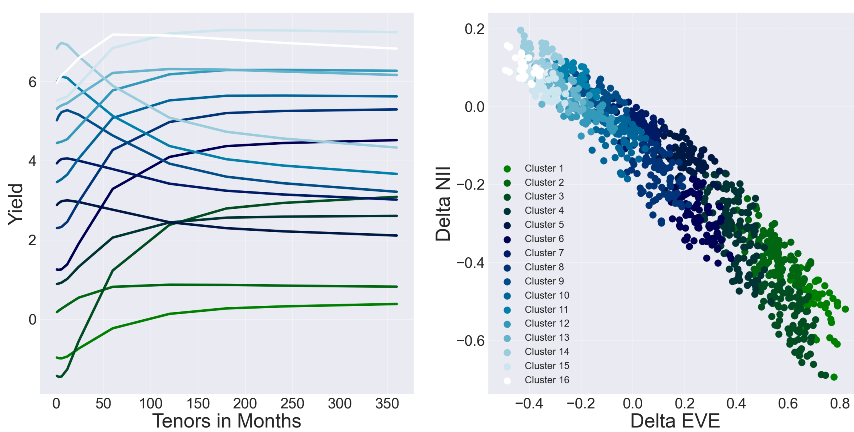

Fig. 20: Left: The average yield curve for each of the 16 clusters in USD. Right: ΔNII and ΔEVE impacts in USD, coloured by cluster. If USD interest rates increased, Reversius Bank - given its balance sheet - stands to lose in terms of ΔEVE though gain in terms of ΔNII and vice versa if rates decreased. If the inversion in USD intensified, Reversius would stand to lose both in terms of ΔEVE and ΔNII. The clustering algorithm helps us understand how every scenario and its ΔNII and ΔEVE impacts in USD (in blue) of Fig.18 relates to the USD simulated yield curves from Fig.11.

3. Putting it all together: how interest rate moves across multiple currencies combine to create joint ΔNII and ΔEVE losses.

Fig.15 clearly reveals that scenarios exist with a joint negative impact on ΔNII and ΔEVE. However, Fig.18 shows that at the individual currency level, there are in fact no scenarios that generate simultaneous ΔNII and ΔEVE losses for Reversius. Fig.19 and Fig.20 in turn help us understand the link between interest rate moves in CHF and USD and their respective impacts on ΔNII and ΔEVE in these two currencies. This begs the question how USD and CHF interest rate moves combine to create jointly adverse scenarios for both metrics?

To investigate this question, we first consider the top-plot of Fig.21. The plot shows the same distribution of ΔNII and ΔEVE impacts as in Fig.15, but in addition we have coloured the scenario points from the top-left (in purple) to the bottom right (in red) along a continuous hue of colours. Hence, each scenario is assigned a specific colour on the rainbow. Next, when plotting CHF and USD only impacts (as in Fig.18), we colour each scenario in the colour that we have fixed in the top-plot of Fig.21. We see that "extreme" scenarios yielding very high ΔEVE losses with high ΔNII gains (e.g. scenario A, or purple scenarios more generally) or very high ΔEVE gains with high ΔNII losses (e.g. scenario B, or red scenarios more generally) arise from combining scenarios of the same colour in each currency. Combining this knowledge with our insights from Fig.19 and Fig.20, we infer that the joint impact marked by scenario A comes about when CHF rates increase, and the (current) inversion in USD subsides and USD rates increase too. Similarly, we can also infer that the impact achieved by scenario B would entail a decrease of both CHF and USD rates.

However, in order to create a simultaneous adverse impact on ΔNII and ΔEVE, we see that this would require CHF rates to fall close to zero in conjunction with the USD yield curve becoming upward sloping again (ending the inversion). This is highlighted by scenario C in turquoise in Fig.21.

What is more, combining the clustering knowledge of Fig.19 and Fig.20 with the information on how scenarios relate to each other from Fig.21, we can in fact infer that this is the *only* way to achieve the ΔEVE and ΔNII impacts highlighted in Fig.21. No other combination of USD and CHF rate moves would achieve a similar effect in terms of combined ΔNII and ΔEVE impacts.

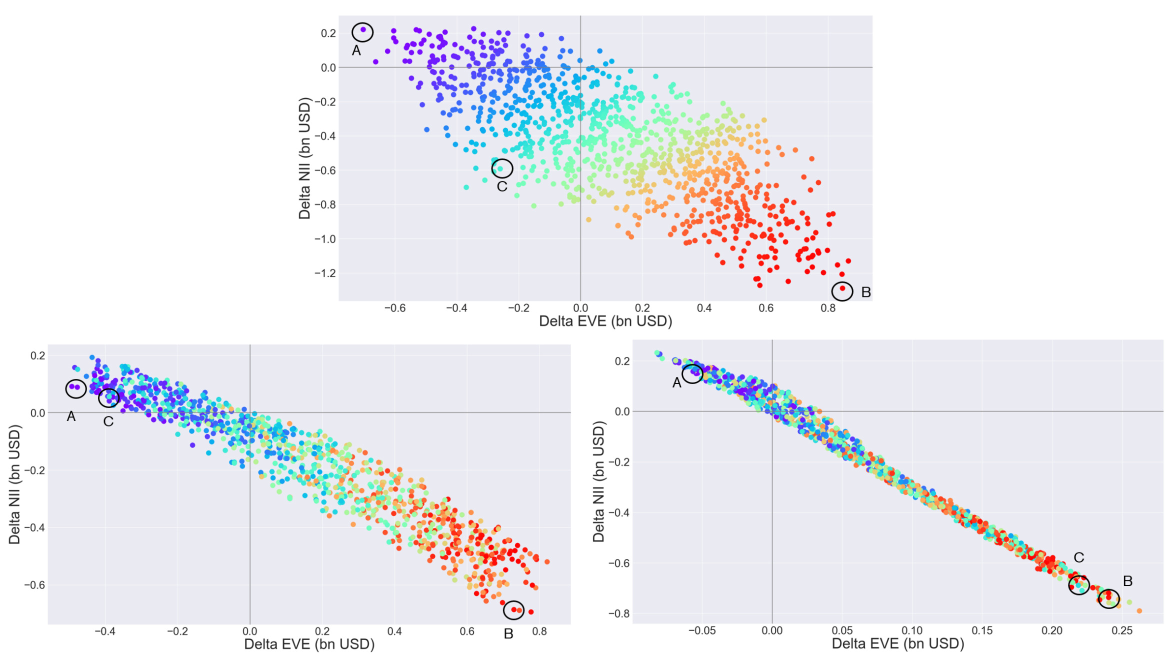

FIG. 21: Cross-currency correlations as drivers of jointly adverse ΔNII and ΔEVE impacts. Top: This chart defines the colouring of each scenario from top-left to bottom-right along the colours of the rainbow. We identify three random scenarios A, B and C, highlighting a bad ΔEVE outcome (A), bad ΔNII outcome (B) and a bad joint outcome (C). Bottom left: Applying the same colouring from the top plot to USD products only and identifying the same scenarios A, B and C. Bottom right: Applying the same colouring from the top plot to CHF products only and identifying the same scenarios A, B and C. Combining these insights with the clustered yield curves from Fig.19 and Fig.20 allows us to conclude that a large loss in terms of ΔNII (scenario B) is achieved only when ΔNII losses occur in both CHF and USD, which we know to happen only if rates in both currencies fall. A similar observation can be made for scenario A, which would result in ΔNII gains combined with ΔEVE losses. Arguably, the most interesting case would be scenario C, where both ΔNII and ΔEVE losses materialize. This would happen if CHF rates were to fall close to or below zero, with the USD yield curve inversion ending but rates remaining broadly at their current levels.

Further applications: reverse stress testing and vulnerability assessments applied to portfolio deep dives and risk appetite setting.

The (aggregate) ΔEVE and ΔNII impacts together with the related interest rate shocks creating these losses above can be used as starting point for further analyses (i.e. step 4 of our proposed framework) and portfolio deep-dives. By way of example, as next step beyond showing aggregate ΔNII impacts summed across all products (as we have done in all scatter plots) one may want to understand how such a total ΔNII impact in a scenario comes about. This can be done for instance by creating so-called "waterfall" charts, splitting the ΔNII impact into various sub-categories, such as fixed rate loans, floating rate loans, fixed term deposits, Lombard lending, non-maturing deposits, CDCPs, interest rate swaps (both hedges and/or structural rate positions such as NII stabilisation) etc. By doing so, a bank will gain a better understanding of the key drivers of ΔNII in each cluster, which further helps understanding whether the observed impacts are indeed blind-spots and may require hedging or remediation or whether in fact these may be due to modelling artefacts. As sensitivity analyses are required by IRRBB regulations, we believe that our proposed framework with its comprehensive scenario analysis can be helpful in identifying model risk and understanding the limitations of certain models.[27]

We conclude our analysis by a comparison of risk appetites for Reversius and YOLO Bank. Fig.22 shows the ΔNII and ΔEVE impacts for both banks (note the different scale of the axis for ΔEVE!). We set for both banks a risk appetite for ΔNII and ΔEVE at a maximum loss of 0.5bn (across our PCA generated scenarios). Generally, banks either set a risk appetite for a sensitivity (e.g. KR01), or for an EVE or NII impact, which in the latter case is conditional on a specific scenario (such as the BCBS scenarios). Our discussion here goes slightly beyond this, as we consider a "risk appetite" against a large "what-if?" distribution of scenarios.

The ΔNII risk constraint is illustrated by the shaded blue area: dots within the shaded blue area lie outside of the risk appetite. A materialisation of such a scenario would thus entail a breach in risk appetite and if any of these scenarios was deemed likely, then risk reducing hedges would need to be executed. Similarly, the ΔEVE risk constraint is illustrated by the shaded red area: dots within the shaded red area lie outside of the risk appetite. Again, a materialisation of such a scenario would correspond to a breach in risk appetite and risk reducing hedges should be executed if these scenarios were deemed likely. Another interesting observation emerges: For the well-hedged Reversius Bank, there is a trade-off between ΔNII and ΔEVE, and the scenarios that lead to a breach in one metric generally do not affect the other. The situation is quite different for YOLO Bank: given its interest rate position, there is no trade-off between these two metrics, but things would either tend to go "well" for both metrics of "wrong" for both metrics at the same time. As a result, the risk appetite set for ΔNII may in fact become the binding constraint for ΔEVE or vice versa, making the interest rate risk management more challenging.

[27]For instance, when a large impact is observed, this may be due to realistic customer behaviour, but it may also be due to the interest rate scenarios being extreme and pushing models to the "boundary" of their applicability.

