# An Analytical Model for Loan Commitments Facing the Material Adverse Change

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Abstract: We propose a new analytical model for the valuation of loan commitments and some of their main features including the MAC (Material Adverse Change) clause. A two-period contingent claims approach in continuous time is developed. The advantage of this approach is that it is based on rational economic considerations that are not based on utility functions.

Keywords: credit line, loan commitment, MAC (Material Adverse Change), Merton's model
JEL classification: G120, G130, G210, G320

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# An Analytical Model for Loan Commitments Facing the Material Adverse Change

## Introduction

Credit line or loan commitments constitute a key instrument in the toolbox of corporate finance. In the US, according to Cooperman et al. (2023), most bank credit to corporate borrowers takes the form of revolving credit lines. These commitments are short-term credit enabling firms to withdraw from their banks, up to a certain, predetermined ceiling (in either a lump sum or as revolving credit), at a certain cost, usually higher than the interest rate on long-term credit. These credit lines serve corporations to cover, primarily short-term financial needs, during periods in which cash outflows are greater than cash inflows. They are used as a buffer against unexpected short-term cash flows gaps and also as a way to take advantage of unanticipated investment opportunities. Thus, loan commitments can be employed offensively as well as defensively by corporations.[1]

Firms require cash for future, uncertain uses. Financial markets and myriad intermediaries provide access to debt and other forms of raising capital. This, however, can be expensive and time-consuming or can require disclosing private information. Most companies keep liquid reserves to serve their immediate needs and provide a buffer for future uses. To safeguard liquidity, firms commonly either adopt a policy of maintaining adequate liquid reserves or enter into loan commitment agreements, including lines of credit.

Since uncertainty surrounds the firm's future needs, it is necessary to design an optimal policy, that takes into account expected cash flows and the uncertainty of these cash flows, the firm's ability and capacity to raise new capital, and the costs associated with the various forms and sources of financing. Part of the problem of enhancing liquidity can be addressed, for example, by issuing callable bonds (raising capital in advance with an option to redeem, should the debt be ultimately deemed unnecessary), adjusting dividend payments and retaining earnings. Liquidity financing through the use of short-term credit instruments, such as lines of credit, however, has become an increasingly important part of the corporate finance landscape.

[1] See also Lins et al. (2010), who surveyed CFOs in 29 countries to examine the major reasons for using credit lines.

From an empirical standpoint, loan commitments are extensively used in both corporate and consumer financing. In Figure 1 we show data from the FDIC depicting the amount of all unused loan commitments on the balance sheets of insured US banks for the period 1984-2024. In 2007, on the eve of the subprime crisis, the total exceeded 8 trillion dollars, comprising a substantial portion of total bank loans at the time. The amount of unused loan commitments fell to below 6 trillion in the post-crisis period 2010-2012, climbing back to close to 10 trillion dollars in 2024.[2] Sufi (2009) reports that firms from virtually all major industries employ lines of credit, with the wholesale and retail industries accounting for the highest proportion of firms with lines of credit.

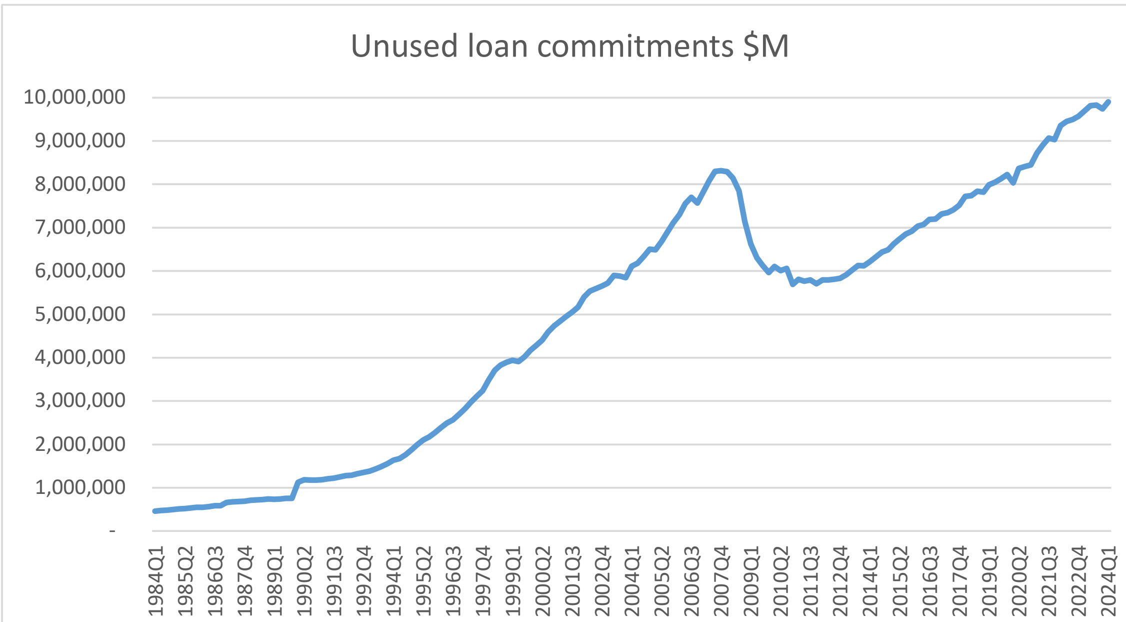


**Figure 1.** Total unused loan commitments issued by US banks, in millions, 1984-2024.

"Unused loan commitments" as defined by the FDIC include credit card lines, home equity lines, commitments to make loans for construction, loans secured by commercial real estate, and unused commitments to originate or purchase loans. (Excluded are commitments after June 2003 for originated mortgage loans held for sale, which are accounted for as derivatives on the balance sheet.)

Source: FDIC, Quarterly Banking Profile.

Corporate lines of credit have been a mainstay of short-term finance for some time. Theoretical studies present alternative explanations for the primary function of credit lines. Campbell (1978), Thakor (1982), James (1982), and Melnik and Plaut (1986) argue that credit lines improve the completeness of financial markets. Boot et al. (1987), Berkovitch and

[2] According to Platt et al. (2020) at the beginning of the Covid crisis the US companies withdrew $124B from credit lines, probably to mitigate the potential liquidity risk.

Greenbaum (1991), Holmstrom and Tirole (1998) and Acharya et al. (2014) argue that lines of credit serve as a means of interest rate or liquidity protection while mitigating moral hazard. Morgan (1993) provides an asymmetric information model in which loan commitments dominate ordinary debt contracts when investment projects have random returns that are costly to observe and random costs that are entirely unobservable. A general review of syndicated loans and their costs can be found in Berg et. al. (2016).

A few studies address the question of pricing loan commitments. Greenbaum et. al. (1989) take into account bank-client relationship and clients' search cost. They determine the loan interest rate policy of a lender who is better informed about a client than other potential lenders. Turnbull (2003) values loans with perfectly foreseen drawdowns using the reduced-form valuation methodology proposed by Jarrow and Turnbull (1995, 1997). Hughston et al. (2002) consider default as a point process the intensity of which depends on both the state of the economy and the unique characteristics of the borrower. They incorporate state-dependent drawdowns in their analysis. The break-even loan spread is such that it equalizes the present value of revenues and the costs of the loan. Loukoianova et al. (2006) model non-committed credit lines as an option on the credit spread combined with a reverse knock-out option. Jones (2001) models credit quality evolution as a jump-diffusion process; process parameters are estimated using monthly corporate bond data for 105 firms. Jones and Wu (2009) examine credit line valuation using the reduced form approach suggested by Duffie and Singleton (1997). They assume that credit quality follows a mixed jump-diffusion process.

Sufi (2009) takes an empirical approach, examining the factors that determine whether firms use credit lines or cash to manage liquidity. He concludes that a loan commitment is a viable liquidity substitute only for firms with high cash flows. Demiroglu and James (2011) also review the evidence on the use of bank loan commitments. They conclude that access to lines of credit does not constitute a perfect substitute for cash, since they are contingent on both the credit quality of the borrower and the financial conditions of the lending institution. Also Kashyap et al. (2002) show the relationship between credit lines and liquid deposits as tools to manage liquidity risk from the bank's perspective.

Cooperman et al. (2023) focus on banks' funding risk and on the effect of the change in the reference rates from LIBOR-based to SOFR-based, where the latter is considered to be a riskless rate and the LIBOR is more credit-sensitive.[3] The point of view in the paper is

[3] LIBOR is the London Interbank Offered Rate and the SOFR is the Secured Overnight Financing Rate. Since 2022, due to Basel Committee instructions, most banks are switching to SOFR as a reference rate.

macro-economic, showing how the choice of reference rate affects the supply of credit lines, concluding that the move to SOFR will lead to heavier drawdowns when the credit spread for the bank rises sharply.

The major cost to credit line borrowers is the interest charged on the drawdown. Interest accrues between payment dates at a fixed contractual spread above the level of a default-free reference rate or at a fixed rate. Hence, credit line contracts include a valuable embedded option for the borrower. When the cost of debt is high, the borrower may use the credit line as a more cost-effective source of capital. The borrower is also charged a fixed fee on the unwithdrawn fund (see examples in Appendix C). These are the potential benefits and costs of credit lines to the borrower.

The majority of published papers model loan commitments based on uncertain market interest rates, or on exogenously assumed process for the firm's credit quality. Our approach is a micro-economic one. In our model, the major source of uncertainty is the value of the levered firm that is purchasing, a loan commitment from a bank, with an option to exercise it at the end of the first period, $T_1$, and repay it at the end of the second period, $T_2$. The starting point is that the firm chooses an initial leverage ratio, $B_0/V_0$, where $B_0$ and $V_0$ denote the value of debt and the value of the firm's assets, respectively (at the initial time zero). The initial assumption is that the loan commitment is for an amount needed to pay at $T_1$, the one-period debt. Hence, the size of the loan commitment is predetermined, and the only question is whether the firm will exercise its option at $T_1$. We show analytically the economic conditions for exercising this option, and as a result, what is the economic value of the option. **The loan commitment in our paper is a tool for short-term liquidity management.**

There are a few issues that we address in a different way from previous papers. The first one is that we fully control for the size of the LC. In previous literature, the amount of debt, $B_1$, is not controlled for, and hence, by exercising the loan commitment, the financial risk of debt and equity may change. The second one is the analytical determination of the decision to exercise the loan commitment option from the corporation's perspective, given the value of the firm at $T_1$, and its expected uncertainty in the second period. We show the economic conditions, that are required for the rational exercise of LC. Third, an analytical solution is developed, using the compound option approach, for the value of the LC is proposed.

The decision of whether to exercise the option is endogenous and based on the (currently uncertain) future value of the firm's assets.[4] The key parameter is the realized future value of the corporation (assets) at the decision point, $V_1$, a value that at present is uncertain and assumed to follow a log-normal process.

In this paper, we propose a two-period contingent claims model for the valuation of loan commitments. This approach allows us to take into account future uncertainty and to derive an analytical closed-form solution for the present value of a loan commitment. This option-like approach to the valuation of credit lines was first introduced by Thakor, Hong and Greenbaum (1981). They price lines of credit as an option when the market interest rate for similar loans is stochastic. We take a similar approach, although we assume that the default-free interest rate is constant and known. The uncertainty we focus on is the riskiness of the assets of the firm, given the desired initial leverage of the firm in the spirit of Merton (1974) and Galai and Masulis (1976) models.[5]

Our approach is unique since the value of the loan commitment is firm-specific. It considers the current leverage of the firm, and also the dynamics of its future leverage and credit risk. The future value of the assets will determine whether the credit line will be used by the firm or it may prefer to raise money directly in the debt market, due to an increase in the market value of the firm and a decrease in its leverage ratio. We endogenize the stochastic credit risk premium of the firm.

