Bridging business cycle dynamics and monetary policy in asset allocation

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Abstract

Multi-asset allocation seeks consistent returns across business and monetary policy cycles phases. This work proposes a new allocation framework embedding market sentiment, business cycle phase and monetary policy stance signals. First, a new way of gauging monetary policy stance in a timely fashion is presented building upon Markov Switching Dynamic Factor Models. Secondly, this paper shows the value to combine this monetary policy signal with existing methodologies identifying in real-time regime switches in both business cycle phases and market sentiment when constructing portfolios. Encompassing macroeconomic and monetary policy regimes in building asset allocation strategies outperforms the well known 60-40 portfolio both in return maximization and risk minimization. During market downturns driven either by recession or monetary policy restriction, the proposed methodology outperforms several benchmark strategies. In more "normal" periods, the signals developed do not significantly hamper the performance of the considered portfolios compared to more "passive" strategies.

Keywords: *Bayesian Estimation; Dynamic Factor; Non-linearity; Asset allocation*

JEL codes: *C11, C32, E37, G11*

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1. Introduction

Navigating through US business cycle and monetary policy phases might be challenging for a multi-asset investor who seeks consistent risk/return couple. There is evidences macroeconomic regimes determine financial returns distributions (Ang and Timmermann (2012)). Expected paths of future monetary policy rates or monetary policy regimes also play a major role in asset prices variations (Rigobon and Sack (2004)). A substantial body of literature has focused on improving portfolio returns by adapting allocation through regime identifications (Ang and Bekaert (2004)). Accounting for business cycle phases into portfolio construction was brought by Brocato and Steed (1998). Jensen and Mercer (2003) propose to rather take into account monetary policy regimes in portfolio construction. Kollar and Schmieder (2019) advocate to consider narrowing asset allocation to both business cycle phases and financial cycles in investment allocation. Kritzman et al. (2012) are the first to deploy a regime identification composed of growth, inflation and financial turbulence regimes to build an adequate allocation strategy. More recently, Kim and Kwon (2023) present an investment framework for dynamic asset allocation strategies based on changes in the growth and inflation environments (implying a monetary policy reaction). In a similar vein, Bouyé and Teiletche (2024) show macro regime-based portfolios (overheating, "goldilock", stagflation, downturn) can outperform traditional asset-based portfolios. This literature mainly faces two drawbacks: one is the frequency at which regimes are identified (mainly monthly or quarterly regimes), the other is the nature of the identification when looking at optimal portfolios, most of it is ex-post.

This paper intends to narrow dynamic long-only asset allocation to business cycle phases real-time identification, existing and new market sentiment measures as well as a novel framework gauging monetary policy stance through long-term expected real rates. Real rates variations are modelled within a four-state Markov-switching dynamic factor model (MS-DFM) encompassing hard and soft hawkish and dovish regimes. Real rates are known to move according monetary policy and information shocks (Nakamura and Steinsson (2018)). When computed as the difference of nominal interest rates and inflation-linked swap they do capture a market assessment of the monetary regime. A new market sentiment is also proposed in the (MS-DFM) framework building on a four-state Markov-Chain allowing for bull correction and bear rallies inter-states. Finally, we use the Asset Swap spreads extension of Aumond (2024) to assess the business cycle regime. The signals are computed on a weekly basis. The macroeconomic turning point signals and market sentiment are used to weight dynamically an equity/bond allocation. The monetary policy stance signal is used as a hedging signal towards cash whenever a restrictive phase is identified. The race-horse 60 equity/40 bond constant portfolio is sensitive to restrictive monetary policy phases where both equity and bond prices fall because of an increase in the discount factor. This salient fact motivates regime-contingent asset allocation in a real-time fashion. Indeed, a multi-asset investor seeking consistent return across business and monetary policy phases has to be able to identify them in a timely manner, this work paves the way for further research.

The paper is organized as follows: Section 2 describes the Markov switching framework as well as the specifications used to build the monetary, business cycle regimes and market sentiment probabilities. Section 3 describes the general Bayesian estimation with details on priors and MCMC algorithms given in Appendices A and B. Section 4 presents in-sample and out-of-sample probabilities of the monetary policy stance identifier and the market sentiment. Section 5 defines the allocation rules as well as the weekly backtest performances of the strategies together with an event study regarding specific macroeconomic and monetary phases. Section 6 concludes.

2. Markov-switching Models

The use of Markov-switching models to capture cyclical dynamics in market returns and macroeconomic aggregates is widespread in the literature. The work of Chauvet (1998), Kim and Nelson (1998), Camacho et al. (2015), among others, using non-linear dynamic factor models to infer business cycle phases are worth to be quoted. Maheu and McCurdy (2000), Maheu et al. (2012) allow for a broad partition of market returns into bull and bear regimes. The latter deploy a methodology to identify bull corrections and bear rallies. Hamilton and Gang (1996) were the first to narrow business cycle phases and market volatility regimes. Regarding Monetary policy regimes, a vast strand of the literature focuses on monetary policy rules switches via a MS-Taylor rule as in Perruchoud (2009), on a simultaneous multivariate system as in Sims and Zha (2004) or in MS-DSGE as in Chang et al. (2021). Worth to note that the first analysis of regime-switching in the term-structure of interest rates was proposed by Hamilton (1988). We introduce a new approach to identify monetary policy regimes which builds upon the market sentiment of equity returns framework presented in section 5.1.3. We will consider in this section three independent Markov-Switching Dynamic Factor Models to identify monetary policy phases, business cycle phases in real-time and markets stressed episodes. A market sentiment signal stemming from a univariate serie of S&P 500 returns will be characterized by a textbook Hidden Markov Model (HMM) as benchmark. The general form of the models used will take the following form.

Let $\boldsymbol{y}_t$ a vector of m monthly or weekly observable time series and let f_t a latent common factor. We have the standard DFM given by :

$$\boldsymbol{y}_t = \boldsymbol{\Lambda} f_t + \boldsymbol{u}_t, \tag{1}$$

where $\boldsymbol{\Lambda}$ denotes the loadings matrix, $\boldsymbol{u}_t$ is orthogonal to f_t and for all $j = 1, \ldots, m$

$$\psi_j(L) u_{j,t} = e_{j,t}, \quad e_{j,t} \sim \mathcal{N}(0, \sigma^2_{e,j}), \tag{2}$$

let us denote $\boldsymbol{\psi}_j = (\psi_{j,1}, \ldots, \psi_{j,l})'$ the coefficients of the lag polynomial $\psi_j(L)$ of order l and the factor follows

$$\begin{aligned} f_t = & \quad \mu_{S_t} + \phi f_{t-1} + \epsilon_t \\ \epsilon_t = & \quad \sigma_t \eta_t \end{aligned} \tag{3}$$

where η_t is iid $(0, 1)$ and S_t is an independent first order n-state Markov chain. $\mathbf{P}$ will denote its constant transition probability matrix with dimensions n $\times$ n. Depending on the specified model, the volatility of the factor σ_t will be conditional or Markov-switching, thus we keep the $_t$ notation in the general notation.

2.1. Monetary policy stance identification

As introduced by Woodford (2003) in the wake of Clarida et al. (1999), in many New Keynesian models, the real interest rate gap is a measure of monetary policy stance. The gap is defined as a difference between a real interest rate and a natural rate of interest. The forward dimension of the real interest rates is paramount as the information content of market expectations concerning inflation gives us a hint regarding the dynamics the market participants are pricing. According to Woodford (2003), the role of monetary policy is to keep agent's inflation expectations anchored at Central Banks' target. This theory builds upon rational expectations. Inflation expectations implied by market prices have now become a pivotal information set entering the monetary policy reaction function. A horse-race model for this reaction function is the Tayor Rule (Taylor (1993)) which specify the short-term nominal interest rate as a function of output gap and and the percentage deviation of inflation from its target.

The intuition behind the novel approach we propose is the need for practitioners to grasp the intensity or amplitude of forward real rates interest dynamics. Financial markets, compared to textbook macroeconomic models are indeed very sensitive to forward real interest rates moves rather than absolute levels or gaps. The drifts and inertia related to real rates are moreover challenging. We draw from Orphanides (2003) approach in modelling interest rates in first difference. At the effective lower bound or very far from natural rate of interest, strong variations of real interest rates had historically a dramatic impact on risky assets, in line with the duration model proposed by Leibowitz et al. (1989) showing that equity prices display a negative sensitivity to real interest rates. We thus need to capture the common dynamics of a set of forward real interest rates. In our approach, because of data availability we will use the weekly 5Y-, 7Y-, 10Y- forward real interest rates. Those forward real interest rates are defined as the difference between the nominal interest rate of a given maturity measured in our case by Overnight Index Swap (OIS) and an inflation expectation component measured by Inflation-Linked Swap (ILS) of corresponding maturity. We take the absolute variations of the negative values of the data so that a negative variation corresponds to a restrictive movement in the forward real interest rates.

We consider the factor model presented in equations (1) to (3). The approach intends to identify four states : high or low volatile hawkish or dovish dynamics in a factor capturing the co-movement of the forward real interest rates. This can also be interpreted as "hard" or "soft" dovish or hawkish moves in markets' participants views. The first order Markov Chain displays four states ($n = 4$). The first element of $\mathbf{\Lambda}$ is set to one for the sake of identification in equation (1). The residuals u_t in (2) follow an AR(1) process. In equation (3), we impose $\mu_1 < \mu_3 < 0 < \mu_2 < \mu_4$. The volatility $\sigma_t = \sigma_{S_t}$ is also Markov-switching with $\sigma_1 > \sigma_3$ and $\sigma_2 > \sigma_4$. $\phi = 0$, the factor does not follow an autoregressive process. $\mathbf{P}$ is unrestricted. The absence of restriction in $\mathbf{P}$ enables to capture the idea that the chain can switch from a hard hawkish surprise to a hard dovish interpretation, as we can have noticed during the 2008-2009 great financial crisis. Given monetary policy surprises or events the market participants face and interpret, dynamics of the monetary policy stance perception can vary a lot. We will refer to the work of Jarociński and Karadi (2020) who perform a thorough analysis of markets reactions to Monetary policy communication in a structural VAR framework.

2.2. Business cycle turning point detection

The business cycle turning point model builds upon Aumond (2024) based on Aumond and Royer (2024). It is an MS-DFM model with an ARCH extension in the volatility process for the factor. The information sample is composed of eleven monthly variables: industrial production, real manufacturing trade and sales, civilian employment, real personal income as well as 7 grade buckets of USD asset swap spreads ranging from Investment grade (AAA) to Speculative grade (CCC).