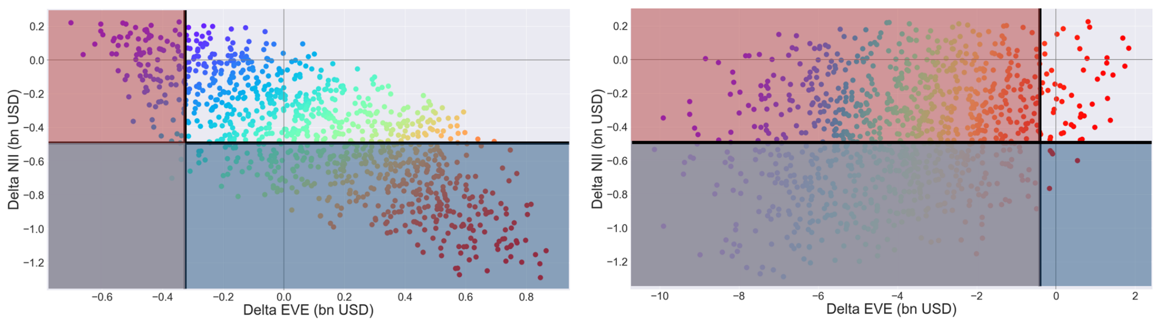

Fig. 22: Risk Appetite setting for Reversius and YOLO Bank: The red and blue shaded areas correspond to "outside of risk appetite" zones for ΔEVE and ΔNII respectively (both set at 0.5bn for the sake of argument). If scenarios falling outside of the risk appetite are deemed likely to materialize, then risk mitigating actions may be required to be implemented. A noteworthy observation is that while for the well-hedged Reversius Bank scenarios that would lie outside of the risk appetite are different from the ones outside of the risk appetite for NII. This is different for YOLO Bank: many scenarios lie outside of both appetites and as a result, the risk appetite set for ΔNII may in fact become the binding constraint for ΔEVE or vice versa, making the interest rate risk management more challenging.

Further analyses. We conduct further case studies, by analysing the behaviour of i) Reversius Bank on HJM scenarios, ii) YOLO Bank across the PCA scenarios, iii) Reversius Bank without NII stabilization on PCA scenarios. For the sake of brevity these plots are all relegated to the annex.

6 Conclusion

In this Section we draw implications for ALM Risk Management, Policy Making and propose suggestions for future research. We refer the reader to the Executive Summary section for an overview of the main results and conclusions.

6.1 Implications for ALM Risk Management

- **Risk Appetite Setting:** Our results indicate a strong dependency between ΔNII and ΔEVE, which depends on the interest rate risk position of the Bank: Indeed, while there was a general trade-off between the two metrics for the well-hedged Reversius Bank, our simulations showed that a more aggressive maturity transformation – as undertaken by YOLO Bank – can in fact lead to jointly positive but also jointly negative ΔEVE and ΔNII outcomes. As we result, banks may want to better understand how their risk positioning affects the joint distribution of ΔNII and ΔEVE across a range of scenarios, and consider this when setting risk appetite. In the case of YOLO Bank, our results showed that a risk appetite constraint on ΔNII may in fact may become the binding constraint for ΔEVE or vice versa.

- **An illusion of precision:** Often banks assess their ΔNII and ΔEVE risks merely across a handful of supervisory and some additional internal scenarios. We believe such analyses are falling considerably short of a holistic risk assessment. Calculating ΔNII and ΔEVE impacts across a hand-full of scenarios may lead to an illusion of precision, which masks how sensitive the respective ΔNII and ΔEVE *estimates* (rather than "results") are to key assumptions around customer behaviour and balance sheet dynamics as well as interest rate dynamics. For this reason, we believe that instead banks should evaluate ΔNII and ΔEVE impacts across a full range of yield curve distributions including a rigorous conducting of sensitivity analyses.

- **A comprehensive assessment of IRRBB risks:** By virtue of i) comprehensively simulating yield curve moves the entire "yield curve space" and ii) conducting a rigorous clustering analysis, a "completeness statement" emerges: it is the combination of adverse scenarios as identified in the Reverse Stress Testing Approach - and only those combinations - that could lead to the extreme ΔEVE and/or ΔNII outcomes; there are no other ways yield curves could move that would generate similarly adverse impacts.

- **Cross currency shocks and ALM strategy:** Banks often assess their ALM strategies such as the term-out of non-maturing deposits or the investment of equity currency by currency. However, our results highlight a very strong cross-currency dependency. As a result, the optimisation of ALM strategies across N currencies should not be solved as N 1-dimensional optimisation problems, but should instead be solved as a single N-dimensional optimisation problem. The global optimum of this N-dimensional problem may differ significantly from the aggregation of the N local optima per currency.

6.2 Policy implications

- **Calibration of ΔNII and ΔEVE SOT Thresholds:** Our simulations indicate that for well-hedged balance sheets the distribution of ΔNII and ΔEVE are roughly the same. Moreover, we have shown that the joint distribution of ΔEVE and ΔNII changes substantially depending on the interest rate risk profile a bank takes. The differential level of thresholds for the EVE SOT and NII SOT at 15% and 5% of Tier 1 Capital respectively may therefore potentially be reviewed.
- **Towards a "Market Risk" regulation for IRRBB?:** Banks assess their market risks mainly through estimating Value-at-Risk and Expected Shortfall for their portfolios across a wide range of scenarios. Our results clearly indicate that assessing ΔEVE and ΔNII risks on six and two scenarios respectively may prove insufficient and that institutions should rather assess their IRRBB risks by i) jointly considering ΔEVE and ΔNII impacts, and ii) holistically across a wide range of scenarios. What is more, we showed that while YOLO Bank stands to lose its entire capital in a range of scenarios due to excessive maturity transformation, this blind spot is not picked up the by the EVE SOT, which the bank would in fact pass. As such, a more holistic assessment of IRRBB risks seems necessary to identify blind spots.
- **Towards a macroprudential reverse stress test?:** Our results illustrate in the simple case of Reversius and YOLO Bank that the worst case scenario changes dramatically, given the balance sheet exposures of the two banks. Current macroprudential stress tests such as CCAR and DFAST in the US or the EBA EU-wide bank stress test focus on subjecting the entire banking sector of a jurisdiction to one single adverse macroeconomic scenario. The scenario is often defined in terms of interest rate moves and other macro variables such as GDP, unemployment and financial indices such as the VIX. We argue that instead of subjecting all banks to a single scenario, a macroprudential regulator should rather in a first step assess the impacts of a distribution of shocks for every bank, and in a second step seek to understand whether "pockets of concentration" can be identified: i.e. are all banks well diversified in the sense of being vulnerable to different shocks, or are all banks exposed to the same scenario which if it materialized might carry systemic repercussions? And if so, what are the interest rate moves that such a systemic scenario would consist of?

6.3 Discussion and future research.

In the spirit of Mark Twain's quote at the beginning of this paper - there are many things that can go wrong, and a big risk may lie in believing one has a full understanding and overview of a banks' risk profiles after having evaluated six IRRBB scenarios. In fairness, the same argument applies to this paper, which only considers Interest Rate risk and thus ignores Credit, Market and Liquidity risks, and the interactions between these different risk stripes, which we recognise can indeed be significant. For example, Coelho et al. (2023) highlight the interconnectivity present between interest rates and banks' liquidity positions. However, we shall leave such a risk stripe interaction analysis for future research. We point out some immediate possible enhancements to our framework:

- **Better scenario generators for (reverse) stress testing purposes:** While our scenario generator is able to cover a wide range of hypothetical scenarios, we have noted that - given historical data - the scenario generator is less prone to generating inversions for CHF than for USD. We believe the yield curve and scenario generating process can be further improved to cover possible yield curve moves even more comprehensively. Generative Adversarial Networks Models such as Cont et al. (2022) may pose a very interesting avenue to explore.

- **Better clustering techniques for yield curves:** While we deem our proposed methodology "fit for purpose" in the sense of being capable to answer the question we set out to answer, we have no doubt that the advances in Machine Learning will (or have already) produce(d) better performing clustering techniques for yield curves.
- **Sensitivity analyses around customer behaviour:** The banking stress of 2023 has shown that in the age of social media and online banking, the speed at which bank runs can unfold have increased substantially compared to one or two decades ago. This poses new challenges for bank risk managers and regulatory authorities alike. We believe large-scale and systematic sensitivity analyses of customer behaviour may help in identifying significant risks ahead of their materialisation.

We hope the present manuscript will contribute to the timely debate in academic and policy making circles around Asset Liability Management and Interest Risk Management and also hope that ALM practitioners may find some useful advice in our paper.

I always pass on good advice. It is the only thing to do with it. It is never of any use to oneself.

Oscar Wilde.

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7 Annex

7.1 Abbreviations

- ALM: Asset Liability Management.
- BCBS: Basel Committee on Banking Supervision.
- BP: Basis point.
- CBS: Constant Balance Sheet.
- DBS: Dynamic Balance Sheet.
- EBA: European Banking Authority.
- EVE: Economic Value of Equity.
- FINMA: FInanz Markt Aufsicht - Swiss Financial Market Supervisory Authority.
- FTP: Funds Transfer Pricing.
- HJM: Heath-Jarrow Morton.
- IRRBB: Interest Rate Risk in the Banking Book.
- KR01: Key rate duration (of 1 basis point).
- LTCM: Long-Term Capital Management.
- NII: Net Interest Income .
- NMD: Non-maturing deposit.
- NMP: Non-maturing product.
- NPV: Net Present Value.
- PCA: Principal Component Analysis.
- PnL / P&L: Profit and Loss.
- PV: Present Value.
- RST: Reverse Stress Testing.
- RTS: Regulatory Technical Standards.
- SOT: Supervisory Outlier Test.
- SVB: Silicon Valley Bank.
- YOLO: You Only Live Once.