We make some simplifying assumptions about the way that loan commitments are used by corporations. This allows us to derive an economic valuation model of the optional elements of loan commitments. We show how banks should price each specific commitment as well as how to determine the materially adverse change (MAC) clause, and how this clause affects the value of the loan commitment. This paper is the first to model MAC and provides tools to determine its economic value. Acharya et al. (2014) are concentrating on the effect of changes in liquidity risk on the use of credit lines versus cash holdings. They derive a binomial model to explain how credit line revocation (i.e. MAC in banking terms) that follows negative profitability or liquidity shocks can be optimal for the bank, as a monitoring tool to reduce the

[4] The model can also be extended for the case of uncertain default-free interest rate as another source of risk. Nevertheless, it will require an additional assumption on the possible correlation between market and assets uncertainty. For the sake of keeping the presentation clear and focused we do not add this source of uncertainty, which is empirically of much less importance compared to the credit spread driven by the uncertainty of the firm's value.

[5] A recent paper by Becker et al. (2024) on callable corporate bonds shows empirically that a major explanatory variable of calling a bond (i.e. exercising the option) is the change in the value of the corporation. An increase in value (everything else is the same) means that the credit quality of the firm has improved and the risk premium has decreased giving an incentive to call the bond and to transfer the added value to shareholders.

incentive of the firm to invest in too risky new projects. Their paper does not derive the value of the credit line revocation option, which is derived in this paper.

## The model

We start with a simple two-period model, with bank providing the loan commitment (LC) to the company. We initially make the following assumptions: the firm at time t=0 has total assets worth $V_0$, and it decides to finance them with a one-period, zero-coupon debt, $B_0$, and equity $S_0=V_0-B_0$. The debt is assumed to mature at $T_1$, with face value $F_1$ (which includes both principal and interest amounts). At maturity, $T_1$, the debt will be refinanced for an additional period $\tau$ (till $T_2=T_1+\tau$).

There are three main options for the company. The first one is to take a loan till $T_1$ and then refinance it for one additional period at the interest rate that corresponds to the creditworthiness of the firm at $T_1$, determined mainly by $V_1$ and $F_1$. The second option is to sign an LC with the bank at the time of taking the initial loan. The valuation of this LC is the key topic of this research. The third option is to take a longer loan from t=0 till $T_2$.

Notice that the third option is not equivalent to the first two. Since we assume that the company is not fully transparent and without a need to take a new loan or to use the LC, it will not reveal its state at $T_1$, the third option will not lead to bankruptcy at $T_1$ even if the value of assets is very low. In equilibrium, this will be taken into account by the bank and will increase the required yield.

The first option is the standard setting of Merton's model. We use the following notations:

$\sigma$ – volatility of the assets V, assumed to be constant in continuous time,

r – the risk-free rate,

$F_1$ – the face value of the debt that must be repaid at $T_1$.

The main equation for determining $F_1$ is given by

$$B_0 = F_1 \cdot e^{-r \cdot T_1} - Put(V_0, F_1, T_1, r, \sigma) \tag{1}$$

This is part of Merton's model that represents debt as a risk-free debt minus a put option on the assets of the firm. Solving this equation numerically relative to $F_1$ determines the terms of the initial debt. Its yield can be calculated as $y_1 = \frac{1}{T_1} ln \frac{B_0}{F_1}$ and the risk-neutral probability of default is given by

$$PD_1 = N(-d_2) = N\left(-\frac{ln\left(\frac{V_0}{F_1}\right)+\left(r-\frac{\sigma^2}{2}\right)\cdot T_1}{\sigma\sqrt{T_1}}\right) \quad (2)$$

If the firm is bankrupt at $T_1$, it will be dissolved and there will be no need to roll over the debt. Otherwise, for every value of $V_1 > F_1$ one can calculate the terms of a recycled debt from $T_1$ to $T_2$ as a function of both $V_1$ and $F_1$.

In the second option, the firm raises the required amount of debt, $B_0$ and in addition, purchases an LC that will help to roll over the debt at $T_1$ at some predetermined rate R. Denote the resulting face value of the second-period debt by $F_2=F_1 \cdot e^{R\tau}$. Specifically, it will work in the following way: If the value of the assets $V_1$ is sufficiently high, that the firm can borrow at $T_1$ at a rate below R, it will forfeit the LC and borrow at the market rate.[6] If the value $V_1$ is below $F_1$, the company is already bankrupt at $T_1$, and the LC can not be used. In addition, we assume that there is a certain area where the MAC clause is applied. This happens when $V_1$ is slightly above but close to the bankruptcy threshold. We introduce a MAC variable $M \geq 1$, such that when

$$F_1 \leq V_1 \leq M \cdot F_1 \quad (3)$$

the bank will refer to the MAC clause and not fulfill its obligation to provide a new loan at the promised yield R. The firm is not bankrupt (but close) and will have to find another way to refinance its debt at probably very high, stressed terms.

In Figure 2 we schematically show the lognormal distribution of the value of $V_1$ and the 4 areas of LC. On the horizontal axes, we show $V_1$, the value of the firm's assets at time $T_1$. The grey area marked by "A" is where the company is in default and the value of its assets is below its debt obligation $F_1$, as a result, the LC cannot be exercised in zone A.

[6] We provide below an analytical solution also in the case when the promised rate is so low that there is no $V_1$ that would lead the firm to abandon the LC.

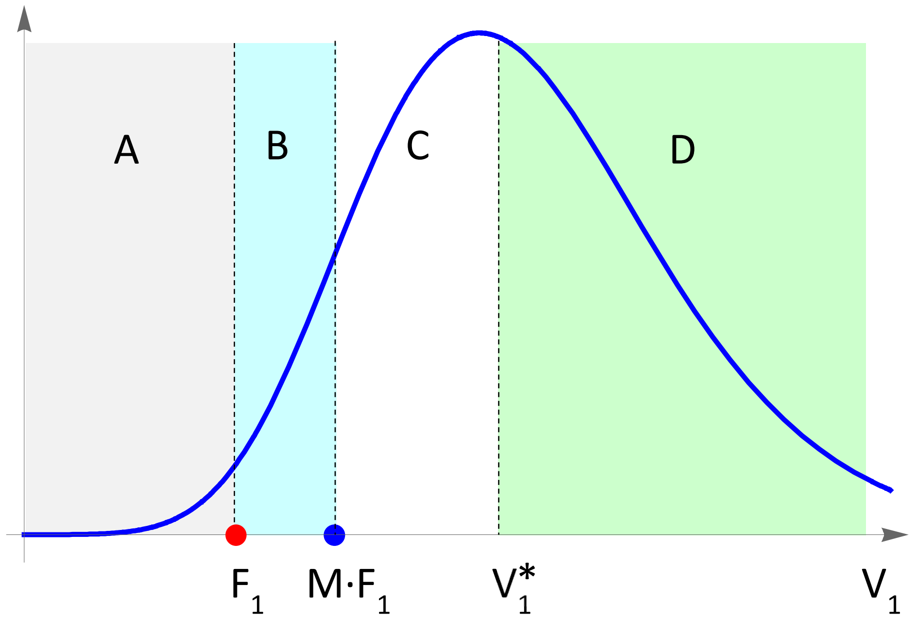


**Figure 2.** The probability distribution of $V_1$ at $T_1$ with the area of bankruptcy (below $F_1$), the MAC area (between $F_1$ and $MF_1$), area of exercise (between $MF_1$ and $V_1^*$), and the area with no voluntary exercise (above $V_1^*$).

The area marked by "B" corresponds to the case when the value of the assets is only slightly above its debt $F_1$ that must be repaid and within the MAC region as defined by equation (1). In this area, between $F_1$ and $M \cdot F_1$ there is no immediate bankruptcy. The LC is very valuable theoretically, but it cannot be exercised due to the MAC clause. The area marked by "C" is when the value of the assets is large enough such that $V_1 > M \cdot F_1$ but below $V_1^*$, which is defined as the value of assets at $T_1$ such that the yield on the new loan of $F_1$ is exactly R (the promised rate). It can be found as a solution of the following equation:

$$F_1 = F_1 e^{(R-r)\tau} - Put(V_1^*, \tau, F_1 e^{R\tau}, \sigma, r) \qquad (4)$$

where $F_2 = F_1 e^{R\tau}$ and the value at time $T_1$ of a commitment to pay $F_2$ at time $T_2$ is exactly $F_1$.

In other words, $V_1^*$, is the value of the assets at $T_1$ that is high enough that it is cheaper to borrow at the market rate rather than to exercise the LC. This area is marked by "D". In this area the LC has a "negative" value and will not be exercised. Specific boundaries between the regions depend on the parameters, and it will be presented below with some illustrative examples. Notice that when $R<r$ equation (4) has no solution. This happens since in this case the promised loan is at a yield lower than the risk-free rate and there is no value of assets $V_1^*$ high enough that can lead to a voluntary abandonment of the LC.

In Figure 3 we show the value of the LC at maturity ($T_1$) as a solid red line as a function of the assets $V_1$. It is defined as the difference between the amount of money provided by the bank as a new loan ($F_1$) and the economic present value of $F_2$, the amount that the company promises to return at $T_2$.

The dashed line depicts the unrestricted value of the LC at $T_1$. It is however unfeasible for $V_1 \leq M \cdot F_1$ and unprofitable when it becomes negative for large values of $V_1$. This figure represents the payoff of LC at its maturity.

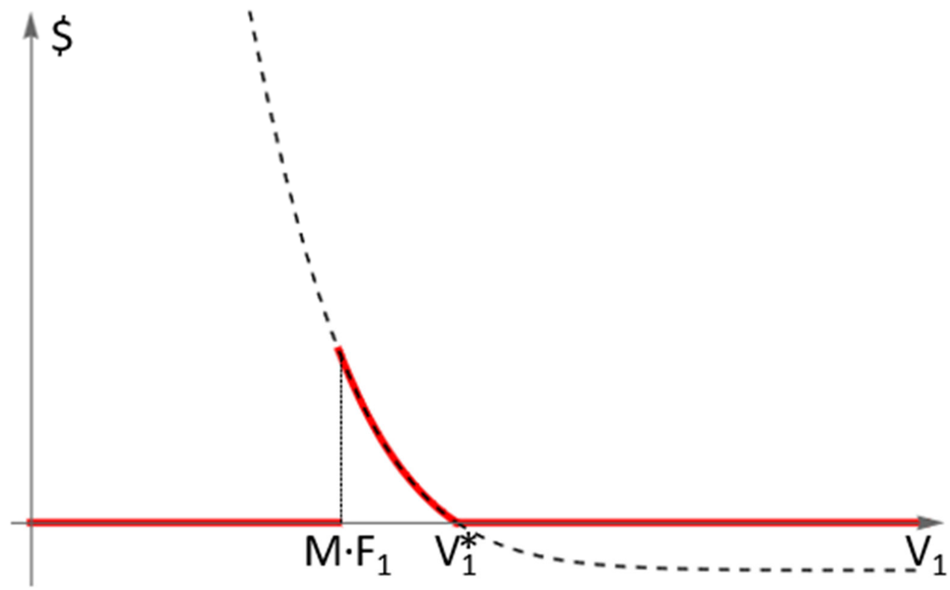


**Figure 3.** Value of LC at maturity, $T_1$, as a function of $V_1$ in the case when R>r.

According to the standard principles of risk-neutral valuation, the price of LC before maturity can be determined by discounting the expected payoff at the risk-free rate. To derive the analytical formula for LC we will use the standard Merton's model of credit risk (1974). According to Meron's model, the value of a corporate bond can be found as the difference between a risk-free bond and a put option on the value of assets, while debt repayment $F_1$ plays the role of the strike price.

$$\text{Bond} = \text{Risk-Free-Debt} - \text{Put(assets)}$$

Figure 4 illustrates the value of the second-period bond before maturity (the solid line) and its payoff at maturity (the dashed line).