We consider again the factor model presented in equations (1) to (3). The first order Markov chain displays two states ($n = 2$). The first element of $\mathbf{\Lambda}$ is set to one for the sake of identification in equation (1). The residuals u_t in (2) follow an AR(1) process. In equation (3), we impose $\mu_1 < \mu_0$. The volatility σ_t follows a ARCH(1) dynamic: $\sigma_t^2 = \omega + \alpha\epsilon_{t-1}^2$. The factor follows an auto-regressive process of order 1. $\mathbf{P}$ is unrestricted. We thus identify two states : a recession regime ($S_t = 1$) and an expansion regime ($S_t = 0$). We can rewrite $\mathbf{P}$:

$$\mathbf{P} = \begin{bmatrix} q & 1-q \\ 1-p & p \end{bmatrix}$$

The transition probabilities are defined as

$$q = \mathbb{P}(S_t = 0|S_{t-1} = 0) \quad \text{and} \quad p = \mathbb{P}(S_t = 1|S_{t-1} = 1),$$

Aumond (2024) shows the ability of this specification to contemporaneously capture economic downturn episodes.

2.3. Market sentiment

We build here on the work of Maheu et al. (2012) who introduce a framework in which bull and bear regimes in the returns of S&P 500 allow for bull corrections and bear rallies. The framework takes into account short-term reversals within each regime of the market. As such, a bull regime can face a series of persistent negative returns (a bull correction), even if the expected long-run return (primary trend) is positive in that regime. We extend this framework by taking into account a dynamic factor model approach enabling to partition the common dynamics of four equity indices : the returns of the S&P 500, the Russell, the Dow Jones and the NASDAQ. The underlying idea is to be able to capture a global stress in the equity market which would not be tilted to a specific market profile, either Large/Small or Value/Growth, given the growing concentration of the S&P 500 index. This factor extension is different from the multivariate extension adopted by Liu et al. (2024) with a hierarchical Markov switching model.

This factor extension extension can be described as follows. Taking equations (1) to (3) into consideration. The first order Markov Chain displays four states ($n = 4$). The first element of $\mathbf{\Lambda}$ is set to one for the sake of identification in equation (1). The residuals u_t in (2) follow an AR(1) process. In equation (3), we impose

$$\text{Bear Regime} \begin{cases} \mu_1 < 0 & \text{(bear market state)} \\ \mu_2 > 0 & \text{(bear market rally)} \end{cases}$$
$$\text{Bull Regime} \begin{cases} \mu_3 < 0 & \text{(bull market correction)} \\ \mu_4 > 0 & \text{(bull market state)} \end{cases}$$

There is no restriction on σ_t. $\phi = 0$, the factor does not follow an auto-regressive process. $\mathbf{P}$ is restricted.

$$\mathbf{P} = \begin{bmatrix} p_{11} & p_{12} & 0 & p_{14} \\ p_{21} & p_{22} & 0 & p_{24} \\ p_{31} & 0 & p_{33} & p_{34} \\ p_{41} & 0 & p_{43} & p_{44} \end{bmatrix}$$

The movement of a bull (bear) regime to a bear rally (bull correction) state is not allowed for identification, but Maheu et al. (2012) justify it through the data. To allow for short-term deviation from the long-term trend the authors use the Hamilton (1994) resolution of unconditional probabilities of $\mathbf{P}$ given by :

$$\mathbf{\Pi} = (\mathbf{A}'\mathbf{A})^{-1}\mathbf{A}'\mathbf{e}$$

with $\mathbf{A}' = [\mathbf{P}' - \mathbf{I}, \boldsymbol{\iota}]$ and $\mathbf{e}' = [0, 0, 0, 1]$ and $\boldsymbol{\iota} = [1, 1, 1, 1]'$. This matrix of unconditional state probabilities $\mathbf{\Pi}$ allows to constrain long-run market returns in the bear and bull regimes:

$$E[f_t \mid \text{bear regime}, S_t = 1, 2] = \frac{\pi_1}{\pi_1 + \pi_2}\mu_1 + \frac{\pi_2}{\pi_1 + \pi_2}\mu_2 < 0$$
$$E[f_t \mid \text{bull regime}, S_t = 3, 4] = \frac{\pi_3}{\pi_3 + \pi_4}\mu_3 + \frac{\pi_4}{\pi_3 + \pi_4}\mu_4 < 0$$

The bull (bear) market regime is defined with long-run positive (negative) returns but the market regimes can have short-term reversals from their long-run mean. We will also consider the univariate version of the above specification as in Maheu et al. (2012) as benchmark.

3. Bayesian estimation

The general model from (1) to (2) can be cast in state space form :

$$\begin{aligned} \boldsymbol{y}_t = &\quad \boldsymbol{H}\boldsymbol{z}_t + \varsigma_t & \varsigma_t \sim \mathcal{N}(0, \boldsymbol{R}) \\ \boldsymbol{z}_t = &\quad \boldsymbol{\delta}_{S_t} + \boldsymbol{\Xi}\boldsymbol{z}_{t-1} + \boldsymbol{\zeta}_t & \boldsymbol{\zeta}_t \sim \mathcal{N}(0, \boldsymbol{Q}_t) \end{aligned} \tag{4}$$

$\boldsymbol{H}$ the $(m) \times (ml+1)$ matrix, $\boldsymbol{\Xi}$ the $(ml+1) \times (ml+1)$ matrix, and $\boldsymbol{\Xi}_{j=1\ldots m}$ the $l \times l$ matrix such that

$$\boldsymbol{H} = \begin{bmatrix} \lambda_1 & \boldsymbol{h}^l & \ldots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ \lambda_m & 0 & \ldots & \boldsymbol{h}^l \end{bmatrix} \quad \boldsymbol{\Xi} = \begin{bmatrix} \boldsymbol{\phi} & & & \\ & \boldsymbol{\Xi}_1 & & \\ & & \ddots & \\ & & & \boldsymbol{\Xi}_m \end{bmatrix}$$

$$\text{with} \quad \boldsymbol{\Xi}_{j=1\ldots m} = \begin{bmatrix} \psi_{j,1} & \psi_{j,2} & \ldots & \psi_{j,l-1} & \psi_{j,l} \\ 1 & 0 & \ldots & 0 & 0 \\ 0 & 1 & \ldots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \ldots & 1 & 0 \end{bmatrix}.$$

$\lambda_1 = 1$ for identification purpose. $\boldsymbol{h}^l$ is a $1 \times l$ vector with the only first element equal to one. $\boldsymbol{\delta}_{S_t} = (\mu_{S_t}, 0, \ldots, 0)'$, $\text{diag}(\boldsymbol{Q}_t) = (\sigma_t^2, \sigma_{e,1}^2 \boldsymbol{h}^l, \ldots \sigma_{e,m}^2 \boldsymbol{h}^l)$. The vector $(ml+1)$ of unobserved variables $\boldsymbol{z}_t$ is given by

$$\boldsymbol{z}_t = (f_t, (1, L, \ldots, L^{l-1})u_{1,t}, \ldots, (1, L, \ldots, L^{l-1})u_{m,t})'.$$

The vector of parameters to estimate is given will be specific to the model specification we use. We denote by $\boldsymbol{\vartheta}^{(\text{MP})}$, the vector of parameters of the Monetary Policy stance model. $\boldsymbol{\vartheta}^{(\text{BC})}$ refers to the parameters vector for the business cycle phases assessor. Finally $\boldsymbol{\vartheta}^{(\text{MS})}$ will refer to the vector of parameters for the Market sentiment model. We have the following set of parameters vectors :

$$\begin{aligned} \boldsymbol{\vartheta}^{(\text{MP})} &= (\mathbf{P}, \boldsymbol{\Psi}', \sigma_{e,1}, \ldots, \sigma_{e,m}, \boldsymbol{\Lambda}', \mu_1, \mu_2, \mu_3, \mu_4, \sigma_1, \sigma_2, \sigma_3, \sigma_4)' \\ \boldsymbol{\vartheta}^{(\text{BC})} &= (\mathbf{P}, \boldsymbol{\Psi}', \sigma_{e,1}, \ldots, \sigma_{e,m}, \boldsymbol{\Lambda}', \mu_0, \mu_1, \phi, \omega, \alpha)' \\ \boldsymbol{\vartheta}^{(\text{MS})} &= (\mathbf{P}, \boldsymbol{\Psi}', \sigma_{e,1}, \ldots, \sigma_{e,m}, \boldsymbol{\Lambda}', \mu_1, \mu_2, \mu_3, \mu_4, \sigma_1, \sigma_2, \sigma_3, \sigma_4)' \end{aligned}$$

Let us denote $\boldsymbol{z}^{(T)} = \{\boldsymbol{z}_1, \ldots, \boldsymbol{z}_T\}$ the unobserved state vector in equation (4), $\boldsymbol{y}^{(T)} = \{\boldsymbol{y}_1, \ldots, \boldsymbol{y}_T\}$ the observed data, and $S^{(T)} = \{S_1, \ldots, S_T\}$ the unobserved Markov Chain. The models are estimated using a Markov Chain Monte Carlo (MCMC) Gibbs sampling algorithm in the spirit of Kim and Nelson (1999) and Bai and Wang (2011) where conditional draws of the state vector, the Markov Chain, and the parameters vector $\boldsymbol{\vartheta}^{(\cdot)}$ are obtained sequentially.

- We generate conditional draws of the state vector from $p(\boldsymbol{z}^{(T)}|\boldsymbol{y}^{(T)}, S^{(T)}, \boldsymbol{\vartheta}^{(\cdot)})$ using the forward-filtering backward-smoothing algorithm of Carter and Kohn (1994).
- We generate conditional draws of the Markov chain from $p(S^{(T)}|\boldsymbol{y}^{(T)}, \boldsymbol{z}^{(T)}, \boldsymbol{\vartheta}^{(\cdot)})$ based on the Hamilton filter (Hamilton (1989)).
- We generate conditional draws for the parameters vector from $p(\boldsymbol{\vartheta}^{(\cdot)}|\boldsymbol{y}^{(T)}, \boldsymbol{z}^{(T)}, S^{(T)})$ by sequentially drawing in the conditional distribution of components of $\boldsymbol{\vartheta}^{(\cdot)}$

Details of priors used and the MCMC algorithms for each model specification can be found respectively in Appendices A and B.