7.2 Additional plots

EVE SOT scenarios in USD

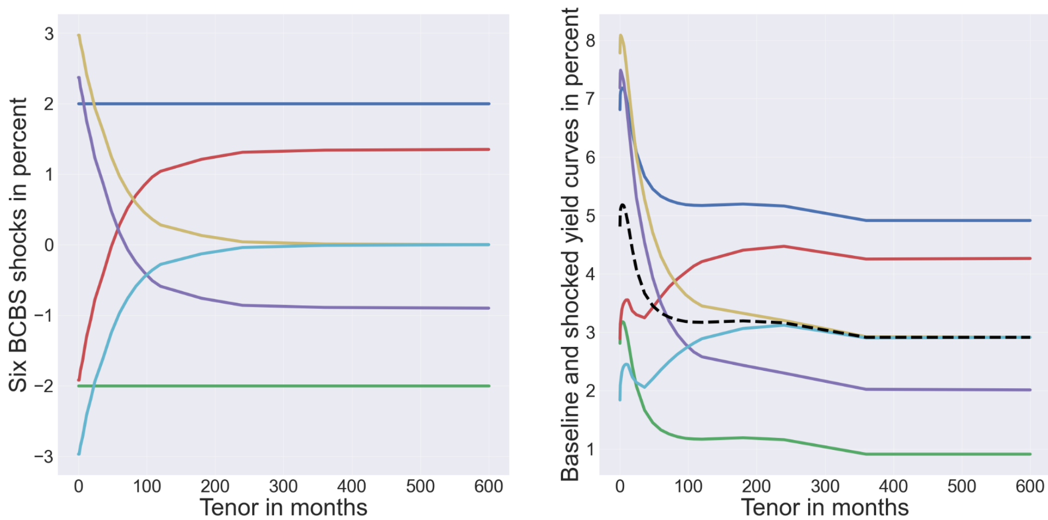


Fig. 23: Left: The six regulatory EVE shocks for USD: Parallel Up (dark blue), Parallel Down (green), Steepener (red), Flattener (violet), Short Rate Up (yellow), and Short Rate Down (light blue). Right: The six regulatory EVE scenarios as resulting from the application of the six shocks to the current yield curve (dashed). The green yield curve in the right-hand plot, for instance, is obtained by adding the shock of the green parallel down scenario in the left-hand plot to the current spot yield curve.

Reversius Bank and HJM scenarios

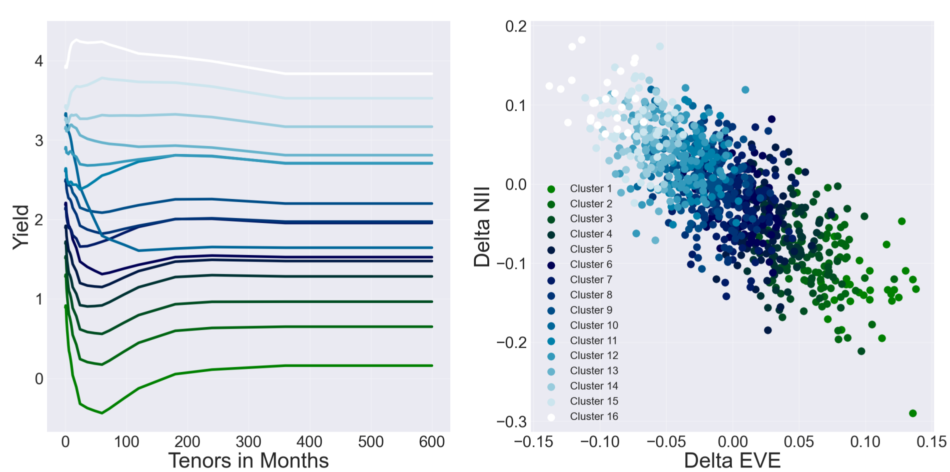


Fig. 24: Left: The average yield curve for each of the 16 clusters in CHF. Right: ΔNII and ΔEVE impacts in CHF, coloured by cluster. If CHF interest rates increased, Reversius Bank - given its balance sheet - stands to lose in terms of ΔEVE though gain in terms of ΔNII and vice versa if rates decreased.

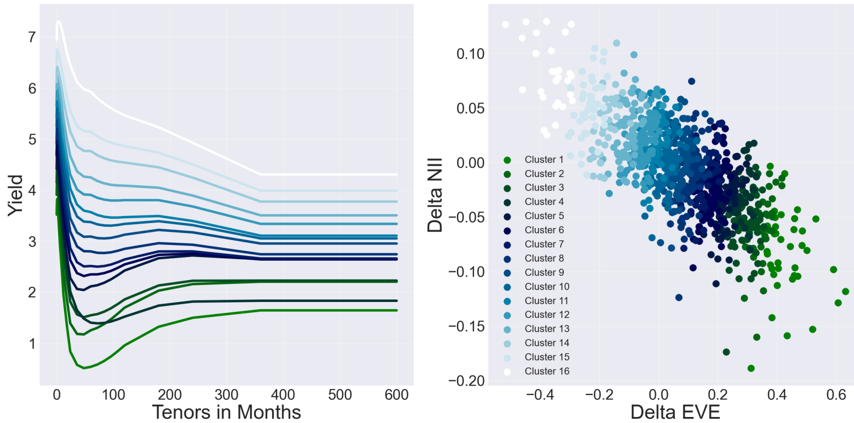


Fig. 25: Left: The average yield curve for each of the 16 clusters in USD. Right: ΔNII and ΔEVE impacts in USD, coloured by cluster. If USD interest rates increased, Reversius Bank - given its balance sheet - stands to lose in terms of ΔEVE though gain in terms of ΔNII and vice versa if rates decreased.

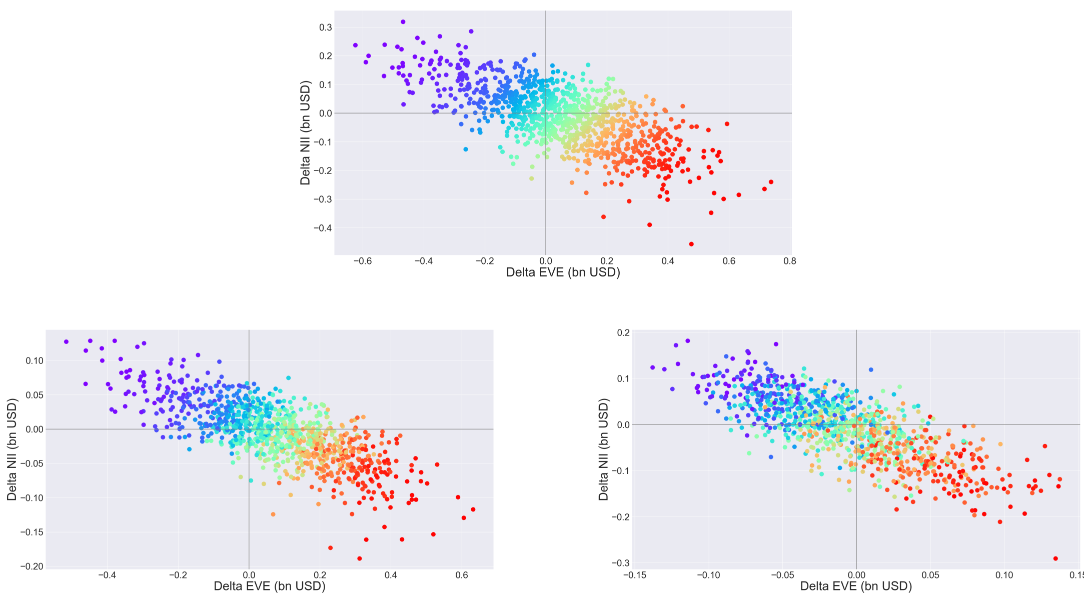


Fig. 26: The top plot defines the colouring of each scenario (from top-left to bottom-right) and the same scenario colouring is applied for USD and CHF impacts (two bottom plots). A large loss in terms of ΔNII (erd scenarios) is achieved only when ΔNII losses occur in both CHF and USD, which we know to happen from Fig.24 and Fig.25 only if rates in both currencies fall. A similar observation can be made for ΔNII gains combined with ΔEVE losses. The HJM scenarios generate fewer adverse cross-currency moves compared to the PCA generator.

YOLO Bank and PCA scenarios

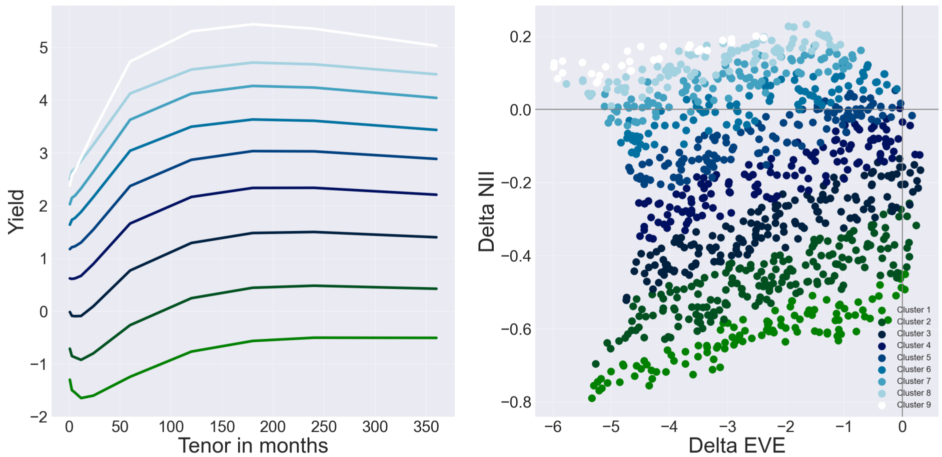


Fig. 27: Left: The average yield curve for each of the 9 clusters in CHF. Right: ΔNII and ΔEVE impacts in CHF, coloured by cluster. If CHF interest rates increased, YOLO Bank - given its balance sheet - stands to lose both in terms of ΔEVE and ΔNII. An increase in rates would also negatively affect ΔEVE, but positively impact ΔNII.

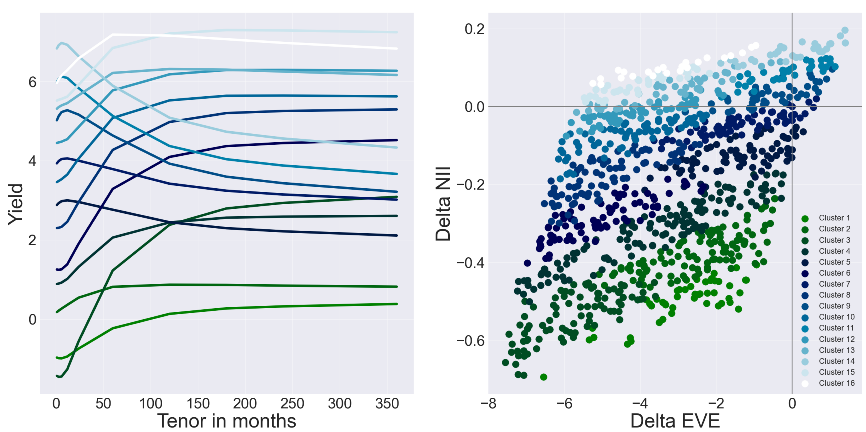


Fig. 28: Left: The average yield curve for each of the 16 clusters in USD. Right: ΔNII and ΔEVE impacts in USD, coloured by cluster. If USD interest rates decreased, YOLO Bank - given its balance sheet - stands to lose both in terms of ΔEVE and ΔNII. Several scenarios in cluster 14 however would lead to YOLO Bank benefitting both in terms of ΔNII and ΔEVE - perhaps the scenario the bank was "yolo'ing" its balance sheet on?