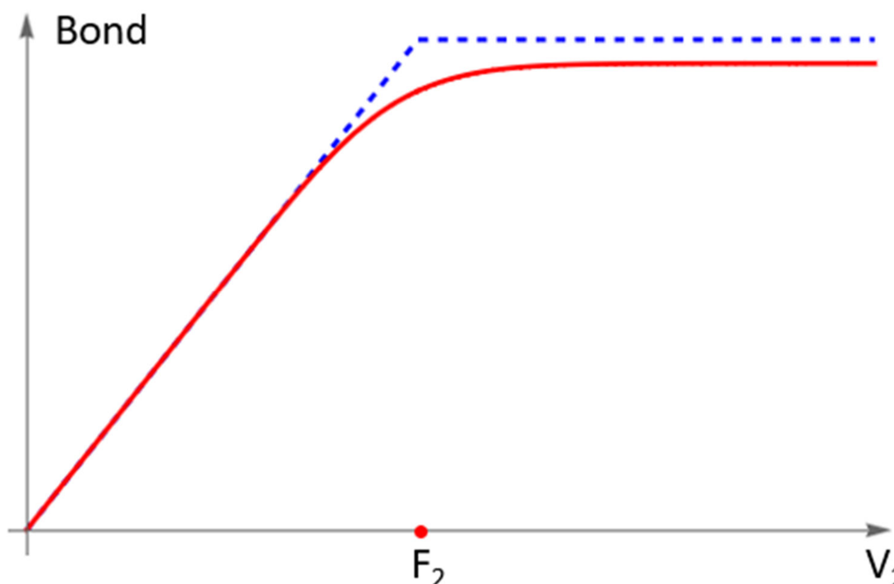


**Figure 4.** Value of a bond before maturity, $T_2$ (the solid red line), and its payoff at maturity (the dashed blue line) as a function of the assets.

LC is similar to an option on the second-period debt (with the additional MAC clause). This allows the firm to get a loan from the bank at terms better than market conditions (otherwise the firm is better off borrowing at a market rate). In other words, the LC allows the firm to sell its debt to the bank at a price set in advance. This is equivalent to a put option on the bond. However, since the bond itself is a combination of a risk-free bond minus a put option on the assets, the LC is equivalent to a call option on the put.

This leads us to the use of the compound option approach developed by Geske (1979) and further extended by Selby and Hodges (1987). Denote the risk-free bond by RFBond, then the payoff of the LC can be written in the following form:

$$\text{Max}[\text{K-Bond}, 0] = \text{Max}[\text{K} - (\text{RFBond} - \text{Put}), 0] = \text{Max}[\text{Put} - (\text{RFBond-K}), 0]$$

The formula on the right-hand side presents the payoff of a Call-on-Put compound option with the strike price equal to the value of the risk-free bond minus $K=F_1$. In contrast with a regular Call-on-Put option, in the case of LC, when $V_1 < M \cdot F_1$, it is not valid anymore and the payoff must be zero. We will achieve it in two steps. First, we consider a similar Call-on-Put option but with a lower strike, so that its value is zero when $V_1 \geq M \cdot F_1$. The left part of Figure 5 presents the payoff of these two Call-on-Put compound options, one with a strike $F_2 \cdot e^{-r\tau} - F_1$ (blue line) and the second one with the strike $\text{Put}(M \cdot F_1, \tau, F_2, \sigma, r)$ (red line). The green line in the right part of Figure 5 depicts a Binary Cash-or-Nothing Put Option with the strike at

$M \cdot F_1$. It gives \$1 if $V_1 < M \cdot F_1$ and zero otherwise. One can see that the combination of these three options gives exactly the same payoff as the original LC.

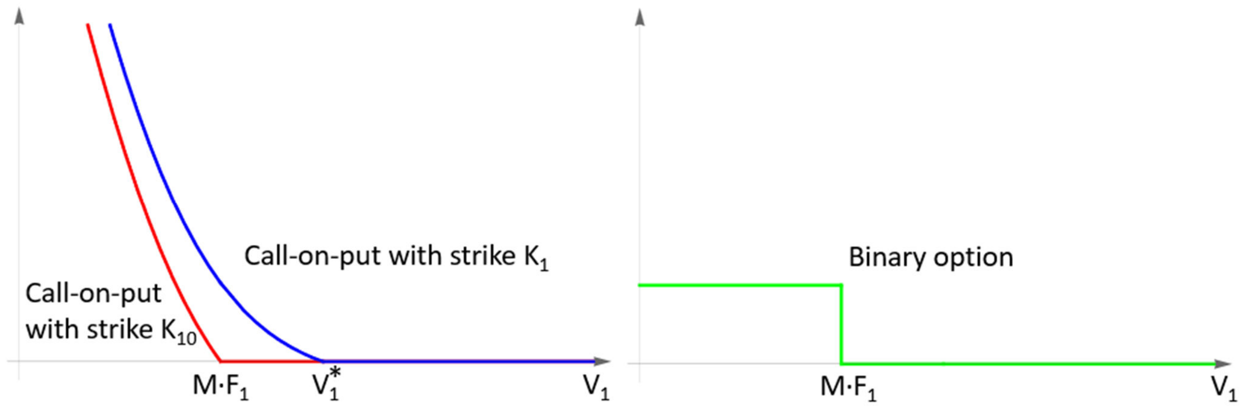


**Figure 5.** The payoff of the two Call-on-Put options with different strikes and a Binary Put option with the strike at $M \cdot F_1$.

Here $K_1 = F_2 \cdot e^{-r\tau} - F_1$ and $K_{10} = Put(M \cdot F_1, \tau, F_2, \sigma, r)$. By subtracting $(K_{10} - K_1)$ units of a binary Cash-or-Nothing option with the strike at $M \cdot F_1$ we get the desired payoff of LC as presented in Figure 6 by the bold black line.

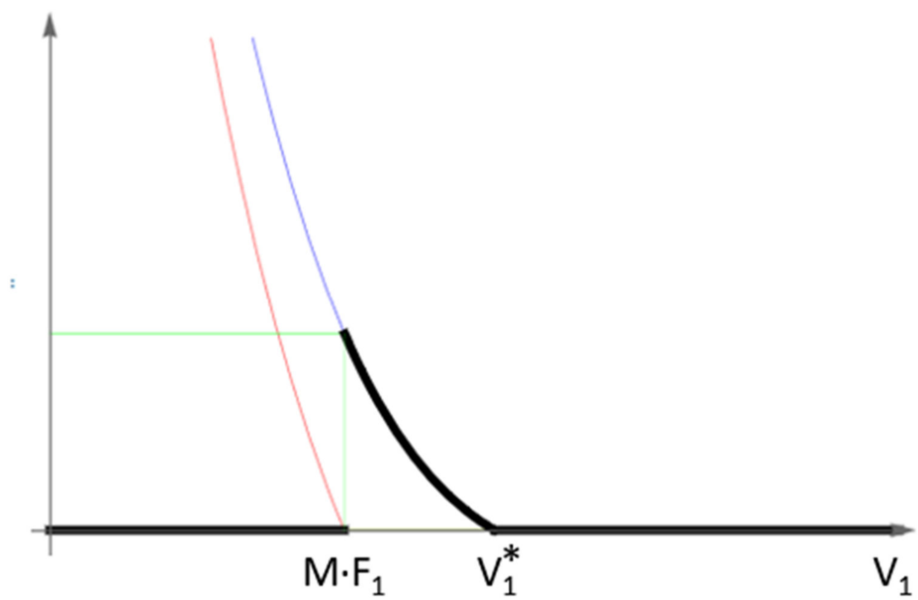


**Figure 6.** LC as a combination of two Call-on-Put options minus a binary Cash-or-Nothing put option.

The final payoff of LC is the same as a combination of the following 3 options.

A. Call-on-Put option with time to maturity of the call $T_1$ and of the put $T_2$. The strike of the call is equal to $F_2 \cdot e^{-r\tau} - F_1$ and the strike of the put is equal to $F_2$.

B. Call-on-Put option with time to maturity of the call $T_1$ and of the put $T_2$. The strike of the call is equal to Put($M \cdot F_1$, $\tau$, $F_2$, $\sigma$, r), and the strike of the put is equal to $F_2$.

C. Put($M \cdot F_1$, $\tau$, $F_2$, $\sigma$, r) – ($F_2 \cdot e^{-r\tau} - F_1$) units of the binary Cash-or-Nothing put option with the strike equal to $M \cdot F_1$.

We use the following notations:

$K_1 = F_2 \cdot e^{-r\tau} - F_1$ the strike of the first Call-on-Put option,

$K_{10}$ = Put($M \cdot F_1$, $\tau$, $F_2$, $\sigma$, r). It is the strike price of the second Call-on-Put option, this is a standard put option with the value of the asset $M \cdot F_1$, strike $F_2$, time to maturity $\tau$, volatility $\sigma$, and the risk-free rate r.

The final analytical formula of the LC is given by

$$LC = CallOnPut(V_0, T_1, K_1, T_2, F_2, \sigma, r) - CallOnPut(V_0, T_1, K_{10}, T_2, F_2, \sigma, r) - (K_{10} - K_1) \cdot BinaryCashOrNothingPut(V_0, T_1, \mathrm{M} \cdot F_1, \sigma, r) \quad (5)$$

In the case of a negative $K_1$ (when the promised rate on LC is below the risk-free rate), it becomes

$$LC = (Put(V_0, T_2, F_2, \sigma, r) - K_1 e^{-rT_1}) - CallOnPut(V_0, T_1, K_{10}, T_2, F_2, \sigma, r) - (K_{10} - K_1) \cdot BinaryCashOrNothingPut(V_0, T_1, \mathrm{M} \cdot F_1, \sigma, r) \quad (5')$$

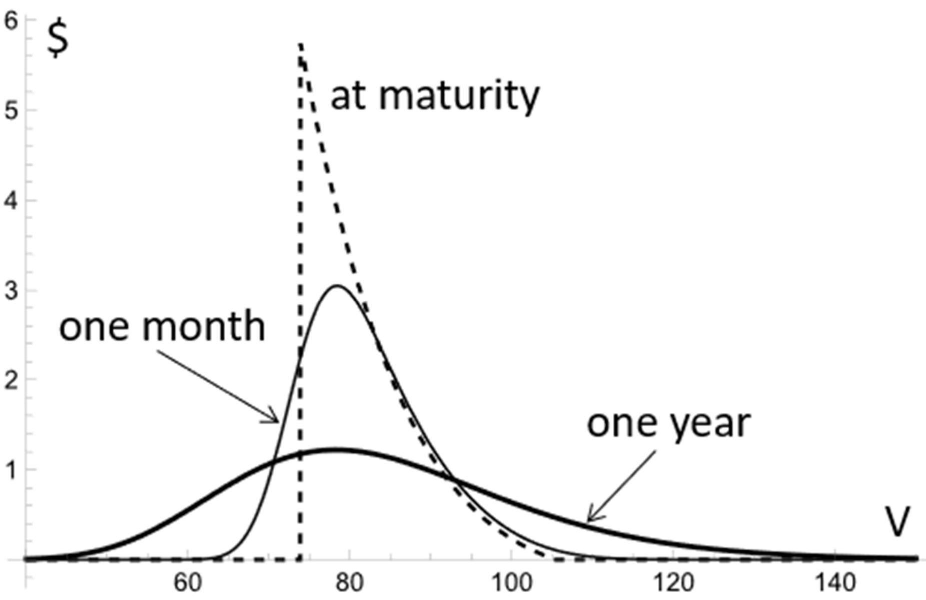


**Figure 7.** Development of LC value with time. The thick line gives the value of LC one year before maturity, the thin line gives the value of LC one month before maturity and the dashed line gives the payoff of LC at its maturity ($T_1$); $F_1$=73.86, yield guaranteed by LC R=5.37%, $\sigma$=20%, r=5%.