4. Results

4.1. In Sample

This section presents the in-sample probabilities of the MS-DFMs presented in Section 2. The Figure 1 shows the 4 regimes identified by our monetary policy stance identifier. Five hard hawkish phases are noticeable. The first phase is the one taking place from mid-2004 until the end of the year. This event materializes policy rates normalisation following the gradual US recovery which took place from June 2004 to June 2006. We catch the early stage of the tightening with the soft hawkish probability which then translates to hard hawkish regime as the market expectations for long-term maturities are more ample at the beginning of a hiking cycles or monetary policy shock as shown by Boeck and Feldkircher (2021). According to the authors this is not the case for short-term maturities yields that under-react first and exhibit then a period of overcompensation called delayed overshooting. The hard hawkish regime then flips again to soft hawkish regime and fades away at the end of 2004. The second hawkish regime identified corresponds to the noisy period surrounding FOMC communications during the Great Financial Crisis. Jarociński and Karadi (2020) identify likewise restrictive monetary policy shocks during this period. Moreover, the information shock as identified by the authors induces a bearish guidance regarding future activity provoking a fall in long-term inflation expectations and thus a rise in real interest rates, which in turn transposes itself into a overarching hawkish regime. From late 2010 to February 2011, the recovery and reflation expected in the US has brought a soft hawkish signal into place. Another hard hawkish period identified is the "taper tantrum" period in may 2013 triggered by Bernanke's speech regarding the possibility the Federal Reserve could reduce the speed of its the balance sheet expansion. It ignited a massive bond sell-off dragging rates upwards, this is a clear hawkish perception the market had regarding the Federal Reserve even though the monetary regime was clearly dovish at the time. This is also the reason why the Federal Reserve has put a lot of effort to divorce expectations of future rate increases from balance sheet reductions afterwards (Smith and Valcarcel (2023)). The Covid outbreak in February 2020, by the uncertainty it has brought regarding economic prospects has brought hard deflationary expectations for a short period of time (before the massive monetary and fiscal stimulus). This is captured by the spike in the hard hawk probability regime and surrounded by a slight move in the soft hawkish probability. The last hawkish period identified by our specification is the one referring to the expeditive action undertaken by the Federal Reserve from 2022 onwards to dampen the supply/demand unbalances provoked by the global economy reopening and amplified by the Russian invasion of Ukraine.

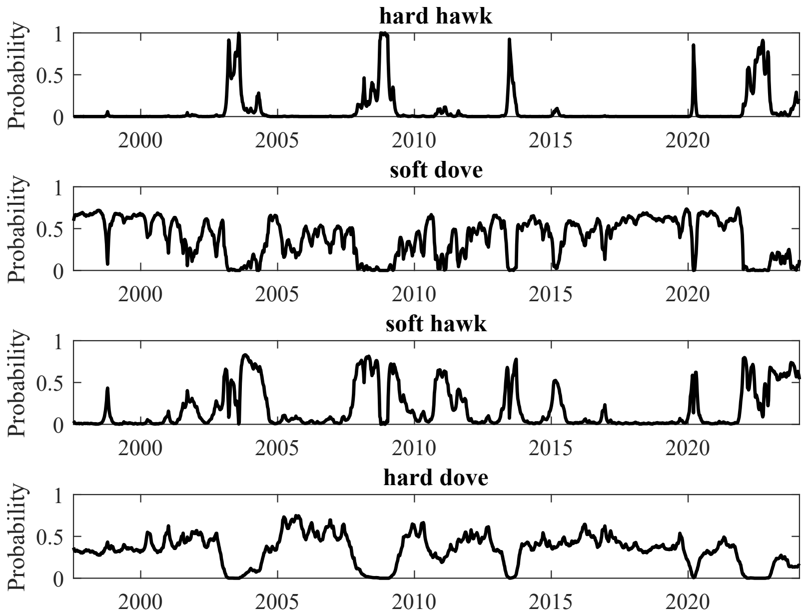

Figure 1: *Dovish/hawkish regime in sample probabilities*

If we recombine the inter-state regimes into two broad measures of hawkishness and dovishness by summing up the soft/hard dovish hawkish probabilities we get the two symmetric probabilities presented in Figure 2. What we are interested in is the capacity of the monetary policy stance signal to detect a hawkish regime (implied by expected long-term real rates) in order to deploy a specific hedging asset allocation strategy.

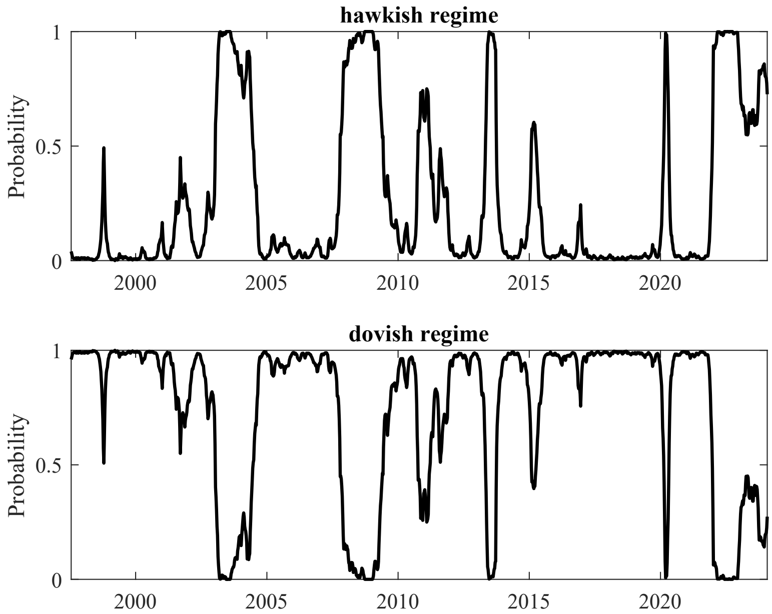

Figure 2: *Dovish/hawkish regime in sample probabilities*

Figure 3 displays the 4-states market sentiment in sample probabilities. The bear regime probability manages to capture seven market ample downturns from 1989 to 2023. Those downturns are the dotcom bubble burst in march 2000 and specific stresses associated to it afterwards, specifically in 2002. The Great financial crisis and the market stress associated to it is labelled as bear regime in early September 2008 until March 2009. Lastly the Covid outbreak in China followed by a broad shutdown in world economy provoked a fall in equity markets from 19th February until the massive Monetary and State supports announced mid-march 2020. Bull correction phases appear more frequently in our dataset. Significant periods are the 1990 recession in the US in the wake of the Gulf crisis and the rise in oil products as consequence. 1998 is worth to be noted as a pre-dotcom bubble burst triggered by the Russian debt crisis. A significant bull correction regime appears during the beginning of June 2004 hiking cycle. The Great Financial Crisis is captured as a bull correction from October 2007 to September 2008. The "flash crash" of May 2010 and the correction of 2011 in the wake of the Greek debt stress and European debt crisis are captured as bull correction events. The "taper tantrum" is also identified as bull correction regime. The 2015 slowing in corporate earnings is labelled as bull correction phase. The 2018 market correction is associated with fears regarding Federal Reserve decisions and the uncertainty in the tariffs war between Trump's administration and China. Finally, one can observe that the correction induced by the Federal Reserve expeditive action to dampen 2022 inflationary pressures is labelled as bull correction regime.

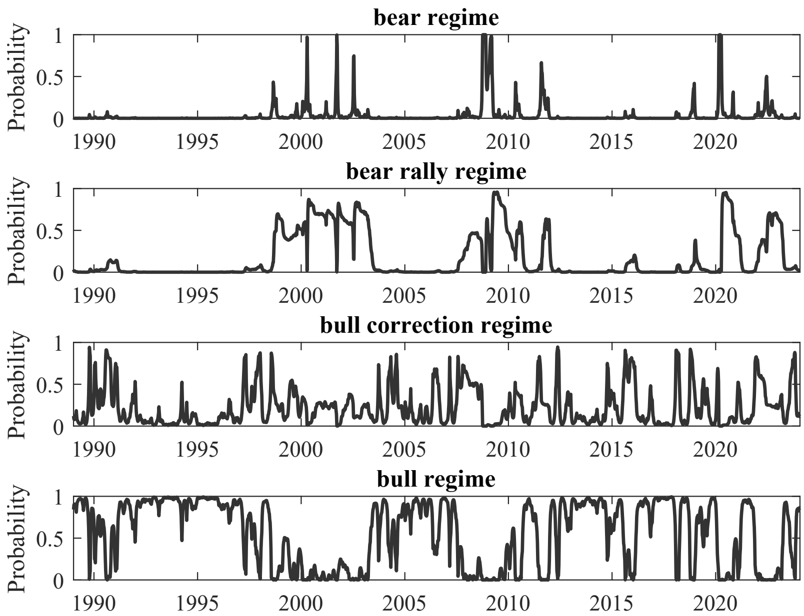

Figure 3: *In sample probabilities from the multivariate specification of the market sentiment*

Figure 4 displays the recombined bull/bear market sentiment in sample probabilities. This partitioning is globally consistent with the multivariate bull/bear approach probabilities obtained by Liu et al. (2024) on this specific time span.

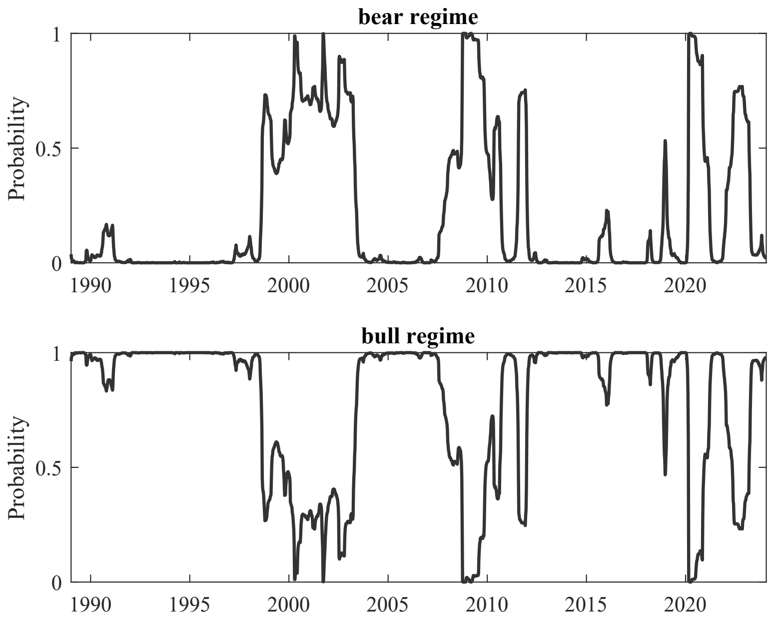

Figure 4: *Bull/bear in sample probabilities from the multivariate specification of the market sentiment*

Figures 13 and 14 in Appendix C display the 4-states market sentiment in sample probabilities and the recombined bull/bear probabilities based on the univariate specification as in Maheu et al. (2012), taking only into account the S&P500 time series.