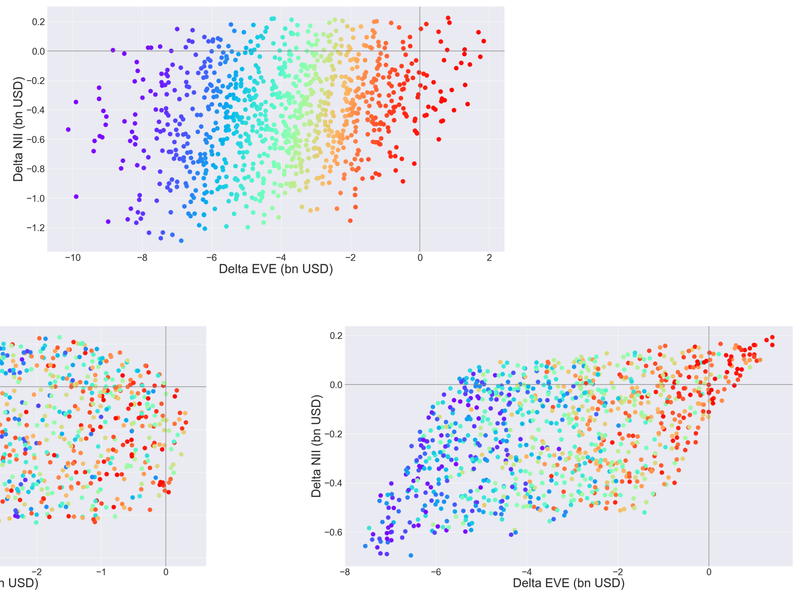


Fig. 29: Aggregate impact summed across USD and CHF (top plot), USD only impact (bottom left plot) and CHF only impact (bottom right plot) for YOLO Bank. The top plot defines the colouring of each scenario (from top-left to bottom-right) and the same scenario colouring is applied for USD and CHF impacts (bottom plots). The different interest rate risk position that YOLO Bank has taken on leads to materially different scatter plots for USD and CHF. Nonetheless, we observe a similar cross currency effect as for Reversius Bank.

Reversius Bank without NII stabilization

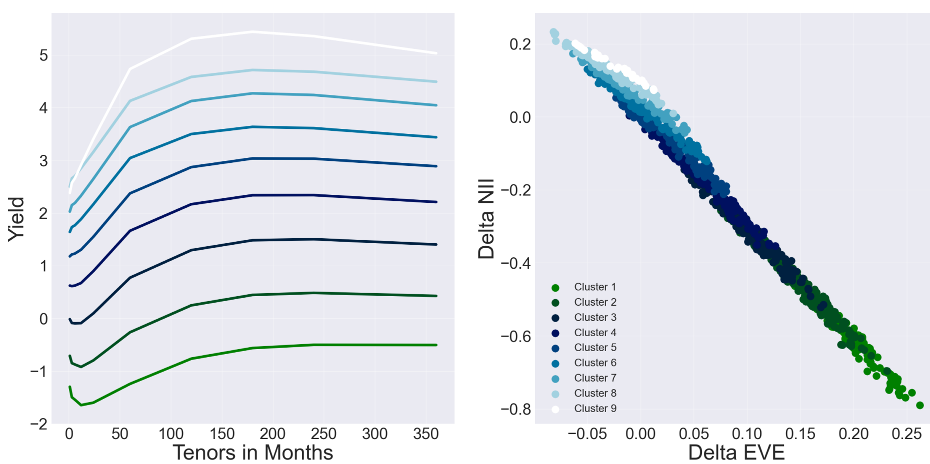


Fig. 30: Left: The average yield curve for each of the 9 clusters in CHF. Right: NII and EVE impacts in CHF, coloured by cluster. These are exactly the same as for Reversius.

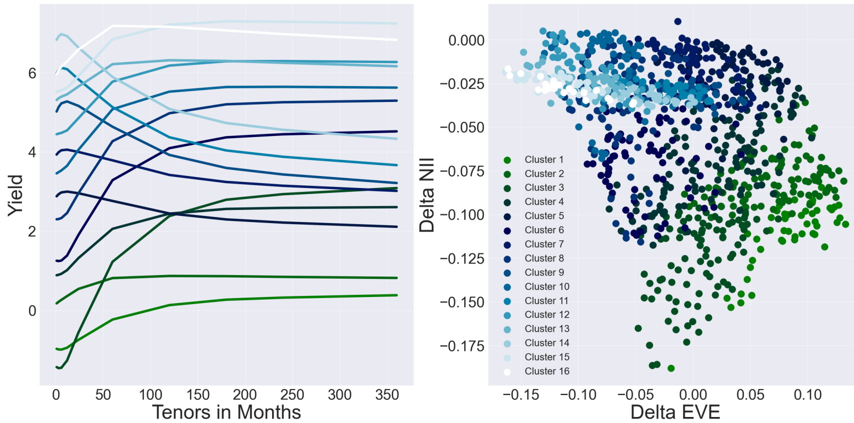


Fig. 31: Left: The average yield curve for each of the 16 clusters in USD. Right: ΔNII and ΔEVE impacts in USD, coloured by cluster. Without the NII stabilisation portfolio, the

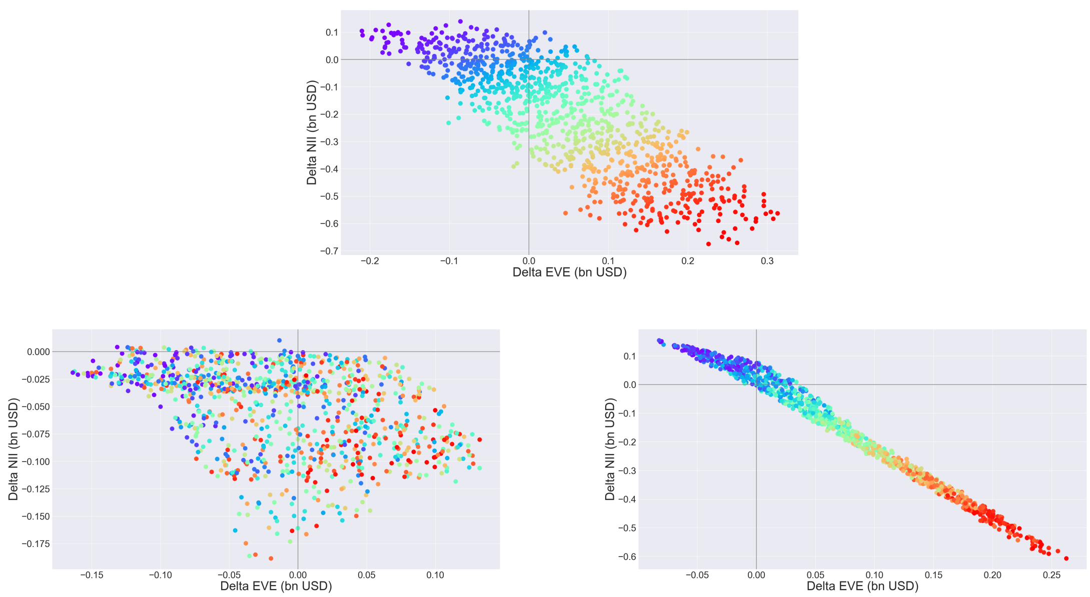


Fig. 32: Aggregate impact summed across USD and CHF (top plot), USD only impact (bottom left plot) and CHF only impact (bottom right plot) for Reversius Bank without NII stabilisation. CHF becomes the dominant

Scenario generation

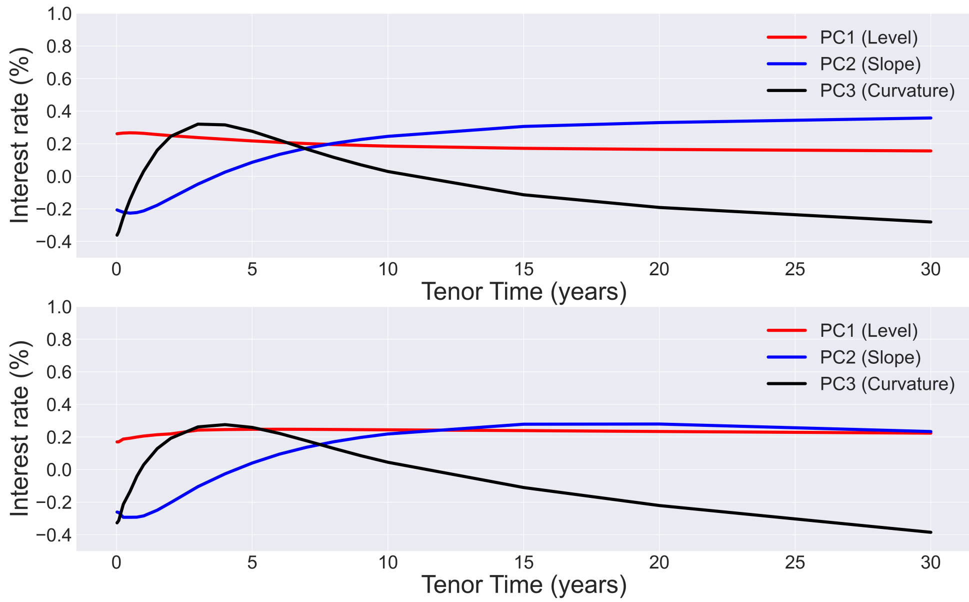


Fig. 33: The first three principal components of USD (top) and CHF (bottom) yield curves respectively, with unity coefficients.

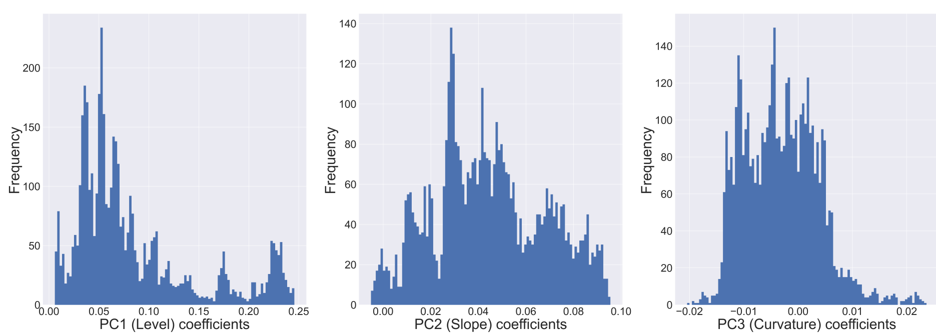


Fig. 34: The distribution of USD coefficient values for the 3 PC's required to reproduce the dataset.