In Figure 7 the time value of the LC is described. It shows the value of the LC as a function of the current value of the firm, V, for a one-year LC, one-month LC, and at maturity of the LC. One can see that the time value is positive in some areas while negative in others. This is a direct result of the combination of different options simultaneously, in our compound options model, where we have both call and put options. This also creates a non-monotonicity in the standard deviation of the underlying asset.

## A Numerical Example

Consider the following numerical example:

$V_0$=100,

$B_0$ = 70,

$T_1$ = 1 year,

$T_2$ = 2 years,

r = 5% - risk-free rate for both one and two years horizon

$\sigma$ = 20% - annual volatility of the assets,

And as a result, $F_1$= 73.86, and the yield on the first-year loan is 5.37%.

In Table 1 below we assume that the firm arranges an LC with its bank for a new loan that it will be able to take after the first year for an additional year ($T_2$=2) at the same interest rate as the first-year yield. Table 1 below shows the value of the LC for various levels of leverage and assets' volatility. Notice that different leverage ratios will imply different implied yields and as a result, the guaranteed yield for the second year will be different as well and will reflect the first-year credit risk.

| Leverage\volatility | 15% | 20% | 25% | 30% |
|---|---|---|---|---|
| 60% | $0.04 | $0.24 | $0.58 | $0.93 |
| 70% | $0.25 | $0.64 | $0.98 | $1.23 |
| 80% | $0.62 | $0.88 | $1.00 | $1.05 |
| 90% | $0.53 | $0.50 | $0.46 | $0.41 |

**Table 1.** Value of LC for different leverage ratios and volatilities in the case when the promised yield for the second period R is the same as the implied yield for the first period.

The values in the table can be interpreted as commitment fees, or, as the fixed fees on undrawn loan commitments. It is assumed in our model that the commitments cannot be used before the end of the first period. Hence, in reality the embedded options in the LC are

American rather than European. But since we deal with 2-sided truncation of the distribution, it's not clear whether the American options are more, or, less valuable compared to the European.

Also, there is a tradeoff between the R and the LC; the bank can ask for a higher R in case the LC is exercised, but it must affect the fair value of the LC as demonstrated in Table 3 in the Appendix. For example, for a leverage ratio of 70 and a standard deviation of 20% the fair value of the fixed cost is \$0.64, based on first year interest rate. However, if the promised rate R is 1% above the first-period interest rate, the cost of LC will drop to \$0.44.

First, we notice that the cost of the LC is very much in the "ballpark" for most firms as observed empirically. (Also, the volatility of most non-bank firms is in the range of 15% to 30%). Second, it is noticed that 90% leverage triggers a cost of LC lower than 80% leverage. This is due to the fact that zone A for the former is expanding, which means a higher risk of default after one period and hence greater probability of not exercising the LC. This is also the explanation for why the cost of the LC declines for a leverage ratio of 90% when increasing the riskiness of the assets. Since we have truncations of the distribution of future benefits of the LC to the firm, the value of the LC is not necessarily an increasing function of the standard deviation. The cost is also much affected by the interest rate that is charged initially and assumed to be the basis for the LC.

Table 2 presents the case when the rate promised by LC is fixed at R=6% and does not depend on the rate set for the first-period loan.

| Leverage\volatility | 15% | 20% | 25% | 30% |
|---|---|---|---|---|
| 60% | \$0.01 | \$0.13 | \$0.43 | \$0.87 |
| 70% | \$0.13 | \$0.50 | \$1.01 | \$1.57 |
| 80% | \$0.51 | \$1.07 | \$1.62 | \$2.12 |
| 90% | \$0.96 | \$1.39 | \$1.76 | \$2.06 |

**Table 2.** Value of LC for different leverage ratios and volatilities in the case when the promised yield for the second period is fixed at R=6%.

Comparing Table 2 to Table 1, we see that setting a fixed yield of 6% for the LC, means higher benefit to riskier firms (i.e., higher volatility) and riskier debt (i.e., higher leverage ratios). In Table 2, by assumption, the fixed rate of 6% does not take the credit risk into account.

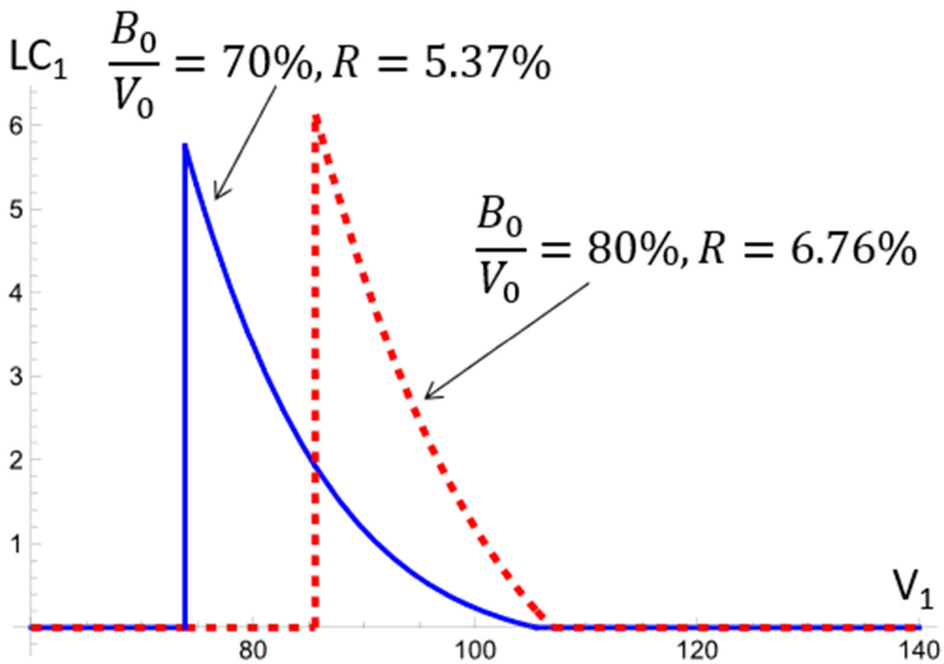


**Figure 8.** Value of LC at $T_1$ as a function of $V_1$ when σ =20% for two leverage ratios. The solid blue line represents the case when the initial leverage is 70% and the promised rate is equal to the rate of the first-period loan 5.37%. The dashed red line is for the case when the initial leverage is 80%, the corresponding initial yield is 6.76% and the LC promised yield is the same.

In Figure 8 we show the value of the LC at $T_1$ as a function of $V_1$ when σ =20% for two leverage ratios, and no effective MAC clause. The solid blue line represents the case when the initial leverage is 70% and the promised rate is equal to the rate of the first-period loan 5.37%. The dashed red line is for the case when the initial leverage is 80%, the corresponding initial yield is 6.76% and the LC promised yield is the same. From Table 1 we learn that the ex-ante economic cost is \$0.64 and \$0.88, per \$100 respectively. Figure 8 shows the ex-post economic cost of providing the LC.

## The Effect of Material Adverse Change Clause on the Model

In many loan commitment agreements, as mentioned above, the bank reserves the right to ignore the LC if the firm's situation materially changes *in the opinion of the bank*. The Material Adverse Change (MAC) clause takes on great importance in real life (see Ergungor (2001)). We have incorporated MAC into our framework by conditioning the LC on the realized value of $V_1$ relative to its debt level $F_1$ at the time of loan maturity $T_1$.

The MAC clause plays an important role in risk mitigation since it allows the bank to fulfill its obligation to provide the loan only if the firm remains solvent and its credit risk does not deteriorate in a major way. We model the MAC clause by introducing a MAC factor (M). When M = 1, we assume that the loan commitment is irrevocable and must be provided if the firm is solvent at $T_1$. A loan commitment of this type is more expensive than with M>1. When M>1, we assume that the loan commitment will be respected by the bank only if $V_1 > M \cdot F_1$. When M >1 we assume that the bank requires a certain cushion in the form of minimal capital in order to extend the new debt.

Figure 9 illustrates the values of LC as a function of M, the MAC term set by the bank, for two cases. The first case is the base case set above and the second one is for enhanced business risk (σ = 30%) and financial risk ($B_0$ = 80). For the base case, when M=1 the value of the LC is 0.64%. It is over 1% for the second, riskier case. It should be noted that for the riskier case a much higher rate R=11.2% is taken into account.

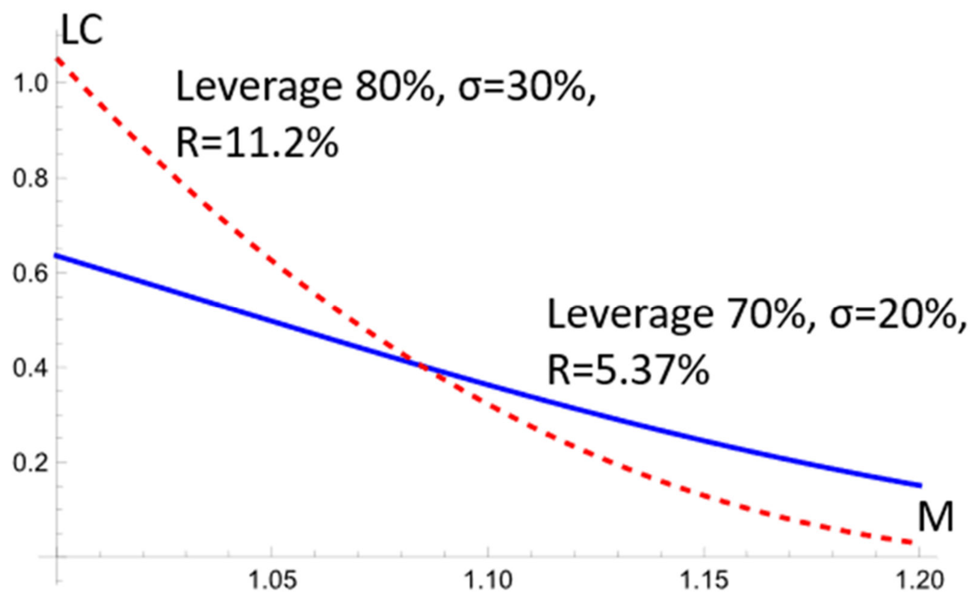


**Figure 9.** Value of LC at $T_0$ as a function of M, when $V_0$=100. The solid line represents the base case with the initial leverage of 70%, σ=20%, and the promised rate is equal to the rate of the first-period loan R=5.37%. The dashed line is for the initial leverage of 80%, σ=30%, both, the initial yield and the promised yield are 11.2%.