4.2. Out of Sample

This section presents the out of sample exercise undertaken on a weekly basis from January the 7th 2000 until 22nd February 2023. Figure 5 displays the real-time probabilities to be in a hard hawkish regimes. The signal manages to capture, from a real-time market perspective the restrictive monetary policy events described in subsection 4.1 and thus, validating the proposed approach.

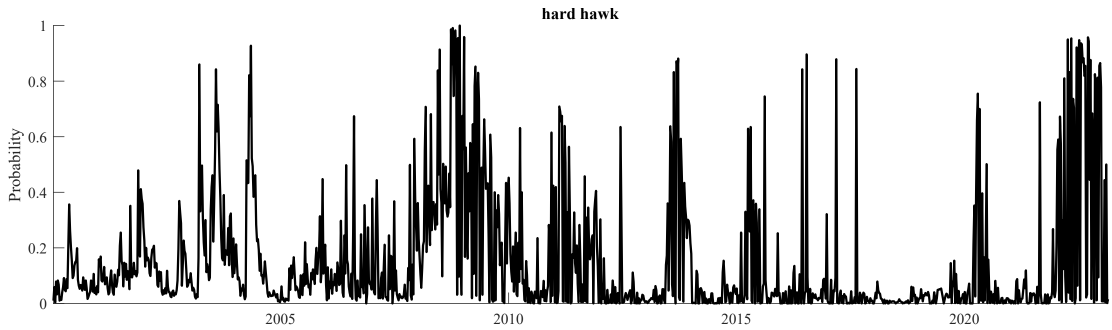

Figure 5: *Hard hawkish regime out of sample probabilities*

Figures 6 and 7 show the real-time probabilities of the market sentiment signal. Compared to the in-sample probabilities displayed in Figures 3 and 4, we get a more sensitive signal which manages to identify in a timely manner the bearish events described in subsection 4.1.

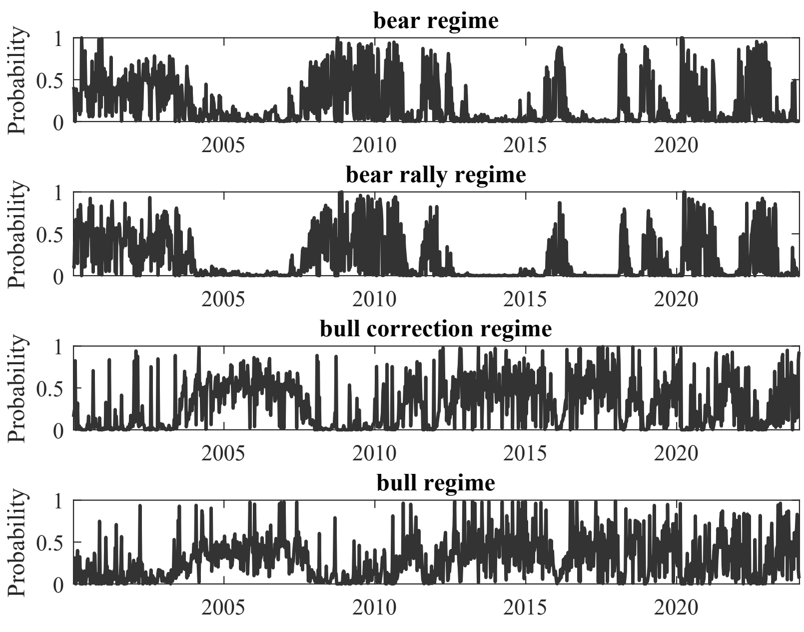

Figure 6: *Real-time probabilities from the multivarite specification of the market sentiment*

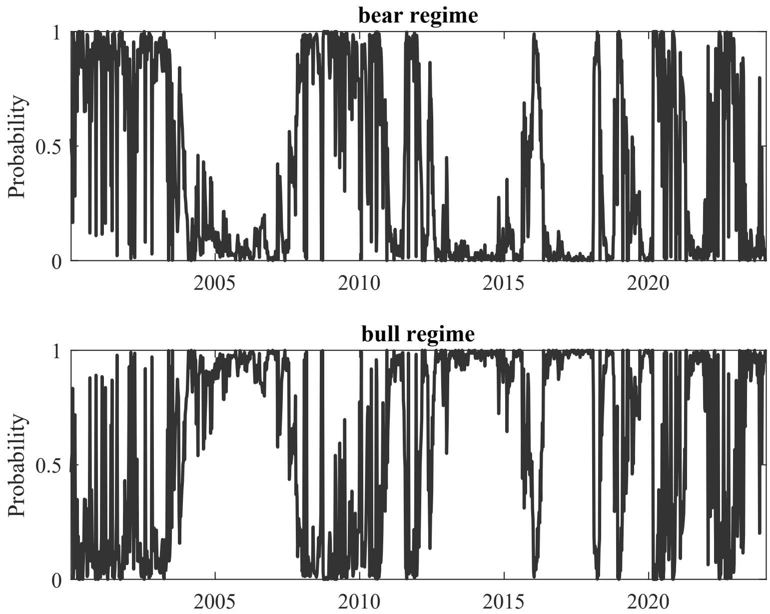

Figure 7: *Real-time bull/bear probabilities from the multivariate specification of the market sentiment*

5. Asset allocation

A substantial litterature has focused on relationships between monetary policy and equity prices (Thorbecke (1997), Bernanke and Kuttner (2005), among others). We can mention Hamilton and de Longis (2015) who bring a coherent allocation strategy along business cycle phases measured on a real-time monthly basis. The need to take into account business cycle phases into portfolio construction has been documented by Brocato and Steed (1998). Jensen and Mercer (2003) advocate taking into account the monetary policy cycle into portfolio optimization would be of a better help than the latter approach. The need to bridge asset allocation to both business cycle phases and financial cycles has been put forward by Kollar and Schmieder (2019). Kritzman et al. (2012) identify monthly or quarterly regimes defined as financial market turbulences, inflation and economic growth and deploy a tactical asset allocation. To the best of our knowledge, this is the first try to narrow in a single allocation framework macroeconomic turning point real-time signals, monetary policy stance regimes and market sentiment on a weekly rebalancing basis, making our investment strategy "live".

This section introduces rules of allocation based on signals extracted in the former section. A race-horse allocation strategy in the asset management industry consists in attributing a constant 60% weight to equity and a 40% weight to bonds. This allocation rule is attributed to the Modern Portfolio theorists in the 50's. This strategy has proven to be efficient to capture equity markets long run returns as well as offering the safe heaven characteristics of the bond market when it comes to risky assets downturns. This induces a less volatile profile of total long run returns. This strategy shows nonetheless a vulnerable return profile during monetary policy restrictive periods or hawkish phases in the market. As interest rates increase, bond prices fall as well as equity prices. We propose an asset allocation to circumvent this issue. By taking into account a monetary policy stance signal, the investor can opt for a hedging strategy to cap the losses incurred by reducing its exposure to the equity/bond market. We define in this section a range of alternative portfolio strategies that mimic the 60/40 strategy by allowing first weights to be dynamic. The turning point probability and the market sentiment probability are used as inputs in the dynamic weighting scheme. Moreover, the monetary stance probability will be used as hedging weight when incorporated to the allocation decision. We compare the constructed portfolios to well-known benchmark strategies : 100% cash, 100% equity, 60% equity / 40% bond. Rebalancing of the portfolio is implemented on a weekly basis, at the time when the market sentiment, business cycle phase assessor and monetary policy stance signal are computed. Finally, we apply a 2 basis point transaction cost to the S&P500 rebalancing and none for cash or 10Y US Treasuries, given the liquidity of those markets.

5.1. Rules

We consider a set of rules taking into account a portfolio universe composed of a risky asset (S&P500 index), a fixed income asset (10 Year US Bond) and Cash (3-month Treasury Bill). Business cycle and market sentiment regimes will serve as weights in the equity/bonds allocation weights, making the 60/40 allocation portfolio dynamic. If a stressed market or a recession signal occur, the weight applied to the risky asset can be reduced up to 0%. High volatile markets are generally associated with negative future returns. This stylized fact, known as leverage effect has been extensively studied in the literature. One can refer to Black (1976), Kim et al. (2000) for the volatility feedback analysis.

Business cycle phases also play a major role in equity returns profiles. When a fear of a downturn or a recession occurs, expected revenues of firms fall triggering equity prices to drop. As mentioned by Siegel (1991) there is a positive relationship between firm profits and earnings as well as dividends for stakeholders. Balance sheet effects through increase of debt ratio also amplify the deterioration in valuation ratios. On the other hand, bond markets are then considered as a safer place for investors, especially for Government debt with good credit grades. This so-called "flight to quality" phenomenon tends to reduce bond yields and increase the market price of the already-issued bonds.

A restrictive monetary policy is carried out in a inflationary regime central banks want to cool down. The main tool is the increase of the cost of credit by setting its policy rates higher. It tames the aggregate demand to put it in a better balance with respect to the aggregate supply of the economy. The impact of such moves is an increase of bond yields across the maturity curve. It implies a drop in bond indices prices. In such events, equity markets tend also to suffer as the discount factors in cash flow models increase. The anticipated slowdown in activity also brings the expected revenues of firms down. One of the only asset class which does not suffer from such restrictive monetary environments is cash. Short-term rates increase, making the carry of money markets attractive compared to deterioration in other asset classes valuations, especially the most risky ones. A 60/40 portfolio, in a restrictive monetary environment faces negative returns on each leg of the asset allocation rule.

We will thus build up some allocation rules taking into account monetary policy stance into account and giving a specific weight to cash. In the remaining part of the paper, we will always consider long-only portfolios. We define w_t^{cash} as the weight allocated to cash at time t, w_t^{equity} the weight for the S&P 500 and w_t^{bond} the weight attributed to the 10Y Treasury bond. $\forall t, w_t^{equity} + w_t^{cash} + w_t^{bond} = 1$. Finally, $P(S_t^{BC} = 1 \mid I_t)$ will denote the filtered probability to be in a recession phase, $P(S_t^{MS} = \{1, 2\} \mid I_t)$ in a bear regime phase, $P(S_t^{MS} = \{1, 3\} \mid I_t)$ the probability to be in a negative return equity market regime and $P(S_t^{MP} = \{1\} \mid I_t)$ to be in a restrictive monetary regime. Five broad asset allocation cases will be considered. One which takes only the probability of being in a bear regime or a negative return market. The second approach will modulate the latter probability by averaging the signal with a business cycle regime. The third approach will take into account Monetary policy stance with the market sentiment. The fourth one will mix the three axes. Finally the latter approach will exclude the market sentiment and look at the utility of business cycle and monetary policy stance only.