Coefficients sampling takes place from the space:

$$\alpha^c \in \prod_{i=1}^{3} \left[\min_t(\alpha_i(t)) - (b(\max_t(\alpha_i(t)) - \min_t(\alpha_i(t))), \max_t(\alpha_i(t)) + (b(\max_t(\alpha_i(t)) - \min_t(\alpha_i(t))) \right],$$

where $b = 0.2$. Sampling 1000 points uniformly from this 3-D hypercube is illustrated in Fig.35, where Fig.11 then illustrates the resulting set of 1000 yield curves in USD (left) and CHF (right).

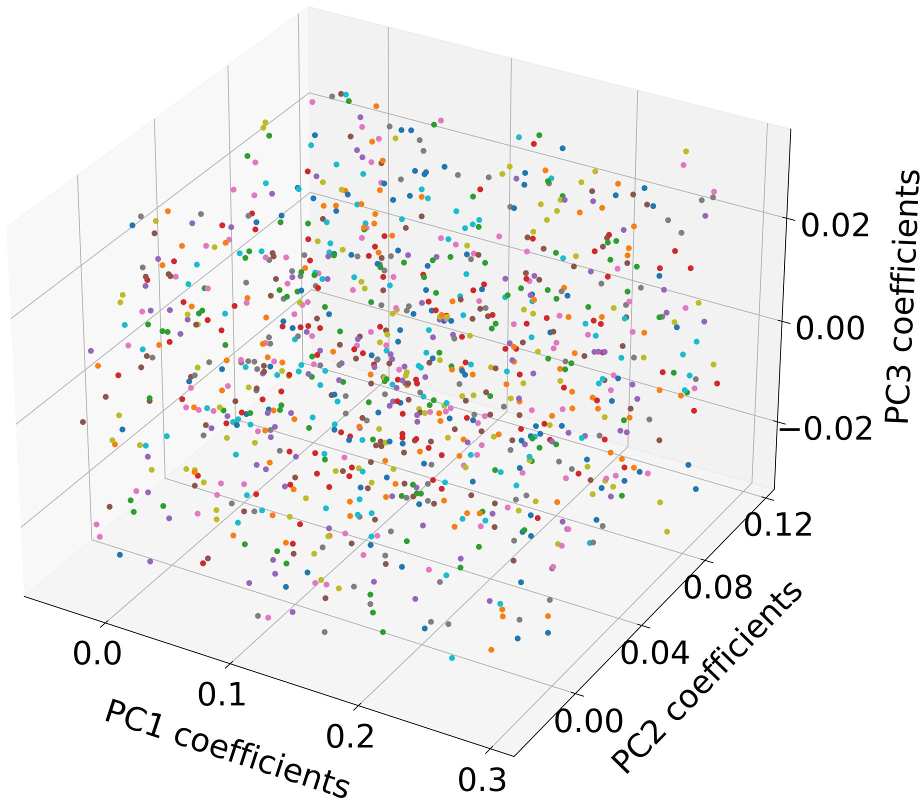


Fig. 35: Illustration of the sampling of USD coefficients as locations within this cube of uniform probability parameter space

7.3 The IRRBB regulatory framework

Capital requirements, SOT and EVE vs NII

There are no regulatory minimum capital requirements (Pillar 1) for IRRBB, which contrasts IRRBB starkly from other risk stripes such as Credit-, Market- or Operational Risk. Instead, IRRBB regulations focus on the Supervisory Review Process (Pillar 2) and Public disclosures (Pillar 3). The Supervisory Review Process sets out Supervisors' expectations on how banks are to manage IRRBB, and is grouped along nine principles, which we copy from BCBS 368 for ease of reference:

- **Principle 1**: IRRBB is an important risk for all banks that must be specifically identified, measured, monitored and controlled. In addition, banks should monitor and assess CSRBB.
- **Principle 2**:The governing body of each bank is responsible for oversight of the IRRBB management framework, and the bank's risk appetite for IRRBB. Monitoring and management of IRRBB may be delegated by the governing body to senior management, expert individuals or

an asset and liability management committee (henceforth, its delegates). Banks must have an adequate IRRBB management framework, involving regular independent reviews and evaluations of the effectiveness of the system.

- **Principle 3**:The banks' risk appetite for IRRBB should be articulated in terms of the risk to both economic value and earnings. Banks must implement policy limits that target maintaining IRRBB exposures consistent with their risk appetite.
- **Principle 4**:Measurement of IRRBB should be based on outcomes of both economic value and earnings-based measures, arising from a wide and appropriate range of interest rate shock and stress scenarios.
- **Principle 5**:In measuring IRRBB, key behavioural and modelling assumptions should be fully understood, conceptually sound and documented. Such assumptions should be rigorously tested and aligned with the bank's business strategies.
- **Principle 6**:Measurement systems and models used for IRRBB should be based on accurate data, and subject to appropriate documentation, testing and controls to give assurance on the accuracy of calculations. Models used to measure IRRBB should be comprehensive and covered by governance processes for model risk management, including a validation function that is independent of the development process.
- **Principle 7**:Measurement outcomes of IRRBB and hedging strategies should be reported to the governing body or its delegates on a regular basis, at relevant levels of aggregation (by consolidation level and currency).
- **Principle 8**: Information on the level of IRRBB exposure and practices for measuring and controlling IRRBB must be disclosed to the public on a regular basis.
- **Principle 9**: Capital adequacy for IRRBB must be specifically considered as part of the Internal Capital Adequacy Assessment Process (ICAAP) approved by the governing body, in line with the bank's risk appetite on IRRBB.

The IRRBB Trinity: Gap-, Basis- and Option Risk

In terms of measurement of IRRBB risk, this is often grouped into gap-, basis- and option risk, which we provide short summaries and relevant examples of below.

Gap risk: The S&L crisis and SVB. We've discussed the mechanics of the S&L crisis and Silicon Valley Bank before. Gap risk arises when cash flows do not reprice at the same time. During the S&L crisis, the short-term funding was repricing quicker than the long-term mortgages that the S&L's had provided. In the case of the S&L the gap risk was primarily of an NII nature: funding kept getting increasingly expensive, while the cash flow stream received from the assets increased only at a much slower rate. The large unrealized losses that SVB had accumulated on its bond portfolio are also an example of gap risk. However, in this case it was not that funding got more expensive, rather it was that the economic value of the bond portfolio declined, thus giving an example of gap risk from an EVE perspective. By using interest rate swaps, SVB could have reduced this gap risk because as the value of the bond portfolio declined, the value of hedging interest rate swaps would have increased in tandem.

Basis risk: LTCM. Even when cash flows reprice perfectly in sync, this does not necessarily mean that interest rate risk has been fully mitigated. Indeed, if assets and liabilities reprice simultaneously, but onto different rates or indices, then this gives rise to basis risk. LTCM took positions in off-the-run and on-the-run U.S. Treasury securities. On-the-run Treasuries, which are newly issued bonds typically have slightly lower yields than off-the-run Treasuries, which are older issuances due to liquidity preferences (as they have essentially the same credit risk). LTCM bet that the spread would narrow and put on a spread-trade going long the off-the-run Treasuries while being short the on-the-run bonds.

However, when the Russian government defaulted on its domestic debt in 1998, this lead to a liquidity crisis and a flight-to-quality across markets, which resulted in a widening of the spread contrary to LTCM's expectations. Due to the large leverage of LTCM, the widening of the basis led to large losses for the fund and its eventual collapse and rescue organized by the Federal Reserve and major banks. While the LIBOR-OIS basis was the most prominent one, post LIBOR reform basis risk still exists in various forms: cross currency, SOFR vs Term-SOFR, intra-curve basis risk (e.g. when a loan and a deposit both reprice on the same internal FTP curve of a bank, but the loan reprices on the 6month tenor, while the deposit reprices on the 1 month tenor, say).

Option Risk: Northern Rock Option risk generally refers to "automatic option risk" (e.g. exercising of swaptions or other options) and "behavioural option risk". Behavioural option risk is commonly subdivided into two categories: i) prepayment risk on mortgages, which is particularly relevant in U.S. markets and ii) withdrawal risk of deposits. As rates fall, customers may choose to refinance their mortgages at lower rates, which leads to pre-payments and thus lower expected interest income. Some mortgages include prepayment penalties, which mitigate this risk, though this is not common in US markets. Contrary, if rates increase, customers tend to withdraw funds from deposits and place them into higher yielding products. Arguably, the most severe form of option risk is clients exercising their right ("option") to withdraw funds not due to interest-rate related dynamics but due to idiosyncratic events. In 2007, as liquidity in interbank markets dried up due to the U.S. subprime mortgage crisis, Northern Rock found itself unable to roll over its financing and sought emergency funding from the Bank of England. However, rather than re-assuring depositors, this led to a full-blown bank run by its depositors, who all "exercised their option" to withdraw funds, thus triggering the U.K.'s first visible bank run in over a century. A similar fate struck Silicon Valley bank and several other banks in 2023, though in the age of online- and web banking, the speed of these bank runs had materially increased compared to 2007.

7.4 What positions determine IRRBB?

The Banking Book is a portfolio of assets and liabilities that are not held for trading purposes but instead intended to be held for longer durations in order to earn net interest income. Net Interest Income is the difference between Interest Income, earned on assets, and Interest Expense, paid on liabilities. From a regulatory perimeter perspective, the "banking book" is often implicitly defined as those assets and liabilities that are not held in the trading book.

Assets.

The most important assets earning interest income in the banking book are loans, cash and fixed income securities (not held for trading purposes).

- **Fixed rate loans:** A fixed rate loan is a loan for which the interest rate to be paid is fixed and does not change as a result of a change in market rates. Fixed rate loans can have the interest fixed either until maturity or for an initial period (often 5, 10, 15 or 20 years) after which the interest rate will be reset.
- **Floating rate loans:** A floating rate loan is a loan whose interest rate is tied to a market interest rate such as SOFR, SARON or Euribor. Customers will generally pay a certain percentage above the market interest rate, such as "SOFR + 100bps", which means that if SOFR currently stands at 3.5%, then the customer will pay 4.5%. The interest rate on a floating rate loan generally may reset either at monthly, quarterly or semi-annual frequency. Lombard loans are a sub-category of floating rate loans.
- **Cash:** Cash placed at the central bank may or may not earn interest, depending on the monetary policy framework of the central bank in question.[28]

[28] See e.g. `https://www.snb.ch/en/mmr/reference/vz_mb1/source/vz_mb1.en.pdf` for an overview of the SNB deposit facilities, `https://research.stlouisfed.org/publications/page1-econ/2020/08/03/`

- **Fixed income securities:** Instead of holding excess funds as cash at the central bank, banks generally seek to improve their returns and may as a result decide to a portion of their high quality liquid assets as in the form of a bond portfolio. This will predominantly consist of governement debt, but may also consist of covered bonds, corporate bonds or securitisations such as residential mortgage-backed securities (RMBS). These products generally all pay a fixed coupon.