From Figure 9 it is clear that the LC is a declining function of M, and the negative slope is steeper the riskier the corporate debt. For higher M the cost of the LC for the second case is below the cost for the base case, due to the fact that the regions for exercising the LC are getting smaller. The probability of exercising the LC is diminishing as a function of M, σ and the leverage ratio.

The relationship between leverage, M, and the cost of LC is depicted in Figure 10. In this figure, the promised rate is set equal to R=6%, so it completely ignores the initial credit risk for each combination of parameters. The functions initially are upward sloping, reach a peak, and then are downward sloping as a function of leverage.

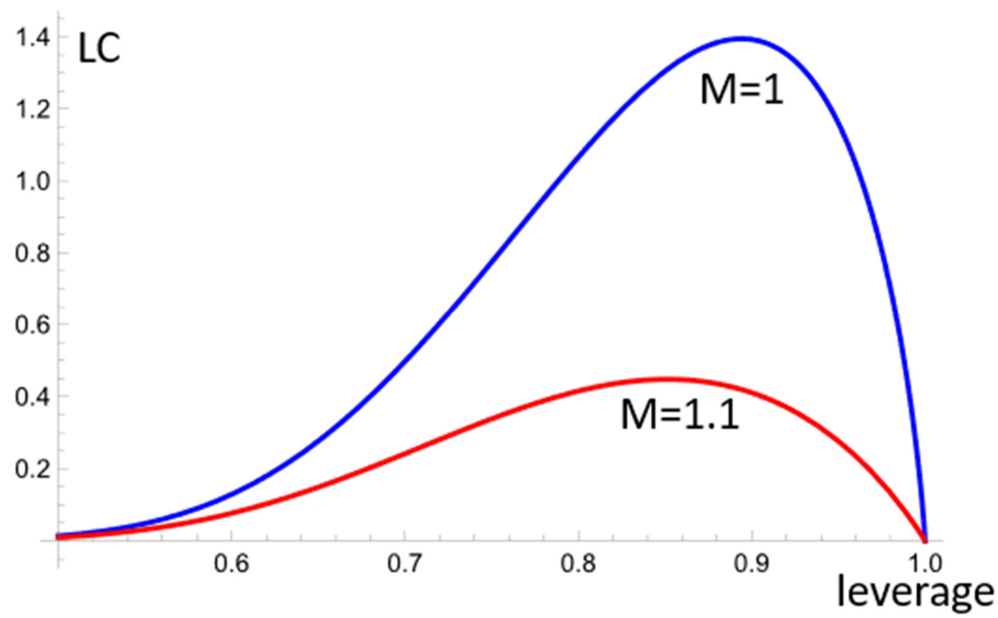


**Figure 10**. Value of LC as a function of the initial leverage. The upper blue line depicts the value of LC without MAC (M=1) and the lower red line is with M=1.1. The other parameters are: $V_0$=100, $T_1$=1, τ=1, yield guaranteed by LC is R=6%, σ=20%, r=5%.

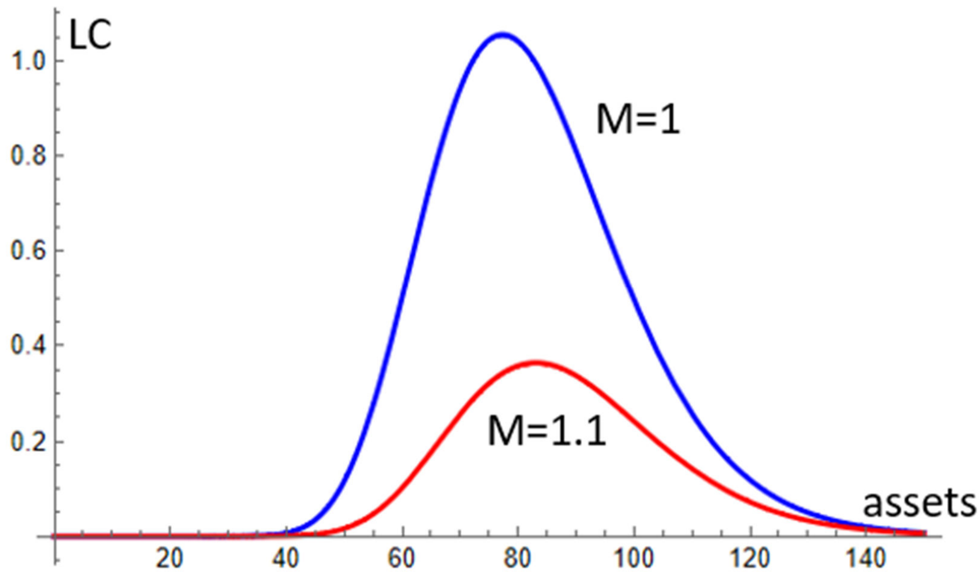


**Figure 11**. The impact of MAC on the value of LC. The upper blue line depicts the value of LC without MAC (M=1) and the lower red line is with M=1.1. The other parameters are: $V_0$=100, $T_1$=1, $\tau$=1, yield guaranteed by LC is R=6%, $\sigma$=20%, r=5%.

Figure 11 shows the relationship between the value of the LC and the current asset, V for the base case M=1, and for the case when MAC is used at 10% above the default value, M=1.1. The shape of the curve is affected by the interaction of the two zones A and D from Figure 2, where the LC=0.

## A Generalized Approach

There is an alternative derivation of the analytical formulas, similar to the approach developed for the valuation of compound options. Applying it directly we get a more general formula that allows using a known term structure of riskless interest rates, while the approach above assumes that r is constant in the two periods.

Denote the risk-free rate at t=0 for time $T_1$ as $r_1$, and the rate for time horizon $T_2$ as $r_2$. The initial value of the assets is $V_0$ and the value at time $T_1$ follows a log-normal process, it depends on a random variable denoted by $z_1$ which is normally distributed in continuous time with zero mean and a constant standard deviation equal to 1.

$$V_1 = V_0 \cdot e^{\left(r_1 - \frac{\sigma^2}{2}\right)T_1 + \sigma \cdot z_1 \sqrt{T_1}} \quad (6)$$

The value of the assets by $T_2$ will become

$$V_2 = V_1 \cdot e^{\left(r_{12} - \frac{\sigma^2}{2}\right)\tau + \sigma \cdot z_2 \sqrt{\tau}} \quad (7)$$

Here $z_2$ is independent of $z_1$ and they both are distributed according to the standard normal distribution with mean zero and standard deviation of one. The forward risk-free (continuously compounded) rate is defined as $r_{12} = \frac{r_2 \cdot T_2 - r_1 \cdot T_1}{T_2 - T_1}$.

Alternatively, the value of assets at $T_2$ can be presented as

$$V_2 = V_0 \cdot e^{\left(r_1 - \frac{\sigma^2}{2}\right)T_1 + \left(r_{12} - \frac{\sigma^2}{2}\right)\tau + \sigma \cdot z_3 \sqrt{T_2}} \tag{8}$$

Note that both variables $z_1$ and $z_3$ are normally distributed, but now they are correlated with the correlation coefficient $\rho = \sqrt{\frac{T_1}{T_2}}$, and $z_3$ can be presented as

$$z_3 = z_1 \sqrt{\frac{T_1}{T_2}} + z_2 \sqrt{\frac{\tau}{T_2}}.$$

The face value of the debt based on the use of the LC is set $F_2 = F_1 \cdot e^{R \cdot \tau}$.

Define $K_1 = F_2 \cdot e^{-r_{12} \cdot \tau} - F_1$. As long as $R > r_{12}$, $K_1$ is positive and we can solve the following equation for $s_1$, which corresponds to the maximal value of assets for which it is profitable to exercise the LC. The solution for a negative $K_1$ appears below.

$$K_1 = Put(s_1, \tau, F_2, \sigma, r_{12})$$

Define also

$$a_1 = -\frac{log\left(\frac{V_0}{s_1}\right) + \left(r_1 + \frac{\sigma^2}{2}\right)T_1}{\sigma\sqrt{T_1}}, \qquad a_2 = a_1 + \sigma\sqrt{T_1},$$

and

$$b_1 = -\frac{log\left(\frac{V_0}{F_2}\right) + \left(r_2 + \frac{\sigma^2}{2}\right)T_2}{\sigma\sqrt{T_2}}, \qquad b_2 = b_1 + \sigma\sqrt{T_2},$$

$$s_{10} = M \cdot F_1,$$

$$K_{10} = Put(s_{10}, \tau, F_2, \sigma, r_{12}),$$

$$a_{10} = -\frac{log\left(\frac{V_0}{s_{10}}\right) + \left(r_1 + \frac{\sigma^2}{2}\right)T_1}{\sigma\sqrt{T_1}}, \qquad a_{20} = a_{10} + \sigma\sqrt{T_1}.$$

We use N(x) to denote the standard normal cumulative distribution function and N(x,y,ρ) for a two-dimensional normal cumulative distribution with correlation ρ. Then the analytical value of LC is given by

$$LC = -V_0 \cdot \big(N(a_1, b_1, \rho) - N(a_{10}, b_1, \rho)\big) + F_2 \cdot e^{-r_2 \cdot T_2}\big(N(a_2, b_2, \rho) - N(a_{20}, b_2, \rho)\big) - K_1 \cdot e^{-r_1 \cdot T_1}\big(N(a_2) - N(a_{20})\big) \tag{9}$$

In the case of a negative $K_1$ the formula becomes

$$LC = -V_0 \cdot \big(N(b_1) - N(a_{10}, b_1, \rho)\big) + F_2 \cdot e^{-r_2 \cdot T_2}\big(N(b_2) - N(a_{20}, b_2, \rho)\big) - K_1 \cdot e^{-r_1 \cdot T_1}\big(1 - N(a_{20})\big) \tag{9'}$$

This formula coincides with equation (5) when $r_1=r_2$ but it is a generalization for the case with a term structure of risk-free interest rates. The derivation of this formula is provided in Appendix A.

This formula can be further generalized for a non-stochastic time dependence of r(t) and σ(t) following the lines developed in Elettra and Rossella (2003) by taking the integral of the forward interest rates and of the variance over time.

In Table 3 we present the values of LC in the case when $r_2$ is different from $r_1$ using the same basic case as an example ($V_0$=100, $B_0$ = 70, $T_1$ = 1 year, $r_1$ = 5%, σ = 20%). As before the first-period loan has a face value, $F_1$= 73.86, and its yield is 5.37%. Set $T_2$ = 2 years, M=1 (no MAC) and R=5.37% (promised yield for the LC equal to the yield of the first-period loan). As one can see the impact of the second-period risk-free rate is significant. Notice that the two right columns correspond to the case when the promised rate is below the forward risk-free rate and as a result the LC is very valuable.

| $r_2$ | 4% | 4.5% | 5% | 5.5% | 6% |
|---|---|---|---|---|---|
| Value of LC | 0.33 | 0.44 | 0.64 | 1.15 | 1.75 |

**Table 3.** Value of LC for different leverage $r_2$ assuming the promised yield is equal to the yield of the first-period loan.

When the LC covers only a part of the debt due at $T_1$, the valuation should be adjusted. There is no analytical formula for this case and one can use Monte Carlo simulation to derive the risk-neutral valuation of LC. More explanations for this case are provided in Appendix B. The analytical case above corresponds to α=1 and α=0 which means there is no LC at all, while Monte Carlo approach will be applied for α greater than 0 and smaller than 1.