5.1.1. Market sentiment only

We consider two alternative signals. The multivariate model presented in section 2.3 and the univariate specification as the one introduced by Maheu et al. (2012). It aims at comparing the added value of considering a broad definition of US equity markets compared to the single S&P 500 total return index when extracting a sentiment index. The weighting scheme of bond and equity is either given by :

$$w_t^{bond} = P(S_t^{MS} = \{1, 2\} \mid I_t) \quad w_t^{equity} = P(S_t^{MS} = \{3, 4\} \mid I_t)$$

looking specifically to bull/bear partition or

$$w_t^{bond} = P(S_t^{MS} = \{1, 3\} \mid I_t) \quad w_t^{equity} = P(S_t^{MS} = \{2, 4\} \mid I_t)$$

when trying to capture negative return phases, allowing for within-regime partitions.

5.1.2. Market sentiment and Business cycle

Based on both univariate and multivariate market sentiment specifications as well as bull/bear or positive/negative return partitions, we then consider using the turning point signal to mitigate market stress not induced by macroeconomic fundamentals. The weighting scheme is given by:

$$\begin{aligned} w_t^{bond} &= \frac{P(S_t^{MS} = \{1,2\} \mid I_t) + P(S_t^{BC} = 1 \mid I_t)}{2} \\ w_t^{equity} &= \frac{P(S_t^{MS} = \{3,4\} \mid I_t) + P(S_t^{BC} = 0 \mid I_t)}{2} \end{aligned}$$

considering bull/bear partition or

$$\begin{aligned} w_t^{bond} &= \frac{P(S_t^{MS} = \{1,3\} \mid I_t) + P(S_t^{BC} = 1 \mid I_t)}{2} \\ w_t^{equity} &= \frac{P(S_t^{MS} = \{2,4\} \mid I_t) + P(S_t^{BC} = 0 \mid I_t)}{2} \end{aligned}$$

looking for a positive/negative short-term returns.

5.1.3. Market sentiment and Monetary Policy Stance

The set of rules described in this section intends to gauge whether adding the monetary policy stance is valuable. The market sentiment rules the equity/bond allocation, the cash is weighted by the monetary policy stance signal, used as a hedging extension. The weighting scheme of bond, equity and cash is either given by :

$$\begin{aligned} w_t^{bond} &= P(S_t^{MS} = \{1,2\} \mid I_t) \times (1 - P(S_t^{MP} = 1 \mid I_t)) \\ w_t^{equity} &= P(S_t^{MS} = \{3,4\} \mid I_t) \times (1 - P(S_t^{MP} = 1 \mid I_t)) \\ w_t^{cash} &= 1 - w_t^{bond} - w_t^{equity} \end{aligned}$$

through the bull/bear partition or

$$\begin{aligned} w_t^{bond} &= P(S_t^{MS} = \{1,3\} \mid I_t) \times (1 - P(S_t^{MP} = 1 \mid I_t)) \\ w_t^{equity} &= P(S_t^{MS} = \{2,4\} \mid I_t) \times (1 - P(S_t^{MP} = 1 \mid I_t)) \\ w_t^{cash} &= 1 - w_t^{bond} - w_t^{equity} \end{aligned}$$

considering specifically the positive/negative return partition.

5.1.4. Business cycle and Monetary policy stance

The rules in this section enable to gauge the benefit of using the market sentiment. The multi-asset allocation is only driven by fundamental macroeconomic signals and the monetary policy stance. The underlying hypothesis is that once looking at macroeconomic or monetary policy stress, there is no need to take into account other kind of market stress not attributable to the former ones.

Hence, the unique set of rules is given by:

$$\begin{aligned} w_t^{bond} &= P(S_t^{BC} = 1 \mid I_t) \times (1 - P(S_t^{MP} = 1 \mid I_t)) \\ w_t^{equity} &= P(S_t^{BC} = 0 \mid I_t) \times (1 - P(S_t^{MP} = 1 \mid I_t)) \\ w_t^{cash} &= 1 - w_t^{bond} - w_t^{equity} \end{aligned}$$

The equity/bond weighting scheme is defined by the real-time turning point signal whereas the monetary policy stance signal enables to migrate the allocation towards cash whenever a monetary policy appears to be increasingly tight.

5.1.5. Market sentiment, Business cycle and Monetary policy stance

Finally, a last set of rules we characterize as all-road will embrace the three signals used concomitantly. The weighting scheme is a mix of the intuitions described in subsections 5.1.2 and 5.1.3. The macroeconomic turning point signal mitigate the market sentiment in the equity/bond allocation while the monetary policy stance signal allows for a dynamic hedging with cash. The weighting scheme is given by:

$$\begin{aligned} w_t^{bond} &= \frac{P(S_t^{MS} = \{1,2\} \mid I_t) + P(S_t^{BC} = 1 \mid I_t)}{2} \times (1 - P(S_t^{MP} = 1 \mid I_t)) \\ w_t^{equity} &= \frac{P(S_t^{MS} = \{3,4\} \mid I_t) + P(S_t^{BC} = 0 \mid I_t)}{2} \times (1 - P(S_t^{MP} = 1 \mid I_t)) \\ w_t^{cash} &= 1 - w_t^{bond} - w_t^{equity} \end{aligned}$$

when focusing on a market bull/bear partition or by :

$$\begin{aligned} w_t^{bond} &= \frac{P(S_t^{MS} = \{1,3\} \mid I_t) + P(S_t^{BC} = 1 \mid I_t)}{2} \times (1 - P(S_t^{MP} = 1 \mid I_t)) \\ w_t^{equity} &= \frac{P(S_t^{MS} = \{2,4\} \mid I_t) + P(S_t^{BC} = 0 \mid I_t)}{2} \times (1 - P(S_t^{MP} = 1 \mid I_t)) \\ w_t^{cash} &= 1 - w_t^{bond} - w_t^{equity} \end{aligned}$$

when concentrating on a positive/negative return real-time identification.

5.2. Performance comparison

We compare the performances of the competing strategies in a real-time backtest exercise. The backtest is ran from 1st January 2000 to 24th February 2023 on weekly basis. The portfolios rebalancing occurs at the end of a given week. The macroeconomic turning point signal stems from Aumond (2024) real-time backtest. The filtered probabilities $P(S_t^{BC} \mid I_t)$, $P(S_t^{MS} \mid I_t)$ and $P(S_t^{MP} \mid I_t)$ are computed at the same date t.
We denote $\boldsymbol{w_t} = [w_t^{bond}, w_t^{equity}, w_t^{cash}]$ and the return of each asset $\boldsymbol{r_t} = [r_t^{bond}, r_t^{equity}, r_t^{cash}]$. The weekly total return of a specific asset a, is given by $r_t^a = (P_t^a / P_{t-1})^a - 1$, P_t^a being the price of asset a at time t. For the Market sentiment strategies and the Market sentiment and

Business cycle strategies in sections 5.1.1 and 5.1.2, $w_t^{bond} = 0$. The strategy s return at time t, $r_{t,s}$ is given by $\boldsymbol{w_{t-1} r_t'}$. In this section, we will focus on annualized returns, volatility, Sharpe ratios (Sharpe (1994)) and maximum drawdowns as well as the same statistics for portfolio holding rolling windows ranging from 1 year to 10 years. Given that first moments of the descriptive statistics are auto-correlated on the rolling window holding periods, we will focus on empirical cumulative distributions albeit we put the average statistics in the performance comparison tables. Moreover, we will use Fleming et al. (2003) approach to measure the economic utility for an investor to hold a specific portfolio. This approach relates to the mean-variance analysis and the quadratic utility framework. The realized weekly utility generated by a strategy s is given by :

$$U(r_{t,s}) = W_0 \left((1 + r_t^f + r_{t,s}) - \frac{\gamma}{2(1+\gamma)} (1 + r_t^f + r_{t,s})^2 \right) \tag{5}$$

W_0 is the initial wealth invested, r_t^f the 3-month cash return, $r_{t,s}$ the portfolio return and γ a fixed aversion parameter. To measure the value of a strategy compared to another one, we can define a constant Δ which equalizes :

$$\sum_{t=1}^{T} U(r_{t,s1}) = \sum_{t=1}^{T} U(r_{t,s2} - \Delta)$$

This constant Δ can be considered as the maximum performance fee that an investor would agree to pay for switching from strategy $s1$ to $s2$ under the hypothesis that he is indifferent between both. The higher the Δ, the more strategy $s2$ is valuable for the investor.

Table 1 displays the annualised performances for the five groups of competing strategies. Bold lines highlight the strategies for which the annualised Sharpe ratio is maximised. From the Modern Portfolio Theory (Markowitz (1952)), an investor is interested in maximizing the risk/return couple, and will not be eager to increase return if it comes at a higher cost in terms of returns dispersion, e.g. volatility. The strategy embedding the bull/bear sentiment, the macroeconomic real-time downturn signal and the Monetary policy stance signal (MS-Bull-Bear-Return-Uni-MP-BC) tend to outperform the other strategies. The strategy only relying on the market sentiment (MS-Bull-Bear-Return-Uni) is the one which performs worst compared to its counterparts incorporating either a monetary policy stance signal, a business cycle phase assessor or both. From the tabled results, the multivariate extension of the equity market sentiment does not improve the performances statistics. Moreover, the strategy consisting in taking exposure to the risky asset whenever a signal of bear rally materialize appears to be inefficient. This is attributable to the fact that short-term reversal identified are not persistent enough to yield higher returns.

Bottom line of those preliminary annualised results is: each highlighted strategy beats the benchmarks of holding a constant 100% exposure to the risky asset, 100% exposure to cash or 60%equity-40%bond. Worth to be noted, the constant 60/40 portfolio tends to be outperformed by a vast majority of competitive specifications, highlighting the necessity to adopt a dynamic weighting scheme based on regime identification. The Market Sentiment-Business Cycle-Monetary Policy strategy is nearly two times higher in terms of annualised Sharpe ratio compared to this race-horse allocation.