Liabilities.

Banks fund themselves through retail markets by taking deposits from retail or private banking customers, as well as from wholesale markets by issuing long term debt and short term instruments such as commercial paper.

- **Fixed term deposits:** A fixed term deposit is a deposit which is placed by the customer for a fixed period of time earning a fixed interest rate. The customer may generally not withdraw the money prior to the maturity of the deposit, and if so only at a fee.
- **Non-maturing deposits:** Current accounts, savings accounts or transactional accounts (e.g. where salaries are received) are examples of non-maturing deposits, i.e. deposits without a contractual maturity where customers can withdraw funds at any point in time. A central topic in ALM Risk Management is the modelling of the *behavioural* duration of these deposits: While contractually these deposits have a duration of 1 day (because funds can be withdrawn at any point in time), behaviourally the deposits have a longer duration as customers tend to leave funds for longer in their accounts.
- **Wholesale funding - Long-term debt and CDCPs:** Banks issue various forms of debt to investors and other wholesale market counterparties. This can take the form of short term instruments such as Commercial Deposits and Commercial Paper (CDCPs) or longer term instruments such as corporate bonds issued as long term debt. The post crisis regulatory reform also expect banks to hold a certain amount of so called "Additional Tier 1" capital, which are debt instruments than can - depending on their structure - be either converted to shares or written off when a so-called "trigger event" occurs.
- **Equity:** Equity corresponds to the initial funds invested by shareholders and increases through retained earnings and decreases when the bank needs to absorb losses. It is the most stable source of funding and does not pay an interest[29]. Instead dividends are paid to shareholders when the bank is making profits.

Off-Balance Sheet items

We will comment on off-balance sheet items and interest rate swaps in particular in Section 3.3, as their use relates to strategic risk management decisions and a few helpful concepts will be introduced first.

7.5 Stochastic Forward Curves

The PCA based yield construction presented in section 4 results in forward curve scenarios with instantaneous shocks applied to the initial yield curve. From a stress testing perspective it is assumed that large shocks ($> 100bps$) can be realized inmediately from the day one of the projection.

However, from a marked to market view, these type of shocks are not always realistic, and therefore short rate models such as Hull-White and Heath-Jarrow-Morton (see Brigo and Mercurio (2001) for a complete review on short interest rate models) define rate dynamics as a diffusion process with increasing volatility over increasing forward rate maturities, providing a closer approximation to the

the-feds-new-monetary-policy-tools for an overview of the Federal Reserve facilities or https://www.ecb.europa.eu/mopo/implement/sf/html/index.en.html for ECB standing facilities.

[29] An alternative view is to say equity is a fixed rate liability with an interest rate of 0 %.

actual interest rate evolution over the projected simulated time horizon.

In this section we implement the Heath-Jarrow-Morton model (HJM) for the generation of instaneous forward curve realizations and associated forward rates, providing a closer marked to market view to be used as reference for the Reverse Stress Testing algorithm presented in section 4.

The HJM model requires as inputs the current instantaneous forward rate curve and a volatility function dependent on the underlying rate tenor. The initial instantaneous forward curve is calculated directly from market rates and it is hence deterministic, on the other hand, the specification of the volatility function allows us to control the variability of the forward rates generated by the HJM instantaneous forward distribution, we can use this feature to help ensure that all relevant scenarios will be included in our Monte Carlo simulation.

The Heath–Jarrow–Morton Model

The Heath–Jarrow–Morton (HJM) model defines the risk neutral dynamics of the instantaneous forward rate over time. The HJM model, in contrast to one-factor short rate models, can incorporate a correlation term structure in between long and short term rates. We present here the main concepts to summarize this model, for more details the reader can consult Heath et al. (1992).

The price of a zero bond at time t with maturity T can be expressed as:

$$Z\left(t;T\right) = \exp\left(-\int_t^T f\left(t;T\right)\right). \tag{2}$$

Given the expresion in Eq.(2), we can derive the expression of the instantaneous forward rate as follows:

$$f\left(t;T\right) = -\frac{\partial}{\partial T}\log Z\left(t;T\right). \tag{3}$$

Using the development followed by Lixin Wu in Wu (2009), the HJM no-arbitage condition and final instantaneous forward rate dynamics are described in Eq.(4):

$$\mathrm{d}f\left(t;T\right) = \left(\vec{\sigma}^T\left(t;T\right)\int_t^T \vec{\sigma}\left(t;s\right)\right)\mathrm{dt} + \vec{\sigma}^T\left(t;T\right)\mathrm{d}\tilde{W}. \tag{4}$$

An important feature in Eq.(4) is that all the dynamics are charaterized by the definition of the instantaneous forward volatility function $\vec{\sigma}\left(t;T\right)$ and the current instantaneous forward $f\left(0,\tau\right)$. In Eq.(4) the time and tenor dimensions are expressed in absolute terms, i.e. t denotes the forward start and T the absolute time elapse since the observation point t_0, being t_0 the current evaluation time. From an implementation perspective is more attractive to deal with a model with relative measures of time, so we introduce the variable τ as the time to maturity from the forward start date t, , i.e.: $\tau = T - t$, and hence the forward rates can be expressed as $f(t_0;t;T) = \tilde{f}(t,\tau)$.

This change of variable in Eq.(4) is known as the *Musiela's parametrization* (see La Chioma and Piccoli (2007)), and its associate multi-factor SDE is depicted in the following SDE:

$$\mathrm{d}f\left(t;\tau\right) = \left(\sum_{i=1}^{n}\sigma_i\left(t;T\right)\int_0^\tau \sigma_i\left(t;s\right)\mathrm{d}s\right)\mathrm{dt} + \sum_{i=1}^{n}\sigma_i\left(t;\tau\right)\mathrm{d}W_i + \frac{\partial}{\partial\tau}f\left(t;\tau\right)\mathrm{dt} \tag{5}$$

where,

n is the number of factors.

$\sigma_i, \forall i \in \{1..n\}$ are the volatility functions associated to each factor.

Monte Carlo Simulation of Correlated Paths

Extending Eq.(5) to simulate correlated Monte Carlo paths for various currencies is straightfoward by converting the single currency dynamics into a multi-currency stochastic scenario generator as depicted in Eq.(6), where it is assumed that the same correlation matrix Γ will be applied to all PCA components (i.e. : *level*, *slope* and *curvature*) and tenors.

$$\begin{bmatrix} \mathrm{d}\mathbf{W_0}^{CHF} \\ \mathrm{d}\mathbf{W_0}^{EUR} \\ \mathrm{d}\mathbf{W_0}^{GBP} \\ \mathrm{d}\mathbf{W_0}^{JPY} \\ \mathrm{d}\mathbf{W_0}^{USD} \end{bmatrix} = \text{Cholesky}\left(\Gamma \in \mathbb{R}^{5\times5}\right) \times \vec{N}\left(\vec{0}; \mathrm{I} \in \mathbb{R}^{5\times5}\right) \tag{6}$$

Eq.(6) includes the following expressions:

- $\mathrm{d}\mathbf{W_0}^{Currency}$ is the correlated brownian shock for each of the currencies under simulation.
- $\text{Cholesky}\left(\Gamma\right)$ is the lower triangular Cholesky matrix decomposition of Γ with dimension $\mathbf{R}^{n\times n}$, being n the number of currencies under simulation.
- $\vec{N}$ is a multivariate normal distribution with dimension n.

With this approach we simulate coherently the correlated evolution of all forward rates (for all currencies) inside the MC engine.

HJM Monte Carlo Simulation

Based on the Eq.(5) we simulate the time evolution of the instantaneous forward rates using a Monte Carlo Engine. The MC simulation engine discretizes time (t) and tenor(τ) dimensions generating N simulation grids as depicted in Figure 36.

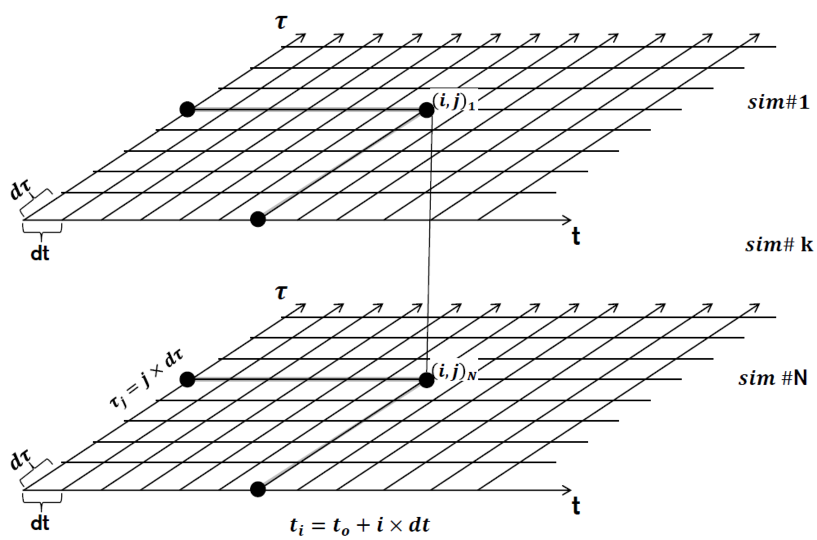


Fig. 36: Monte Carlo Simulation Cube

In Fig.36 each grid corresponds to a MC path realization of the entire instantaneous forward cuve $f\left(t;\tau\right)$ for each dicrete simulation step t_i and τ_j.

The MC discretized update scheme based on Eq.(5) is depicted in the following expression:

$$f\left(t+idt, \tau_j\right) = f\left(t+(i-1)dt, \tau_j\right) + \Delta f\left(t+idt, \tau_j\right),$$

where $\Delta f\left(t+idt,\tau_j\right)$ corresponds to last term in the Musiela's parametrization in Eq.(5).

Hence, the HJM framework generates for each time $t_i = t_o + i\Delta t, i \in \{0,..,n-1\}$, a range of m tenors $\tau = j\Delta_\tau, j \in \{0,..,m-1\}$, over N simulations, generating thus a total simulation cube of $N \times n \times m$ instantaneous forward rates $f_{n\in N}\left(t_i,\tau_j\right)$.

HJM volatility calibration (PCA)

The function $\vec{\sigma}\left(t;\tau\right)$ in Eq.(5) needs to be computed based on market conditions. This work generates the components of $\vec{\sigma}\left(t;\tau\right)$ by applying principal component decomposition to the instantaneous forward historical daily diffences over a predefined period of time.