## Conclusions

We model the LC as a contingent claim on the future value of the firm. It is the first time an analytical solution is proposed, as a special case of a compound option. The bank providing the LC is exposed on one hand to the current risk that the borrower will default on its obligation to pay back the loan, and to a potential increased credit risk of the firm in the future on the other hand. The lender takes into account that the LC will be exercised if the interest rate the borrower will have to pay on a similar loan in the market is higher than the LC rate. So, in our model, the firm purchasing the LC is hedging its credit risk. The bank often tries to cap its exposure to the borrower's credit risk by introducing the MAC. The

MAC mitigates the risk of the bank and may expose the firm to its credit risk when it is at its peak.

Most authors, look at LC as a hedge against adverse change in **market interest rates**, while we focus on LC as a hedge against the firm's **credit risk**. In other words, the initial loan is given to the firm at the yield that equals the risk-free rate and its current spread. Previous literature focused on the risk-free rate as the main source of uncertainty, while our approach assumes that the major source of uncertainty (and the option value) comes from the credit spread. The LC is effectively a hedge against the next period spread, which is uncertain at present.

We offer an analytical solution for pricing the LC, using the compound option approach of Geske, within the contingent claim approach of Merton. It is the first paper to offer a solution that is utility-free.

The bank charges the potential borrower a fixed commitment fee for an unwithdrawn credit line. In the example of J.C. Penney Company (see Appendix C) the commitment fee was 0.375% per annum. This fee is the only fee for unused credit line, and there is another fee for using the credit line. In Appendix C we include additional contracts.

It is straightforward to introduce uncertainty concerning the future risk-free rate. Then, we can have two sources of uncertainty: default-free rate and credit risk. It will change the numerical results of the model but not the qualitative results presented in the paper. The major factor in extending the model is the correlation between default-free interest rate and the firm's credit risk.

Future research on loan commitments should also look at the counter-party credit risk. Current models ignore the credit riskiness of the potential lender and its ability to supply the needed credit upon demand during a liquidity crisis or when the lender faces distress. The basic assumption in most of these models is that the issuer is risk-free. However, such models do not fully capture the clustering in default correlations, sometimes referred to as "credit contagion".

Credit contagion stood at the heart of the financial crisis that started to evolve in 2007. In 2008, Federal Reserve Chairman Bernanke justified the rescue of the investment bank, Bear Stearns by explaining that "the company's failure could also have cast doubt on the financial positions of some of Bear Stearns' thousands of counterparties and perhaps of companies with similar businesses."

Second-generation models attempt to provide structural explanations for default clustering. For instance, Duffie et al. (2009) developed a "frailty" model, in which defaults are driven by a time-varying latent variable, which partially explains the observed default clustering. Another extension would be to consider a multiple-factor effect or industry factors. When a firm defaults, other firms in the same industry could suffer from contagion, reflecting shocks to cash flows that are common in that industry, see Lang and Stulz (1992) and Jorion and Zhang (2007).

Acharya et al. (2020) claim that banks grant LC and become exposed to the liquidity risk of the individual firms. They investigate how covenant violations offer banks to renegotiate the terms of the credit lines. Our paper focuses on the cost of LCs as a function of the credit risk of the firm, and the risk mitigation of the banks by limiting the supply of LCs by imposing the MAC. Acharya et al. (2020) provide empirical evidence to support their claims; this evidence from the US during the GFC is also fully consistent with our theoretical model of LCs.

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# Appendix A

## Analytical formula

The value of LC can be written as the discounted expected value of its risk-neutral distribution at maturity of LC, that is $T_1$. Define a normally (mean zero and standard deviation one) distributed variable $z_1$. Then we can write

$$V_1 = V_0 \cdot e^{\left(r_1 - \frac{\sigma^2}{2}\right)T_1 + \sigma \cdot z_1 \sqrt{T_1}}$$

Each value of $z_1$ leads to some value of the assets and the firm must decide whether to exercise the LC or not.

First, define $a_{20}$ as a value of $z_1$ that leads by $T_1$ to the assets to be equal exactly $M \cdot F_1$ by solving the following equation

$$V_0 \cdot e^{\left(r_1 - \frac{\sigma^2}{2}\right)T_1 + \sigma \sqrt{T_1} z_1} = MF_1$$

Its solution is

$$a_{20} = -\frac{ln\left(\frac{V_0}{F_1 \cdot M}\right) + \left(r_1 - \frac{\sigma^2}{2}\right)T_1}{\sigma\sqrt{T_1}}.$$

As long as $z_1 \leq a_{20}$, the LC has zero value at $T_1$ due to the MAC clause.

Second, define $a_2$ as a value of $z_1$ that leads to the assets $V_1$ to be so high that the firm will prefer a free market loan instead of the rate R, guaranteed by the LC. Denote the corresponding value of assets at $T_1$ by $s_1$. In this case the value of a new debt is given by

$$F_2 e^{-r_{12}\tau} - Put(s_1, F_1 e^{R\tau}, r_{12}, \sigma) = F_1$$

Or using $F_2 = F_1 e^{R\tau}$, we can write it as

$$F_1\left(e^{(R - r_{12})\tau} - 1\right) = Put(s_1, F_1 e^{R\tau}, r_{12}, \sigma),$$

since the right-hand side is a monotonic function of $s_1$, it has a unique solution (as long as $R > r_{12}$).[7] Denote by $a_2$ the value of $z_1$ that leads to the value of assets to be equal $s_1$ by $T_1$ by solving the following equation

$$V_0 \cdot e^{\left(r_1 - \frac{\sigma^2}{2}\right)T_1 + \sigma \sqrt{T_1} a_2} = s_1$$

Its solution is

$$a_2 = -\frac{ln\left(\frac{V_0}{s_1}\right) + \left(r_1 - \frac{\sigma^2}{2}\right)T_1}{\sigma\sqrt{T_1}}.$$

[7] The case of $R < r_{12}$ leads to the situation when $K_1$ is negative. The meaning is that the promised yield is below the forward risk-free rate and the firm will be willing to exercise the LC for all values of assets at $T_1$ ($V_1^*$ does not exist). This corresponds to the case when the equation for $s_1$ has no solution and all three $s_1$, $a_1$, and $a_2$ are at Infinity.

As long as $z_1 \geq a_2$, the LC has zero value at $T_1$ since it is cheaper to take a free loan rather than using the LC that guarantees rate R.

The value of LC is equal to the discounted expected value of its payoff under risk-neutral distribution. At the maturity of LC ($T_1$) the firm will receive the amount of $F_1$ and give in exchange an obligation to pay $F_2=F_1 \cdot e^{R\tau}$ at $T_2$. The discounted expected payoff of LC is thus equal

$$LC = e^{-r_1 T_1} \int_{a_{20}}^{a_2} \left( F_1 - Bond\left( V_0 e^{\left(r_1 - \frac{\sigma^2}{2}\right)T_1 + \sigma \cdot z_1 \sqrt{T_1}}, F_1 e^{R\tau}, \tau \right)\right) \frac{e^{-\frac{z_1^2}{2}}}{\sqrt{2\pi}} dz_1$$

To shorten notations we will use $\varphi(z)$ for normal density function (mean zero, standard deviation one) and $\varphi(z,u)$ for a two dimensional normal density with correlation ρ.

$$\varphi(z) = \frac{e^{-\frac{z^2}{2}}}{\sqrt{2\pi}}, \text{ and } \varphi(z,u) = \frac{e^{-\frac{z^2 - 2\rho z u + u^2}{2(1-\rho^2)}}}{2\pi\sqrt{1-\rho^2}}$$

Using Merton's model the value of LC can be written as

$$LC = e^{-r_1 T_1} \int_{a_{20}}^{a_2} \left( F_1 - F_1 e^{(R - r_{12})\tau} + Put\left( V_0 e^{\left(r_1 - \frac{\sigma^2}{2}\right)T_1 + \sigma \cdot z_1 \sqrt{T_1}}, F_1 e^{R\tau}, \tau, r_{12}, \sigma \right)\right) \varphi(z_1) dz_1$$

Or alternatively

$$LC = F_1 e^{-r_1 T_1}\left(1 - e^{(R - r_{12})\tau}\right)\left(N(a_2) - N(a_{20})\right) + e^{-r_1 T_1} \int_{a_{20}}^{a_2} Put\left( V_0 e^{\left(r_1 - \frac{\sigma^2}{2}\right)T_1 + \sigma \cdot z_1 \sqrt{T_1}}, F_1 e^{R\tau}, \tau, r_{12}, \sigma \right) \varphi(z_1) dz_1$$

The value of the last term (the integral) can be again calculated based on the risk-neutral approach.

Introduce a new variable $z_2$ that is in charge of uncertainty during the second period. Then

$$V_2 = V_1 \cdot e^{\left(r_{12} - \frac{\sigma^2}{2}\right)\tau + \sigma \cdot z_2 \sqrt{\tau}}$$

Or by using a new variable $z_3 = z_1\sqrt{\frac{T_1}{T_2}} + z_2\sqrt{\frac{\tau}{T_2}}$, it can be also written as

$$V_2 = V_0 \cdot e^{\left(r_1 - \frac{\sigma^2}{2}\right)T_1 + \left(r_{12} - \frac{\sigma^2}{2}\right)\tau + \sigma \cdot z_3 \sqrt{T_2}}$$

Note that $z_1$ and $z_3$ are not independent but have a correlation $\rho = \sqrt{\frac{T_1}{T_2}}$. Using these two variables we can describe the payoff of LC on the plane $z_1$, $z_3$ which represents all possible outcomes.

The joint distribution of these two variables is shown in the following Figure 12. Due to the correlation, the level curves are ovals and not circles. We assumed here $T_1$=1 and $T_2$=2.

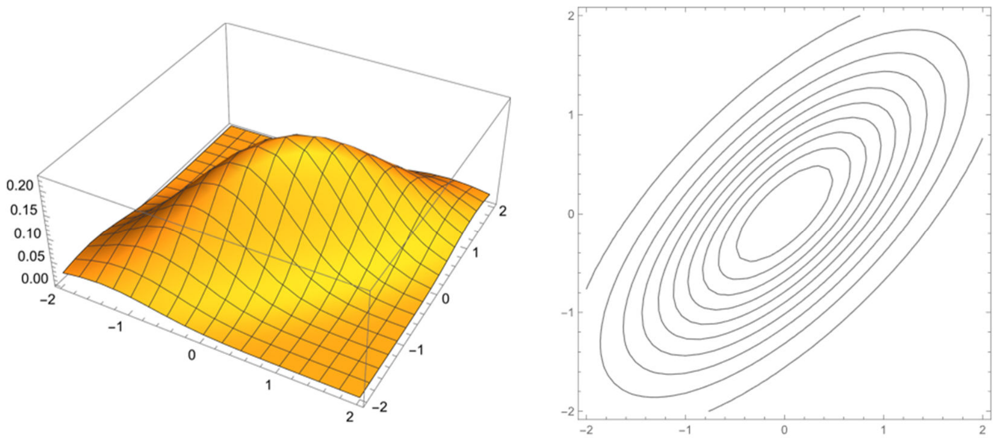

**Figure 12.** The joint distribution of $z_1$ and $z_3$ shown as a probability density function (on the left) and contour levels (on the right).

Each point on the $z_1$, $z_3$ plane corresponds to a certain scenario and leads to a certain LC value.