The 1 year to 10 year portfolio holding rolling windows depicted in Appendix D.1 validate partially this first assessment. Tables 8 to 11 show the superiority of the highlighted strategies for each holding period considered. The strategy based on the univariate bull/bear market sentiment, the business cycle phase assessor and the monetary policy stance signal maximize the rolling Sharpe ratios. For 1 year- to 2 year- rolling holding periods, the strategy composed of the equity market sentiment and the monetary policy stance is the second best. For longer rolling holding periods however, the strategy based on the univariate bull/bear market sentiment performs quite well even though it remains below the 3-signal allocation approach. Finally, the more the holding period increases (e.g. from rolling 1 year to rolling 10 year) the better strategies incorporating monetary policy stance and business cycle phase assessors perform.

Table 1: Annualised performances for the five groups of competing strategies and benchmarks from January 2000 to February 2023

	Ann. Return	**Ann. Vol**	**Ann. SR**	**Ann. Max DD**
MS-Sign-Return-Multi	5.1%	9.8%	0.33	25.4%
MS-Bull-Bear-Return-Multi	5.6%	10.0%	0.37	22.8%
MS-Sign-Return-Uni	2.6%	11.0%	0.06	40.3%
MS-Bull-Bear-Return-Uni	**5.7%**	**9.3%**	**0.41**	**25.8%**
MS-Sign-Return-Multi-BC	5.9%	12.1%	0.33	37.2%
MS-Bull-Bear-Return-Multi-BC	7.4%	12.8%	0.43	36.9%
MS-Sign-Return-Uni-BC	5.4%	11.9%	0.30	36.8%
MS-Bull-Bear-Return-Uni-BC	**6.8%**	**10.6%**	**0.46**	**27.6%**
MS-Sign-Return-Multi-MP	4.1%	9.4%	0.23	24.8%
MS-Bull-Bear-Return-Multi-MP	6.3%	10.7%	0.41	21.9%
MS-Sign-Return-Uni-BC	3.6%	8.1%	0.21	21.4%
MS-Bull-Bear-Return-Uni-MP	**5.8%**	**8.2%**	**0.48**	**15.9%**
MP-BC	**7.4%**	**12.8%**	**0.43**	**42.6%**
MS-Sign-Return-Multi-MP-BC	5.8%	10.4%	0.38	33.3%
MS-Bull-Bear-Return-Multi-MP-BC	6.9%	11.3%	0.44	32.8%
MS-Sign-Return-Uni-MP-BC	5.6%	9.4%	0.40	27.9%
MS-Bull-Bear-Return-Uni-MP-BC	**6.8%**	**9.3%**	**0.52**	**22.9%**
S&P500	6.5%	18.1%	0.26	54.7%
60%Equity/40%Bond	4.8%	10.4%	0.28	32.1%
Cash3m	1.7%			

The Figure 8 displays the cumulative distribution of the rolling window Sharpe Ratios from 1 year to 10 year holding periods. An investor seeks thin left tails and the most right-skewed cumulative distributions. Albeit 1 year and 2 year rolling Sharpe ratios tend to display quite similar distributive patterns, longer rolling holding periods show that strategies implementing a broader set of signals yield a greater Sharpe ratio on a large part of the distribution support. The Market Sentiment-Business Cycle-Monetary Policy strategy yield on each point of the support a higher Sharpe ratio on a 10 year holding period. The longer the holding period, the more likely the investor will face an adverse shock, arising from a monetary policy restriction or a macroeconomic downturn. Thus, strategies incorporating business cycle or monetary policy regimes will be able to cope with those negative shocks by rebalancing the portfolio in a timely manner.

Figure 17 in Appendix D.2 shows the cumulative distribution of rolling returns for the 1 year to 10 year portfolio holding periods. Again we seek strategies with thin left tails (capped drawdowns) and right-skewed profile. Strategies taking into account market sentiment and either business cycle regimes, monetary policy stance or both highlight globally lower downside risks for the entire holding periods considered.

Figure 9 shows the log-total cumulative return of the strategies backtested on a weekly basis from 1st january 2000 to end of february 2023. Strikingly, being capable of timely identifying monetary and macroeconomic regimes yield far better long-term returns than the considered benchmarks. The log-scale enables to capture accelerations in the cumulative returns, this is the reason why fixed income assets (bond or cash) display flattened curves. Hence the pay-off is linear as the composed yield is timely incremental.

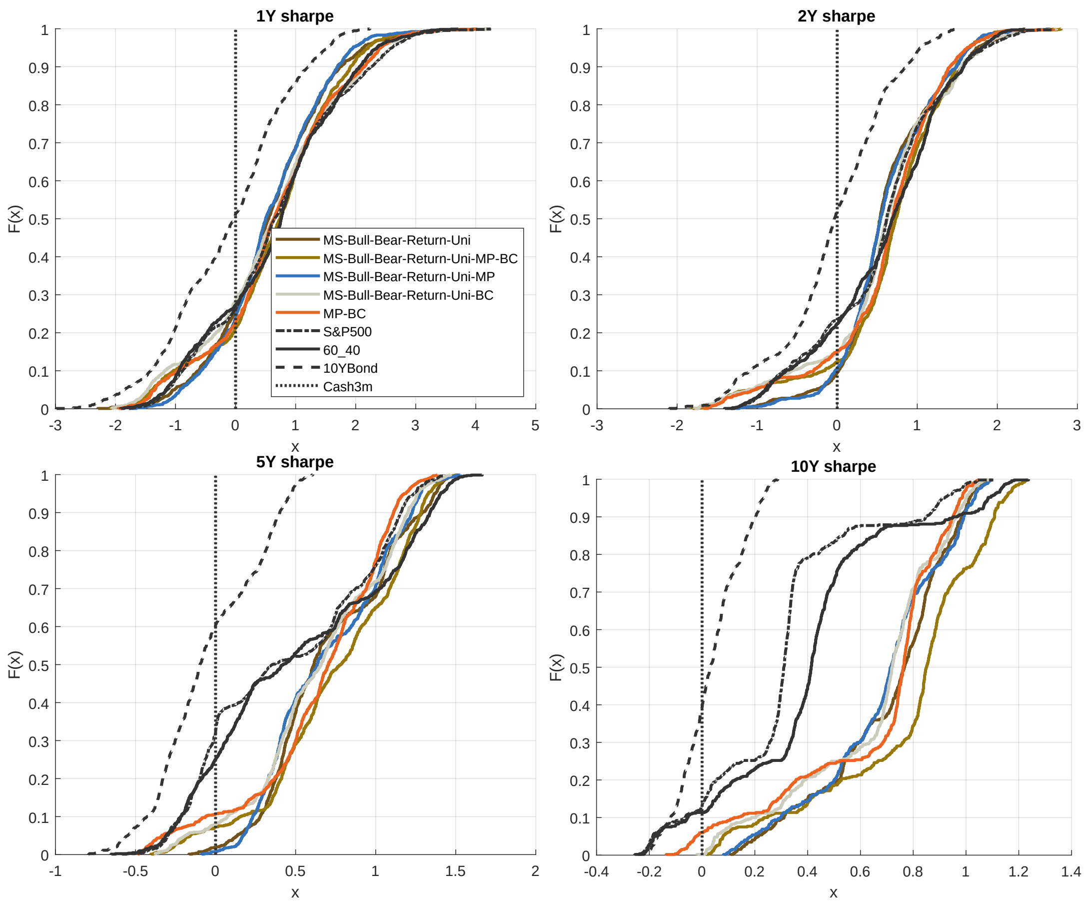

Figure 8: *Cumulative distribution functions of rolling window Sharpe ratios from 1Y to 10Y holding horizons*

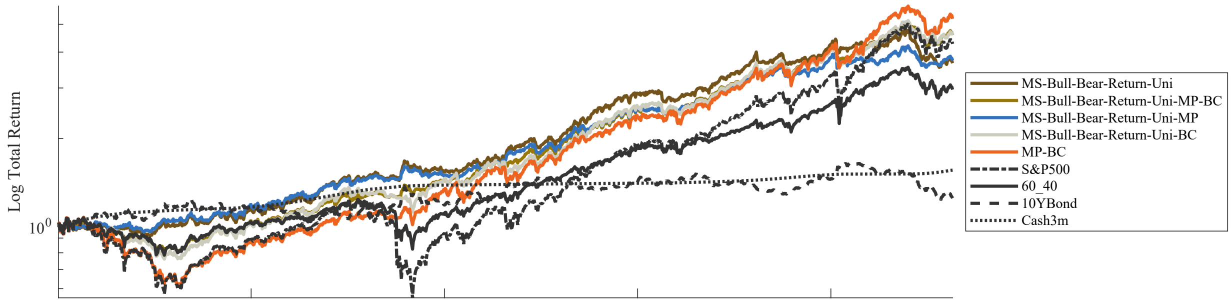

Figure 9: *Log total cumulative return of the selected strategies and benchmarks from 1st January 2000 to February 2023.*

We now focus on the economic utility for the investor to invest in a specific strategy. Tables 2 and 3 report the rolling annual average fees Δ for respectively 1- and 10 year holding periods. The values in the columns are the fees an investor would accept to pay to switch from the benchmark strategies towards the alternative ones. For each benchmark, two columns are presented referring to two aversion parameters γ in the quadratic utility in equation (5). On a 1 year rolling holding period (2), and for each aversion coefficient, an investor will tend to prefer any competitive strategy compared to holding a constant 100% exposure to S&P500 or a fixed 60/40 allocation. Ranking strategies based on absolute values of annual average fees Δs, one can highlight that an investor with low risk aversion ($\gamma = 1$) will tend to prefer a strategy mainly composed of the business cycle phase assessor and the monetary policy stance signal (MP-BC). An investor with a higher risk-aversion ($\gamma = 10$) will prefer the strategy adding the market sentiment to the monetary policy stance signal (MS-Bull-Bear-Return-Uni-MP). The 3-signal approach (MS-Bull-Bear-Return-Uni-MP-BC) compares fairly well to the other strategies as it is globally always the second preferred except for a switch with low aversion from a 100% cash strategy, making it the most robust strategy.