Using as input historical daily instantenous forward curves, the calculation $\vec{\sigma}\left(t;\tau\right)$ in Eq.(5) is performed through principal component analysis (PCA).

Mathematically, the volatility function the calibration algorithm follows these steps:

1. Given the matrix of daily rate samples $X \in \mathbb{R}^{n\times m}$, calculate its covariance matrix.

$$\mathbf{C} = \frac{1}{n-1}\left(\mathbf{X}-\vec{\mu}\right)^T\left(\mathbf{X}-\vec{\mu}\right)$$
$$\in \mathbb{R}^{m\times m}$$

2. Extraction of the associated eigenvalues and eigenvectors applying Singular Value Decomposition (SVD). The daily rates covariance matrix is decomposed as $\mathbf{C} = \mathbf{U}\mathbf{\Sigma}\mathbf{V}^T$, where
   - $\mathbf{V} \in \mathbb{R}^{m\times m}$ holding in the columns the eigenvectors $\vec{e}_i \in \mathbb{R}^m$, with $i \in \{1..m\}$.
   - $\mathbf{\Sigma} \in \mathbb{R}^{m\times m}$ with a diagonal vector $\mathbf{\Lambda} \in \mathbb{R}^m$ holding the eigenvalues (in descendent magnitud order $\lambda_i \geq \lambda_{i-1}$)

$$\mathbf{\Sigma} = \begin{bmatrix} \lambda_1 & 0 & .. & 0 & 0 \\ 0 & \lambda_2 & 0 & .. & 0 \\ 0 & 0 & \lambda_3 & .. & 0 \\ .. & .. & .. & \lambda_k & 0 \\ 0 & 0 & .. & 0 & \lambda_m \end{bmatrix}$$

   - $\mathbf{U} \in \mathbb{R}^{m\times m}$ includes the null space eigenvectors. This latest matrix is not used.

3. Arranging in descending order the obtained eigenvalues and its assocaited eigenvectors, we reduce the SVD system taking only the first K columns and transforming the SVD by reducing their respective ranks as follows:

$$\begin{aligned}\tilde{\mathbf{V}} &= \left[\vec{e}_1,..,\vec{e}_k\right] \\ \tilde{\mathbf{V}} &\in \mathbb{R}^{m\times K} \\ \tilde{\mathbf{\Lambda}} &= diag\left[\lambda_1,...,\lambda_K\right] \\ \sqrt{\tilde{\mathbf{\Lambda}}} &= diag\left[\sqrt{\lambda_1},...,\sqrt{\lambda_K}\right] \\ \tilde{\mathbf{\Sigma}} &= \tilde{\mathbf{\Lambda}} \in \mathbb{R}^{K\times K} \\ \mathbf{U} &\in \mathbb{R}^{n\times K}\end{aligned}$$

4. The resulting filtered volatility matrix is scaled by $\sqrt{260}$, adapting our brownian motion to yearly units of time.

5. The final K-filtered covariance matrix $\tilde{\mathbf{C}}$ is then used as input for the stochastic term in section 5, namely, the following expresion is used in the HJM update, reproduced for convenience in the following equation:

$$\sqrt{\tilde{\mathbf{C}}} = \tilde{\mathbf{\Lambda}}\tilde{\mathbf{V}}^T\sqrt{260}$$
$$\mathrm{d}f(t;\tau) = m(t;\tau) + \sqrt{\tilde{\mathbf{C}}}\mathrm{d}\mathbf{W} + \frac{\partial}{\partial\tau}f(t;\tau)\,\mathrm{d}t$$
$$m(t;\tau) = \left(\sum_{i=1}^{n}\sigma_i(t;T)\int_0^{\tau}\sigma_i(t;s)\,\mathrm{d}s\right)\mathrm{d}t$$

Analysis of the PCA Volatility Functions

The PCA algorithm covered in the previous section has a financial interpretation when applied to historical interest rate curves and hence most relevance when looking after the inclusion of specific risk scenarios. Namely, the first three factors represent the *level*, *slope* and *curvature* functionals of the underlying term structure of the interest rates, while their respective eigenvalues the weights or loads of such functionals. The relation in between Nelson-Siegel solutions to the PCA factor functional shapes are empirically observed in the market (see Bolder and Streliski (1999)), and hence when applying the proposed volatility calibration we can directly connect the expected interest rates moves in our MC simulation by simply observing the selected factors of the volatility functions.

Fig.37 depicts USD daily instantaneous forward yield curves from April 2022 until June 2024 which are used as inputs for the PCA calculation.

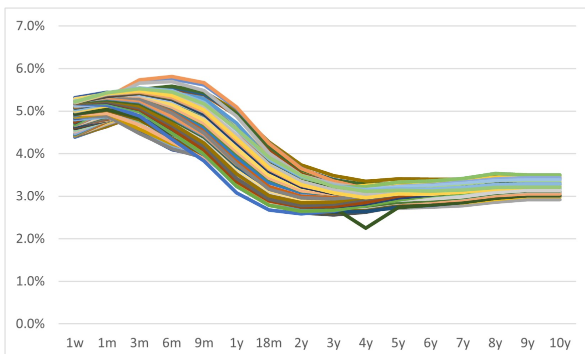


Fig. 37: USD Instaneours forward curves

Applying the algorithm provided in section 7.5 to the historical instananous USD forwards curves in Fig.37 we obtain the three first PCA components plotted in Fig.38, where each line represents the volatility functions $\sigma_i(t;T)$ in Eq.(5).

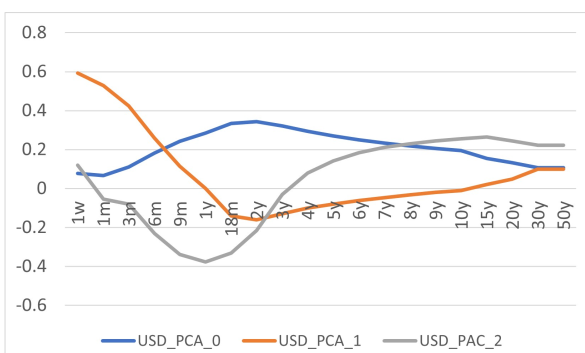


Fig. 38: USD PCA components

PCA Severity Calibration

In order to guarantee that specific interest rate shocks are reachable in the HJM Monte Carlo simulation, the volatility level (or first PCA component) can be stressed to guarantee that the required forward rate levels are simulated at a given quantile, e.g. : we might want to guarantee that the USD 2y forward rate with maturity in 6m (i.e.: $f(t_0, t = 6m, \tau = 2y)$) will reach 100bps up shock with a confidence level of 99%.

Provided a forward rate shock and associated quantile, the following calibration algorithm solves how much it is required to stress the volatility level to match the required 99% distributional forward rate value:

$$\xi(\alpha) = \min_{\alpha}\{|\Delta F_q(\alpha, t_0; t, T) - shock_q|\} \tag{7}$$

$$\alpha > 0.0$$

$$\textbf{with, } \Delta F_q(\alpha, t_0; t, T) = \left\langle \int_0^{T-t} f_i(\alpha, t, \tau)\, \mathrm{d}\tau - \mathbb{E}\left[\int_0^{T-t} f(\alpha = 0, t, \tau)\, \mathrm{d}\tau\right] \right\rangle_q \tag{8}$$

$$f_i(\alpha, t, \tau) = f(t, \tau) \textbf{ as in eq 5 with } \sigma_0(t, \tau) = \tilde{\sigma_0}(t, \tau)(1 + \alpha) \tag{9}$$

, where

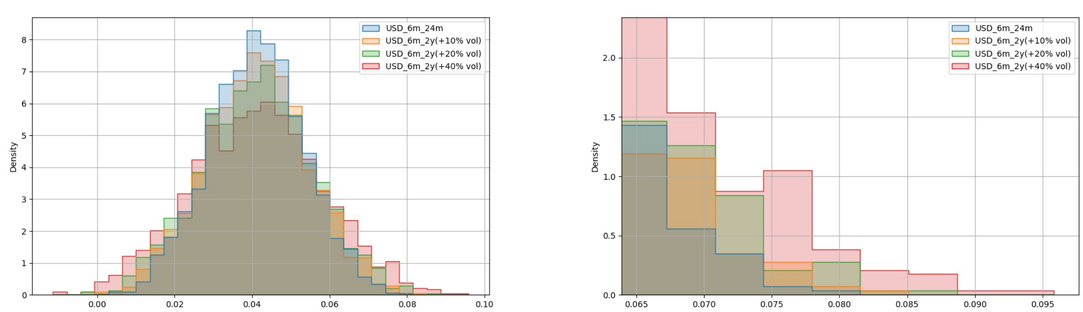
- Eq.(7) defines the fitness function, $\xi(\alpha)$, which minimizes the forward curve spreads for a given reference start date and maturity t and T respectively.
- Eq.(8) depicts the q-quantile MC forward spread, $\Delta F_q(\alpha, t_0; t, T)$, computed as the difference in between the reference (not shocked) forward and the stressed fowards simulated with $\alpha > 0$.
- Eq.(9) expresses the dependency of $f_i(\alpha, t, \tau)$ with α, which acts as a constant positive factor to the volatility level (or first PCA volatility eigenvector).

Volatility Level Calibration Example

Taking as reference the USD forward rates $F(t = 6m, \tau = 2y)$, we proceed to compute the forward rate quantiles at 99% confidence level for values of $\alpha \in (0\%, 10\%, 20\%, 40\%)$ (Eq.(9)).

For each α, the corresponding forward rate densities are plotted in Fig.39a, with a detailed zoom into the tails of each distribution in Fig.39b.

As expected, the tails of the different foward distribution increase proportionally to the shock applied to the volatility function used in each calculation, allowing hence to determine the optimimum value of α required to match a specific rate quantile using Eq.(7).



(A) USD 2y Forwards with Maturity 6m

(B) USD 2y Forward distributions tail detail.

Fig. 39: USD 2y Forward forward distributions for several volaility shocks.

Taking a confidence level of 99%, the associated rates for different shocks of α are plotted in Fig.40.