Notice that the payoff of the Put option depends on $z_3$ but it does not depend on $z_1$ (as long as $a_{20} < z_1 < a_2$). Then we can define $b_2$ as insolvency point at $T_2$

$$V_0 e^{\left(r_2 - \frac{\sigma^2}{2}\right)T_2 + \sigma \cdot b_2 \sqrt{T_2}} - F_1 e^{R\tau} = 0$$

or

$$b_2 = -\frac{ln\left(\frac{V_0}{F_1 e^{R\tau}}\right) + \left(r_2 - \frac{\sigma^2}{2}\right)T_2}{\sigma\sqrt{T_2}}$$

With this notation the integral of a Put option can be written as

$$e^{-r_1 T_1} \int_{a_{20}}^{a_2} Put\left(V_0 e^{\left(r_1 - \frac{\sigma^2}{2}\right)T_1 + \sigma \cdot z_1 \sqrt{T_1}}, F_1 e^{R\tau}, \tau, r_{12}, \sigma\right) \varphi(z_1) dz_1$$

$$= e^{-r_2 T_2} \int_{a_{20}}^{a_2} \int_{-\infty}^{b_2} \left(F_1 e^{R\tau} - V_0 e^{\left(r_2 - \frac{\sigma^2}{2}\right)T_2 + \sigma \cdot z_3 \sqrt{T_2}}\right) \varphi(z_1, z_3) dz_3 dz_1$$

$$= F_1 e^{R\tau} e^{-r_2 T_2} \int_{a_{20}}^{a_2} \int_{-\infty}^{b_2} \varphi(z_1, z_3) dz_3 dz_1$$

$$- V_0 e^{\left(r_2 - \frac{\sigma^2}{2}\right)T_2} e^{-r_2 T_2} \int_{a_{20}}^{a_2} \int_{-\infty}^{b_2} e^{\sigma \cdot z_3 \sqrt{T_2}} \varphi(z_1, z_3) dz_3 dz_1$$

This is a sum of two terms. The first term is equal to

$$F_1 e^{R\tau - r_2 T_2}\left(N(a_2, b_2, \rho) - N(a_{20}, b_2, \rho)\right)$$

The second term can be calculated using a linear transformation of variables in order to account for the exponent under the double integral and the new variables (similar to $d_1$ and $d_2$ in the standard Black-Scholes formula).

Figure 13 below presents the area over which the integration takes place. Notice that the function under the double integral depends on $z_3$ but it does not depend on $z_1$ due to the choice of correlated variables.

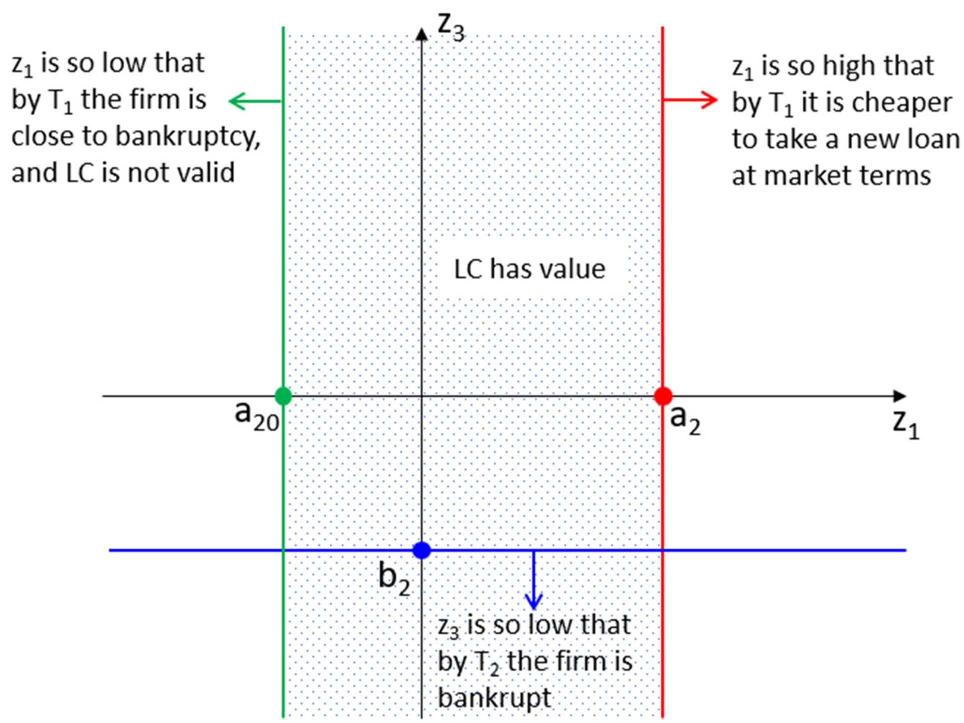


**Figure 13.** The area of double integration.

Define

$$a_1 = a_2 - \sigma\sqrt{T_1}$$

$$a_{10} = a_{20} - \sigma\sqrt{T_1}$$

$$b_1 = b_2 - \sigma\sqrt{T_2}$$

Use a linear transformation of variables and define

$z_1 = u + \sigma\sqrt{T_1}$ and $z_3 = v + \sigma\sqrt{T_2}$ , we will also use $p = \sigma\sqrt{T_1}$ and $q = \sigma\sqrt{T_2}$ to shorten formulas.

Notice also that $\rho = \sqrt{\frac{T_1}{T_2}} = \frac{p}{q}$ in the new variables and $\varphi(z_1, z_3) = \frac{e^{-\frac{z_1^2 - 2\rho z_1 z_3 + z_3^2}{2(1-\rho^2)}}}{2\pi\sqrt{1-\rho^2}}$.

Substituting the new variables, we have

$$\frac{z_1^2 - 2\rho z_1 z_3 + z_3^2}{2(1-\rho^2)} = \frac{u^2 + 2up + p^2 - 2\rho(u+p)(v+q) + v^2 + 2vq + q^2}{2(1-\rho^2)}$$

$$= \frac{u^2 - 2\rho uv + v^2}{2(1-\rho^2)} + \frac{2up - 2\rho(uq + vp) + 2vq}{2(1-\rho^2)} + \frac{p^2 - 2\rho pq + q^2}{2(1-\rho^2)}$$

$$= \frac{u^2 - 2\rho uv + v^2}{2(1-\rho^2)} + \frac{2up - 2\left(up + v\frac{p^2}{q}\right) + 2vq}{2\frac{q^2 - p^2}{q^2}} + \frac{p^2 - 2p^2 + q^2}{2\frac{q^2 - p^2}{q^2}}$$

$$= \frac{u^2 - 2\rho uv + v^2}{2(1-\rho^2)} + \frac{2v\left(q - \frac{p^2}{q}\right)}{2\frac{q^2 - p^2}{q^2}} + \frac{q^2 - p^2}{2\frac{q^2 - p^2}{q^2}} = \frac{u^2 - 2\rho uv + v^2}{2(1-\rho^2)} + vq + \frac{q^2}{2}$$

And the double integral can be written as

$$-V_0 e^{-\frac{\sigma^2}{2}T_2} \int_{a_{20}}^{a_2} \int_{-\infty}^{b_2} e^{\sigma \cdot z_3\sqrt{T_2}} \varphi(z_1, z_3) dz_3 dz_1 =$$

$$-V_0 e^{-\frac{q^2}{2}} \int_{a_{20}-p}^{a_2-p} \int_{-\infty}^{b_2-q} e^{vq+q^2} \varphi(u,v) e^{-vq-\frac{q^2}{2}} dv du =$$

$$-V_0 \int_{a_{10}}^{a_1} \int_{-\infty}^{b_1} \varphi(u,v) dv du = -V_0\big(N(a_1, b_1, \rho) - N(a_{10}, b_1, \rho)\big)$$

The final analytical formula for LC is:

$$LC = -V_0\big(N(a_1, b_1, \rho) - N(a_{10}, b_1, \rho)\big) + F_1 e^{R\tau - r_2 T_2}\big(N(a_2, b_2, \rho) - N(a_{20}, b_2, \rho)\big)$$
$$+F_1(e^{-r_1 T_1} - e^{R\tau - r_2 T_2})(N(a_2) - N(a_{20}))$$

In the case when $R \leq r_{12}$ the formula takes the following form:

$$LC = -V_0\big(N(b_1) - N(a_{10}, b_1, \rho)\big) + F_1 e^{R\tau - r_2 T_2}\big(N(b_2) - N(a_{20}, b_2, \rho)\big)$$
$$+ F_1(e^{-r_1 T_1} - e^{R\tau - r_2 T_2})(1 - N(a_{20}))$$

# Appendix B

## Partial LC

In the case when only a portion of the old debt is covered by LC, the area of integration becomes more complex. Now at the end of the first period, when there is a need to repay the initial debt and to raise $F_1$, only part of it (denoted by $\alpha$) is covered by the LC. The rest $(1-\alpha)F_1$ must be covered by a new debt raised in a free market.

To determine the face value of the new market based debt one should solve the following equation

$$(1-\alpha)F_1 = Xe^{-r\tau} - \frac{X}{X+\alpha F_2} Put(V_1, \tau, X+\alpha F_2, r, \sigma)$$

Here X is the face value of the new debt that will cover the missing part of $(1-\alpha)F_1$. X is an explicit function of $V_1$ and the following graph in Figure 14, assuming $V_0=100$, $B_0=70$, $r=5\%$, $R=6\%$, $T_1=1$, $T_2=2$, $\sigma=20\%$, $\alpha=0.5$.

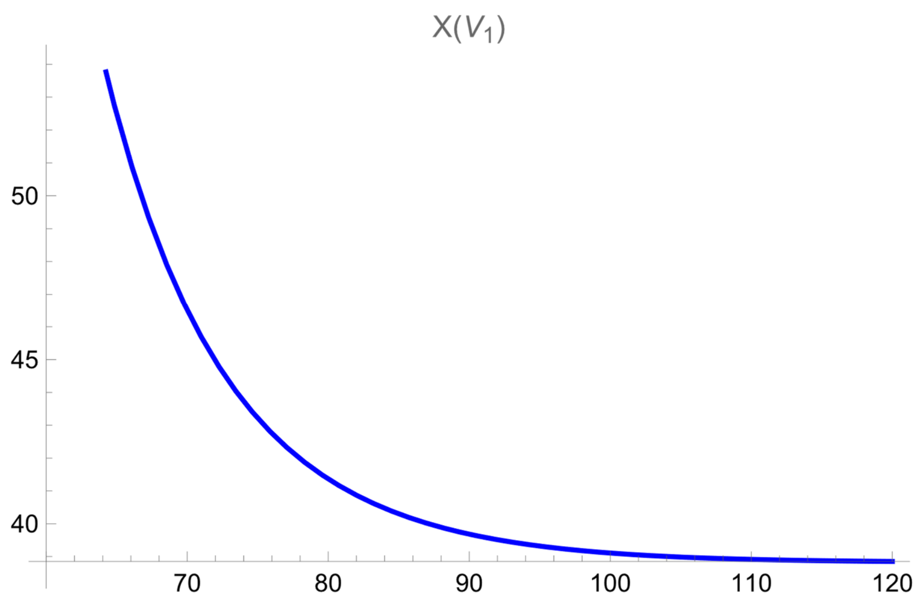


**Figure 14.** Face value of a new debt in the case of a partial LC.

In the case of a partial LC the new debt depends on the value of firm's assets at $T_1$ and the higher the value of $V_1$ the smaller will be the amount that it promises to repay at $T_2$ (in other words the required yield on the loan will be lower). As a result, the integration area will be as shown in Figure 15. Here $a_2$ and $a_{20}$ still provide boundaries for LC to have a positive value. However, since the size of the loan depends on the realization of $z_1$, now the final payoff also depends on $z_1$ (and not only on $z_3$ as in the previous case). The boundary that was flat at $b_2$ in the previous case becomes curved as shown in Figure 15 below.