Table 2: 1Y Rolling window fees

1Y	**S&P500**		**60/40**		**Cash**	
γ	1	10	1	10	1	10
MS-Bull-Bear-Return-Uni	0.92%	13.80%	1.69%	2.60%	5.17%	1.16%
MS-Bull-Bear-Return-Uni-BC	0.66%	11.98%	2.00%	1.72%	5.99%	0.84%
MS-Bull-Bear-Return-Uni-MP	0.65%	14.45%	1.38%	3.14%	4.75%	1.55%
MP-BC	1.26%	10.59%	2.65%	0.42%	6.81%	-0.32%
MS-Bull-Bear-Return-Uni-MP-BC	0.97%	13.93%	2.03%	3.03%	5.78%	1.84%

On a 10 year rolling holding period (3), and for each aversion coefficient, an investor prefer any competitive strategy compared to holding a constant 100% exposure to S&P500 or a fixed 60/40 allocation. Ranking strategies based on absolute values of annual average fees Δs again, an investor with low risk aversion ($\gamma = 1$) prefer a strategy mainly composed of the business cycle phase assessor and the monetary policy stance signal. An investor with a higher risk-aversion ($\gamma = 10$)

will prefer the strategy adding the market sentiment to the two signal mentioned above (MS-Bull-Bear-Return-Uni-MP-BC). Globally, the strategy applying the 3-signal approach compares very well and is the most reliable strategy when considering different aversion profiles and holding periods.

Table 3: 10Y Rolling window fees

10Y	**S&P500**		**60/40**		**Cash**	
γ	1	10	1	10	1	10
MS-Bull-Bear-Return-Uni	1.20%	13.21%	2.49%	3.32%	6.37%	2.40%
MS-Bull-Bear-Return-Uni-BC	1.41%	12.46%	2.72%	2.63%	6.63%	1.73%
MS-Bull-Bear-Return-Uni-MP	0.28%	13.24%	1.56%	3.36%	5.40%	2.43%
MP-BC	2.24%	11.79%	3.57%	2.00%	7.52%	1.12%
MS-Bull-Bear-Return-Uni-MP-BC	1.39%	14.04%	2.70%	4.09%	6.60%	3.18%

5.3. Event study

In this subsection we focus on specific monetary and macroeconomic events to understand precisely the added value of the 3-signal strategy compared to the alternative ones. In our dataset, there are three occurrences of macroeconomic downturns. The Dot com recession in 2001, the Great Financial Crisis from December of 2007 to May 2009 and the Covid recession. From the monetary policy standpoint, our dataset is composed of four rate hiking cycles as well as four cutting cycles. We have first the January to May 2000 monetary restriction phase, which in fact started in June 1999, intended to deflate the dot-com growing bubble and cooling down an economy at the time around its potential. The equity bubble started to crack in March 2000 and slowly to diffuse into the real economy, hence forcing the Federal Reserve to react and cut rates from January 2001 to December 2001, fighting the 8-month long recession and following a Taylor-type rule (Taylor (2007)). The recovery at the time was muted (jobless recovery as mentioned by Bernanke (2010)) and the geopolitical uncertainty brought by middle east wars made the Federal Market Committee decide two rate cuts in November 2002 and June 2003 to fight deflation fears. Starting from this final cut, US growth picked up again in 2004 and 2005. In the wake of this economic expansion, real estate imbalances started to appear obliging the Fed to dampen inflationary pressures from June 2004 to June 2006. The following housing burst started to spread to real economy with rising unemployment and triggered an easing cycle starting from September 2007 to April 2008. The Fed paused the easing cycle afterwards to evaluate the effects of its action. Recession began in December 2007 inducing financial confusion translating into the worst banking crisis in history deepening the macroeconomic downturn and forcing the Fed to reach the zero-lower bound (first-time in history) and to provide ample liquidity to the markets by implementing balance-sheet tools. By its amplitude and duration, the great financial recession became the worst economic downturn in US economic history. Hence bringing tthe necessity for monetary policy to use a set of new tools such as forward guidance and quantitative easing (Bernanke (2020)). After a false hawkish signal known as "taper tantrum" in May 2013 and in the wake of a recovering economy the FOMC proceeded to a gradual policy normalisation from December 2015 to December 2018. Growing concerns about inflation path and economic uncertainty induced by the Trade War between the US and China drove the FOMC to proceed to a "mid-cycle policy adjustment" in 2019. The outbreak of Covid-19 led a two-month recession far fom historical standards by its duration and amplitude but obliged the FOMC to reach the zero-lower bound and provide historically ample liquidity to the markets again. The historic expansive policy mix in a supply-constrained world economy, amplified by the energy shock provoked by Ukraine War in February 2022 forced the FOMC to proceed to an expeditive monetary restriction from March 2022 to July 2023.

Figure 9 shows us that S&P500 was the asset class benefiting the most of the sequence described above compared to cash or long-term treasury bonds. The 60/40 portfolio suffers from being constantly underexposed to the equity market. This fact is also depicted in Table 1. We now turn to the analysis of the strategies during "normal" and "abnormal" times. We characterize "abnormal" times as periods during which macroeconomic downturns occur or tightening monetary policy are implemented. "Normal" times are periods of economic expansion and either neutral monetary policy stance or easing ones. Only two years over our 23 year long sample have experienced a monetary easing cycle: 2007 and 2019.

Table 4 displays annual Sharpe ratios during recession years. Given the "safe heaven" nature of long-term Treasury bonds, this is the only asset class which performs on average positively during recession years. Moreover, recession periods are accompanied with monetary policy easing to counter deflationary pressures or unsustainable high employment with respect to the Federal Reserve dual mandate. Yet, the profile remain very heterogeneous across downturn years as the 2009 recovery has pushed long term yields up,as part of market reflation expectations mechanism. Equity indices suffer from macroeconomic headwinds, on average the Sharpe ratios are slightly negative. Yet, 2009 recession exit and ample monetary support made the S&P500 skyrocket in the second part of the year, thus yielding a positive annual Sharpe ratio (the 2009 market recovery was steady as displayed in Figure 10). Likewise, the 2020 recession brought the Sharpe ratio in positive territories given the historical monetary and budgetary stimulus in US economy (as the cumulative returns show in Figure 11). The constant 60/40 portfolio also display a slight negative Sharpe ratio on average during recession years. Two competitive strategies beat the 60/40 constant portfolio on average during recession years: the strategy implementing a monetary stance/business cycle detection (MP-BC) with a positive Sharpe ratio on average. The second best remains the strategy adding the Market sentiment to the above mentioned 2 signals (MS-Bull-Bear-Return-Uni-MP-BC). This result is also even more striking when looking at returns themselves in Table 13 showed in Appendix D.3.

Table 4: Annual Sharpe Ratios during recessions

	2001	2008	2009	2020	Average
MS-Bull-Bear-Return-Uni	-1.24	1.16	-0.86	-0.11	-0.26
MS-Bull-Bear-Return-Uni-BC	-1.34	-1.53	1.29	0.13	-0.36
MS-Bull-Bear-Return-Uni-MP	-1.15	1.05	-1.05	-0.24	-0.35
MP-BC	**-1.29**	**0.44**	**1.15**	**0.39**	**0.17**
MS-Bull-Bear-Return-Uni-MP-BC	**-1.56**	**0.84**	**0.54**	**0.15**	**-0.01**
S&P500	-0.65	-1.29	1.20	0.50	-0.06
60/40	-0.78	-1.17	0.87	0.83	-0.06
10YBond	-0.63	1.54	-1.28	1.28	0.23

Table 5 displays annual Sharpe ratios during monetary tightening cycles. Long term Treasuries generally suffer from Monetary policy tightening cycles as yield rise along the maturity curve. This explains the average negative Sharpe ratio. The 60/40 portfolio suffers from this Bond dynamic. Surprisingly the S&P500 index resists quite well to those monetary restriction periods with an average positive Sharpe ratio. The Table shows nonetheless the ability of strategies incorporating business cycle phases and monetary policy stance or the 3-signal approach to outperform the 60/40 portfolio. When looking at yearly returns in Table 14, Appendix D.3, adding the monetary policy stance signal always enable to cap losses. 2022 is a text-book case of restrictive monetary policy. Table 14 shows the ability of strategies incorporating either the market sentiment and monetary policy stance or the 3-signal approach to drastically reduce the loss incurred. The profile of the cumulative returns shown in Figure 12 confirm this fact. Adding the monetary policy stance to the market sentiment or to the market sentiment and business cycle phase assessor combined during 2022 allows the investor with a quadratic utility function to always prefer being invested in those strategies compared to the S&P500 or the 60/40 portfolio (Table 6). The results in the Table 6 are quite intuitive: the investor would always have chosen to remain fully cash invested, no matter his aversion profile. Nonetheless, compared to the other 2 benchmark strategies and for each aversion parameter γ, he would have chosen a strategy capable of tracking the monetary policy risk. This is in line with Cochrane (1999) who highlights that risk-averse investors might favour a portfolio with lower Sharpe ratio in a context of time-varying risk and return, if it is able to offer a hedge during times of financial distress.

Table 5: Annual Sharpe Ratios during monetary policy tightening

	2000	2004	2005	2006	2015	2016	2017	2018	2022	Average
MS-Bull-Bear-Return-Uni	-0.73	0.73	0.10	1.03	-0.61	1.45	3.56	-0.40	-2.40	0.30
MS-Bull-Bear-Return-Uni-BC	-0.59	0.94	0.12	1.05	-0.21	0.94	3.56	-0.44	-1.18	0.47
MS-Bull-Bear-Return-Uni-MP	-0.69	1.06	0.09	0.76	-0.42	1.55	3.40	-0.37	-1.66	0.41
MP-BC	**-0.46**	**1.31**	**0.11**	**0.77**	**0.17**	**0.81**	**3.41**	**-0.37**	**-0.95**	**0.53**
MS-Bull-Bear-Return-Uni-MP-BC	**-0.55**	**1.19**	**0.10**	**0.77**	**-0.06**	**1.22**	**3.40**	**-0.39**	**-1.38**	**0.48**
S&P500	-0.62	1.02	0.14	1.05	0.03	0.80	3.57	-0.41	-0.89	0.52
60/40	-0.52	0.94	-0.21	0.41	0.05	0.76	3.19	-0.59	-1.33	0.30
10YBond	0.32	-0.30	-1.00	-1.78	-0.07	-0.28	-0.28	-1.14	-2.11	-0.74

Table 6: 2022 annual fee

2022	S&P500		60/40		Cash	
γ	1	10	1	10	1	10
MS-Bull-Bear-Return-Uni	-7.39%	10.13%	-7.18%	-2.63%	-25.32%	-29.34%
MS-Bull-Bear-Return-Uni-BC	6.02%	23.53%	6.27%	9.19%	-14.45%	-20.82%
MS-Bull-Bear-Return-Uni-MP	8.91%	32.70%	9.17%	17.45%	-12.11%	-14.52%
MP-BC	8.13%	23.88%	8.38%	9.45%	-12.74%	-20.73%
MS-Bull-Bear-Return-Uni-MP-BC	8.77%	30.94%	9.03%	15.85%	-12.22%	-15.76%

Table 7 displays the annual Sharpe ratios of the competing strategies in normal times. At first glance, the 60/40 portfolio is the one yielding the higher risk-scaled return on average. S&P 500 is the second asset class with regard to historical standards. The strategies we propose exhibit Sharpe ratios superior to 1 and beat the 100% Bond portfolio. This result validate the idea the investor pays an insurance fee against "abnormal episodes". If the economic and monetary cycles were only composed of steady expanding periods one could consider the 60/40 portfolio as the best solution for asset allocation. This is also the reason why this allocation strategy has been the race-horse for decade in asset management industry. The annual returns in "normal" periods displayed in Appendix D.3, Table12 confirm the S&P500 yields the better performance. At a cost of higher volatility, all alternative strategies except the one incorporating the risk sentiment and the monetary policy stance signal tend to outperform the 60/40 portfolio on average.