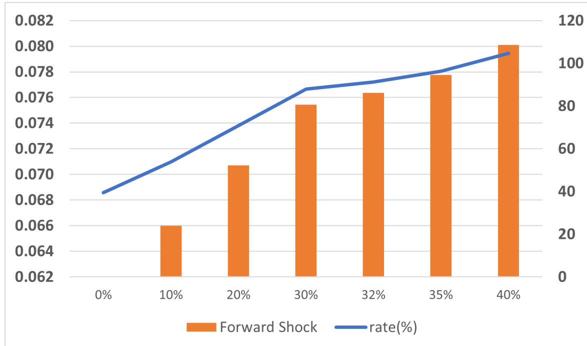
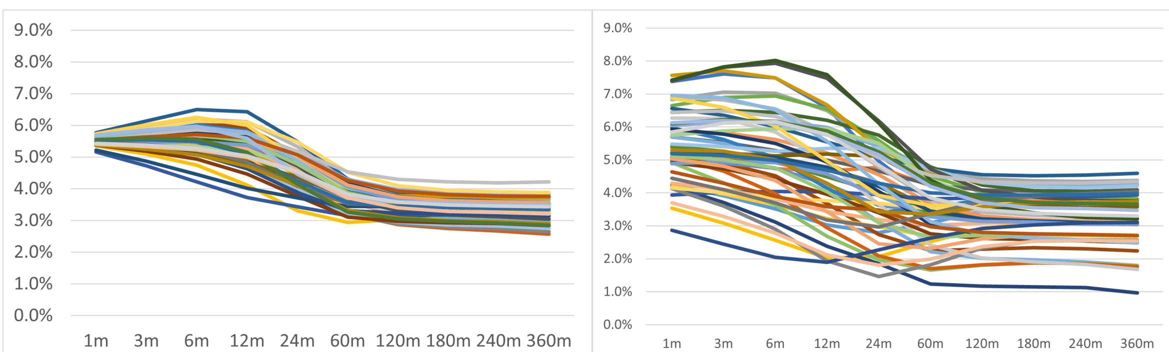


Fig. 40: USD 2y Forward rates (maturing in 6m) quantiles at 99% confidence level. The blue line is the forward rate values (left y-axis) at 99% confidence level as a function of α . The organge bars are the forward rate increase (right y-axis) in bps with respect to the reference forward rate ($\alpha = 0$)

The calibration algorithm of α consists therefore on a trivial optimization problem solving α for a given forward rate quantile value. As an example, in Fig.40, a value of $\alpha = 0.37$ generates a shock to the USD 2y forward with maturity 6m of $\tilde{1}00bps$, hence, if in our scenario assumptions we require that a 100bps shock must be reachable for 2 year rates in 6 months, the values of α must be set to 37%.

Simulation Results



(a) MC paths for USD forward curve with maturity 1m. **(b)** MC paths for USD forward curve with maturity 6m.

Fig. 41: Monte Carlo paths for USD forward curves with maturities 1m and 6m.

Fig.41 shows the Monte Carlo paths for instantaneous forward rate curves simulated with maturities 1m (Fig.41a) and 6m (Fig.41b). As expected, for short maturites, the standard deviation of the realized yield curves is smaller than those for longer maturities, following hence the expected HJM diffussion process with increasing volatilites in which longer projection points in the simulation time horizon allow the materialization of more severe scenarios, both in terms for levels of the rates as well as the variaty of "shapes" the yield curves will take.

Fig.42 depicts the 2y Forward Rate distributions for 2000 Monte Carlo paths. Each subpicture corresponds to the 2y USD forward rate at the given maturity, starting in 1m and ending in 12m. Again, the simulation path follows a difusion process in which the uncertainty of the rate increases with the maturity of the same.

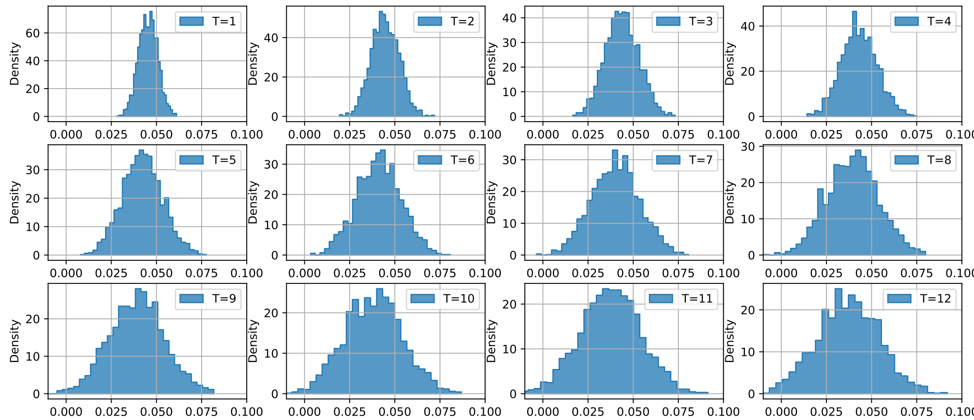


Fig. 42: 2y USD forward distributions with maturities from 1m up to 12m months.

Fig.43 plots the 99% quantile values for the 2y forward rate for maturities from 1m to 12 months.

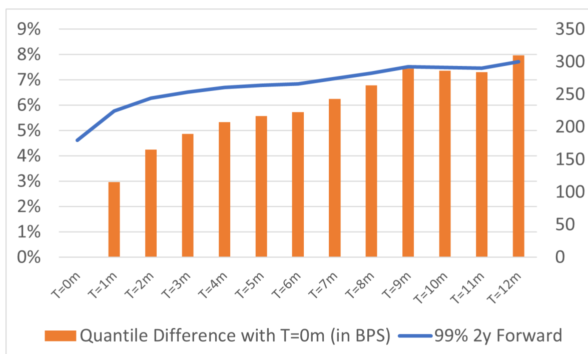
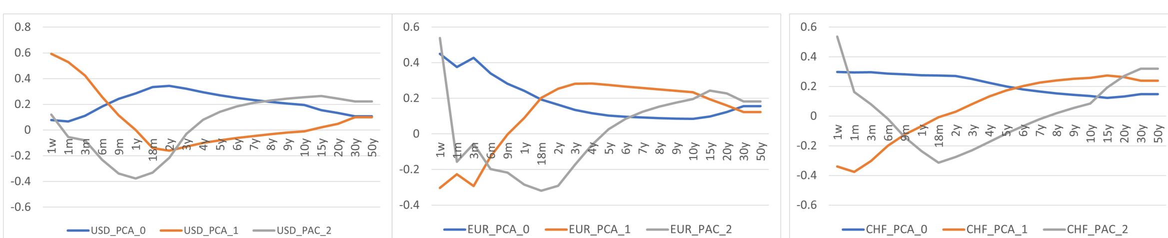


Fig. 43: 99% quantiles for 2y USD forward distributions with maturities from 1m up to 12m months. Left y-axis forward rate values. Righ y-axis difference of T=i quantile with reference values at T=0m.

Finally, Fig.45 depicts a multicurrency Monte Carlo simulation as per the methodology introduced in section 7.5, setting a correlation value for EUR,CHF and USD levels, slopes and curvature risk factors of 80%. The calibrated volatility functions (PCA eigenvectors) for EUR, USD and CHF are depicted in Fig.44.



(a) USD PCA components **(b)** EUR PCA components **(c)** CHF PCA components.

Fig. 44: USD, EUR and CHF 2y Forward forward distributions for several volaility shocks.

It is relevant to highlight that the correlation matrix Γ in Eq.(6) is referring to instantaneous forward rates and the actual forward rate correlations across currencies need to be calculated from the simulation paths using for instance Eq.(2) for the computation of the desired forwards. For the case of long tenors the imposed instaneous forward correlation tends to match those measured from the simulation paths, however for shorter tenors showing more variablity in the level, slope and curvature functions around the selected point, the correlation can be significantly different. In Fig.45 it is plotted the correlation structure for USD, EUR and CHF 2y forward rates with maturity in 6m.

The numerical correlation values for the forward rates in Fig.45 are also depcited in Table 2, matching the fixed 80% instananous correlation factors imposed in our simulation.

Tab. 2: Simulated Monte Carlo Correlation Matrix

	EUR	USD	CHF
EUR	1.0	0.79	0.78
USD	0.79	1.0	0.76
CHF	0.78	0.76	1.0

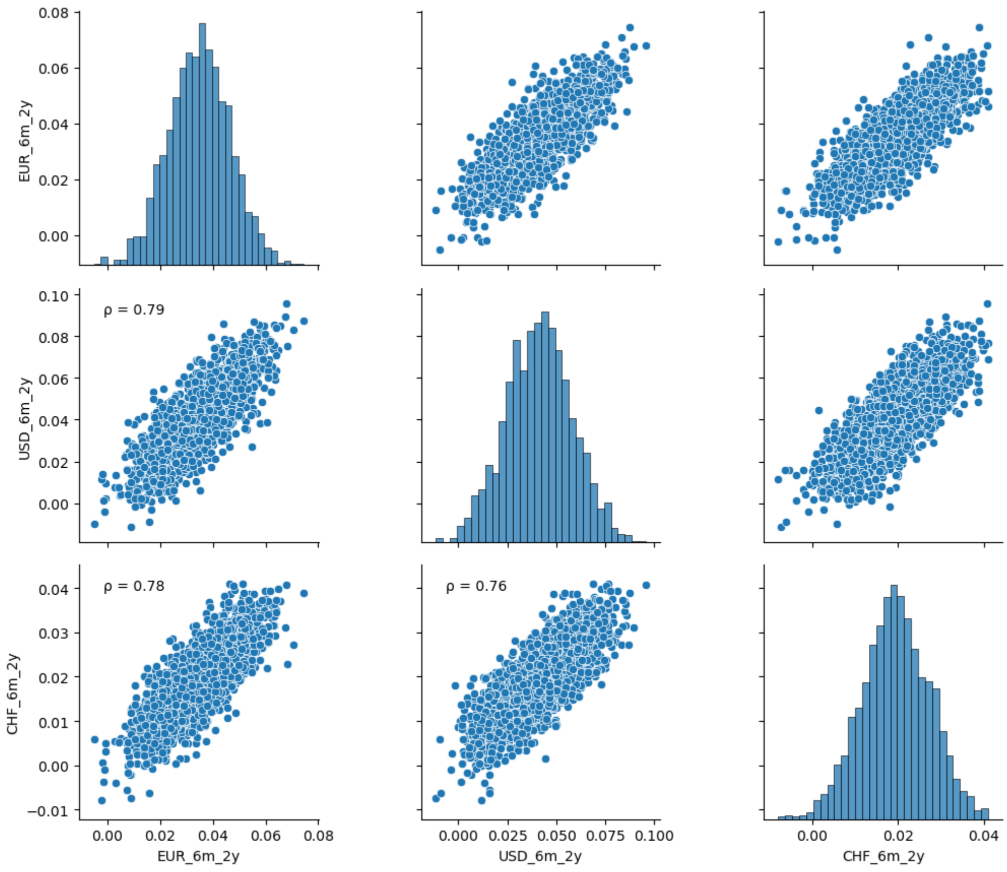


Fig. 45: Multi-currency MC simulation (2000 paths) for 2y forwards in EUR,USD and CHF