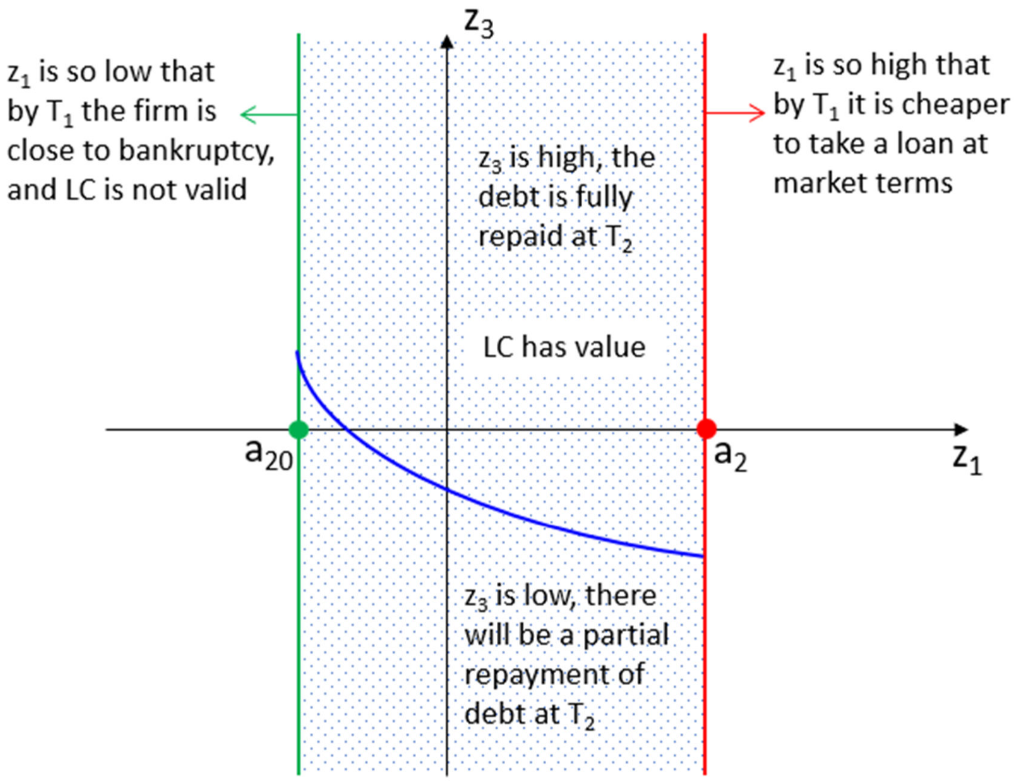


**Figure 15.** The integration area in the case of a partial LC.

We do not think there is a general analytical formula in the case of a partial LC, however, a Monte Carlo simulation can be used to price it. The key disadvantage of Monte Carlo simulation is a slow speed of convergence (as $1/\sqrt{n}$). This can be significantly improved by using a variance reduction technique based on the analytical formula developed in this paper.

Denote by α the portion of debt that is covered by the LC. The analytical case above corresponds to α=1 and α=0 means there is no LC at all. In Figure 15 below we present the values of LC derived for 4 values of LC with α equals 1, 0.75, 0.5, and 0.25, when the other parameters are given by $F_1$= 73.86, $T_1$ = 1 year, $T_2$ = 2 years, $r_1 = r_2$ = 5%, σ = 20%, R = 6%, M=1.1.

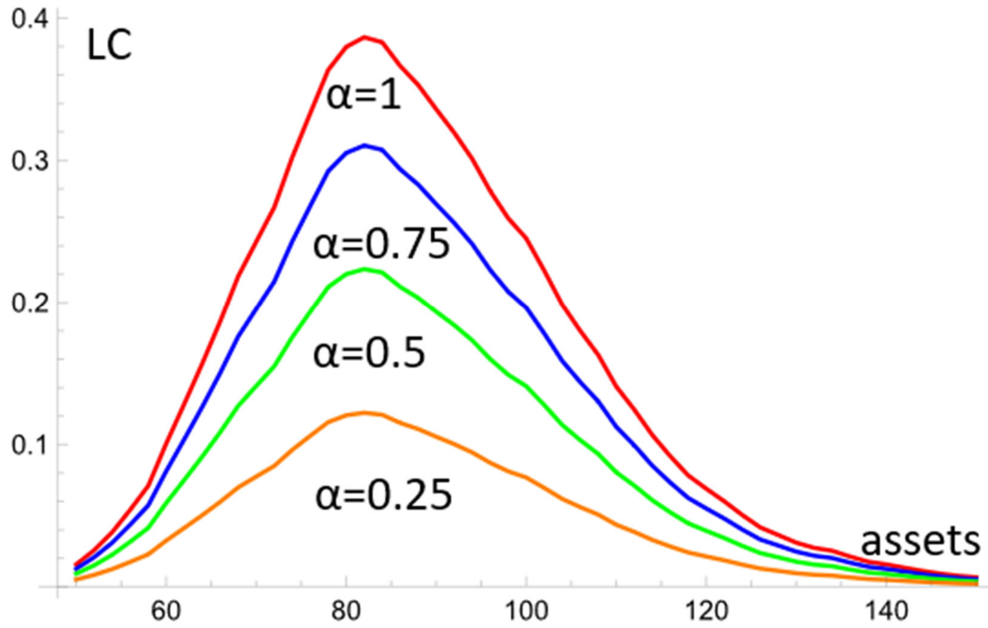


**Figure 15.** The value at t=0 of a partial LC with α having values of 1, 0.75, 0.5, and 0.25.

The value of LC declines with α, but it is not linear and the value of LC with α=0.5 is significantly higher than half of the value with α=1. The rationale for this is that the LC is treated so far in this paper as a free option to the firm since the cost of LC is not internalized yet. The LC evaluated above, measures the present value of its benefits to the firm. The bank, of course, charges a fee for providing the LC.

# Appendix C

## Example of loan commitment financing – J.C. Penney Company

**11. Credit Facility**

The Company has a $2,350 million senior secured asset-based credit facility (2014 Credit Facility), comprised of a $2,350 million revolving line of credit (Revolving Facility). During 2015, the Company amended the 2014 Credit Facility to increase the Revolving Facility from $1,850 million to $2,350 million , and in connection with upsizing the Revolving Facility, the Company prepaid and retired the $494 million outstanding principal amount of the $500 million term loan under the 2014 Credit Facility. The 2014 Credit Facility matures on June 20, 2019 .

The 2014 Credit Facility is secured by a perfected first-priority security interest in substantially all of our eligible credit card receivables, accounts receivable and inventory. The Revolving Facility is available for general corporate purposes, including the issuance of letters of credit. Pricing under the Revolving Facility is tiered based on our utilization under the line of credit. JCP's obligations under the 2014 Credit Facility are guaranteed by J. C. Penney Company, Inc.

The borrowing base under the Revolving Facility is limited to a maximum of 85% of eligible accounts receivable, plus 90% of eligible credit card receivables, plus 90% of the liquidation value of our inventory, net of certain reserves. Letters of credit reduce the amount available to borrow by their face value. In addition, the maximum availability is limited by a minimum excess availability threshold which is the lesser of 10% of the borrowing base or $200 million, subject to a minimum threshold requirement of $150 million.

As of the end of 2015 , we had no borrowings outstanding under the Revolving Facility. In addition, as of the end of 2015 , we had $1,848 million available for borrowing, of which $280 million was reserved for outstanding standby and import letters of credit, none of which have been drawn on, leaving $1,568 million for future borrowings. The applicable rate for standby and import letters of credit was 2.50% and 1.25% , respectively, while the required commitment fee was 0.375% for the unused portion of the Revolving Facility.

75

Source: Annual report of **J. C. Penney Company**.

**Amazon**, 10K report for 2023

We have a $1.5 billion secured revolving credit facility with a lender that is secured by certain seller receivables, which we may from time to time increase in the future subject to lender approval (the "Credit Facility"). The Credit Facility is available until August 2025, bears interest based on the daily Secured Overnight Financing Rate plus 1.25%, and has a commitment fee of up to 0.45% on the undrawn portion. There were $1.0 billion and $682 million of borrowings outstanding under the Credit Facility as of December 31, 2022 and 2023, which had an interest rate of 5.6% and 6.6%, respectively. As of December 31, 2022 and 2023, we have pledged $1.2 billion and $806 million of our cash and seller receivables as collateral for debt related to our Credit Facility. The estimated fair value of the Credit Facility, which is based on Level 2 inputs, approximated its carrying value as of December 31, 2022 and 2023.

**Tesla**, 10K report for 2023

**Credit Agreement**

In June 2015, we entered into a senior asset-based revolving credit agreement (as amended from time to time, the "Credit Agreement") with a syndicate of banks. Borrowed funds bear interest, at our option, at an annual rate of (a) 1% plus LIBOR or (b) the highest of (i) the federal funds rate plus 0.50%, (ii) the lenders' "prime rate" or (iii) 1% plus LIBOR. The fee for undrawn amounts is 0.25% per annum. The Credit Agreement is secured by certain of our accounts receivable, inventory and equipment. Availability under the Credit Agreement is based on the value of such assets, as reduced by certain reserves.

In January 2023, we entered into a 5-year senior unsecured revolving credit facility (the "RCF Credit Agreement") with a syndicate of banks to replace the existing Credit Agreement, which was terminated. The RCF Credit Agreement contains two optional one-year extensions and has a total commitment of up to $5.00 billion, which could be increased up to $7.00 billion under certain circumstances. The underlying borrowings may be used for general corporate purposes. Borrowed funds accrue interest at a variable rate equal to: (i) for dollar-denominated loans, at our election, (a) Term SOFR (the forward-looking secured overnight financing rate) plus 0.10%, or (b) an alternate base rate; (ii) for loans denominated in pounds sterling, SONIA (the sterling overnight index average reference rate); or (iii) for loans denominated in euros, an adjusted EURIBOR (euro interbank offered rate); in each case, plus an applicable margin. The applicable margin will be based on the rating assigned to our senior, unsecured long-term indebtedness (the "Credit Rating") from time to time. The fee for undrawn amounts is variable based on the Credit Rating and is currently 0.125% per annum.

**McDonald's Corp**., MCD 10K report for 2023

LINE OF CREDIT AGREEMENTS

At December 31, 2023, the Company had a line of credit agreement of $4.0 billion, which expires in June 2028. The Company incurs fees of 0.08% per annum on the total commitment, which remained unused. Fees and interest rates on this line are primarily based on the Company's long-term credit rating assigned by Moody's and Standard & Poor's. In addition, the Company's subsidiaries had unused lines of credit that were primarily uncommitted, short-term and denominated in various currencies at local market rates of interest.

**Boeing Co.,** BA 10K report for 2023 (numbers in $M)

In the third quarter of 2023, we entered into a $3,000 five-year revolving credit agreement expiring in August 2028 and a $800 364-day revolving credit agreement expiring in August 2024. The 364-day credit facility has a one-year term out option which allows us to extend the maturity of any borrowings until August 2025. The legacy three-year revolving credit agreement expiring in August 2025, which consists of $3,000 of total commitments, and the legacy five-year revolving credit agreement expiring in October 2024, as amended, which consists of $3,200 of total commitments, each remain in effect. As of December 31, 2023, we had $10,000 currently available under credit line agreements. We continue to be in full compliance with all covenants contained in our debt or credit facility agreements.