Table 7: Annual Sharpe Ratios during "normal times"

	2002	2003	2007	2010	2011	2012	2013	2014	2019	2021	Average
MS-Bull-Bear-Return-Uni	0.88	0.33	0.05	0.91	1.55	0.25	3.1	1.16	1.91	1.48	1.17
MS-Bull-Bear-Return-Uni-BC	-1.56	1.68	-0.04	0.72	0.23	0.71	3.2	1.27	2.29	2.36	1.09
MS-Bull-Bear-Return-Uni-MP	1.23	0.07	-0.04	0.84	1.81	0.18	2.5	1.13	1.89	1.51	1.12
MP-BC	**-1.62**	**1.63**	**-0.07**	**0.61**	**0.14**	**1.06**	**2.6**	**1.38**	**2.70**	**2.55**	**1.10**
MS-Bull-Bear-Return-Uni-MP-BC	**-1.39**	**1.51**	**-0.06**	**0.92**	**0.99**	**0.71**	**2.6**	**1.25**	**2.47**	**2.16**	**1.12**
S&P500	-1.25	1.69	0.06	0.77	0.08	1.16	3.3	1.38	2.74	2.51	1.24
60/40	-1.17	1.70	0.08	1.11	0.70	1.72	2.1	2.12	3.36	2.18	1.39
10YBond	1.17	-0.33	-0.14	0.64	1.51	0.34	-1.4	1.30	0.82	-0.71	0.32

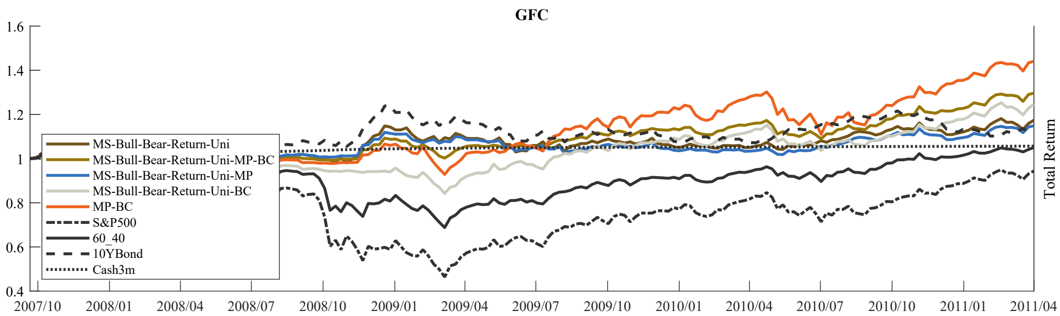

Figure 10: *Cumulative return of the selected strategies and benchmarks during the Great Financial Crisis and its aftermath*

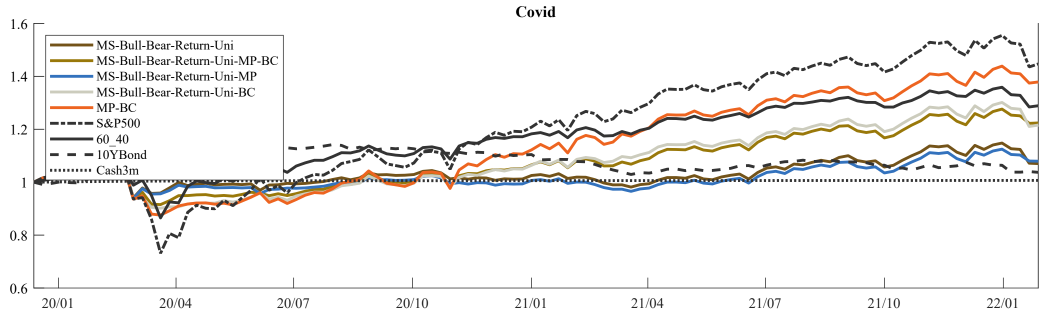

Figure 11: *Cumulative return of the selected strategies and benchmarks during the Covid crisis and its aftermath*

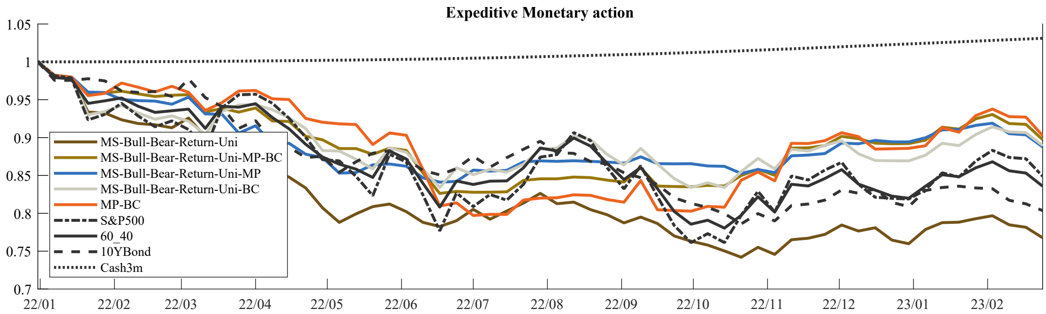

Figure 12: *Cumulative return of the selected strategies and benchmarks during the 2022 monetary policy tightening*

6. Conclusion

Investor seeking consistent returns across economic phases, both from a business cycle and monetary policy perspective, face challenging questions regarding portfolio construction. Trying to time the occurrence of the phases on each of those axes can be of a great relevance to that aim. This work shows the added value for an investor who wants to diversify its portfolio to take into account business cycle dynamics, market sentiment and monetary policy stance. The paper contributions to the regime-contingent asset allocation literature are three-fold.

It first introduces a novel approach to gauge the monetary policy regime through real interest rates dynamics using a Markov-switching dynamic factor model capturing the co-movement of long-term maturities real yields. This model proves to be reliable in identifying monetary policy restriction signals both in-sample and out-of-sample using weekly market data.

It then extends Maheu et al. (2012) bull/bear specification into a dynamic factor model in order to capture a multivariate equity market sentiment allowing for bull corrections and bear rallies. This signal succeeds in capturing an underlying market sentiment across four major US equity indices.

The paper finally combines the monetary policy stance signal with the market sentiment and a weekly real-time business cycle phase assessor within a long-only asset allocation framework. The benefit for an investor to take into account those three dimensions is paramount to weigh dynamically its portfolio. The regimes along those axes and their underlying market prices dynamics warrant allocations beyond the traditional fixed 60/40 equity/bond split. The backtests implemented show that taking into account the three-signal approach in a dynamic equity/bond/cash hedging strategy maximizes the Sharpe ratio between the 7th of January 2000 to the 24th February 2023. Moreover, no matter the portfolio holding period considered (rolling 1 year to 10 year windows) or the monetary/economic regime faced the returns and Sharpe ratios are higher than the 60/40 benchmark advocating this approach is reliable for an investor who seeks steady returns. The economic utility of the investor in strategies incorporating those three cycles shows to be maximal on 1Y to 10Y rolling windows regardless of his/her risk aversion profile.

This promising framework could be used in a bigger investment universe. One could also consider the assets weighting scheme being defined by traditional portfolio construction optimizations such as mean-variance, constant volatility or a risk parity approach. A multi-country adaptation of this model would be relevant to allow for a geographical diversification. Those questions are left for further research.

References

J. H. Albert and S. Chib. Bayesian analysis of binary and polychotomous response data. *Journal of the American Statistical Association*, 88(422):669–679, 1993.

A. Ang and G. Bekaert. How do regimes affect asset allocation. *Financial Analysts Journal*, 60: 86–99, 03 2004.

A. Ang and A. Timmermann. Regime changes and financial markets. *Annual Review of Financial Economics*, 4(1):313–337, 2012.

R. Aumond. Asset swap spreads as business cycle assessors : a markov switching dynamic factor with time-varying variance extension. *2nd thesis chapter*, 2024.

R. Aumond and J. Royer. Improving the robustness of Markov-switching dynamic factor models with time-varying volatility. (2024-04), Mar. 2024.

J. Bai and P. Wang. Conditional markov chain and its application in economic time series analysis. *Journal of Applied Econometrics*, 26(5):715–734, 2011.

B. Bernanke. Monetary policy and the housing bubble: a speech at the annual meeting of the american economic association, atlanta, georgia, january 3, 2010. (499), 2010.

B. Bernanke and K. Kuttner. What explains the stock market's reaction to federal reserve policy? *Journal of Finance*, 60(3):1221–1257, 2005.

B. S. Bernanke. The new tools of monetary policy. *American Economic Review*, 110(4):943–83, April 2020.

F. Black. Studies of stock price volatility changes, proceedings of the business and economic statistical section. *American Statistical Association*, pages 177–181, 1976.

M. Boeck and M. Feldkircher. The impact of monetary policy on yield curve expectations. *Journal of Economic Behavior and Organization*, 191:887–901, 2021.

E. Bouyé and J. Teiletche. Regime-based strategic asset allocation. *The World Bank, Treasury, RAMP*, 04 2024.

J. Brocato and S. Steed. Optimal Asset Allocation over the Business Cycle. *The Financial Review*, 33(3):129–148, August 1998.

M. Camacho, G. Perez-Quiros, and P. Poncela. Extracting nonlinear signals from several economic indicators. *Journal of Applied Econometrics*, 30(7):1073–1089, 2015.

C. K. Carter and R. Kohn. On gibbs sampling for state space models. *Biometrika*, 81(3):541–53, 1994.

J. C. Chan and A. L. Grant. Modeling energy price dynamics: GARCH versus stochastic volatility. *Energy Economics*, 54:182–189, 2016.

Y. Chang, J. Maih, and F. Tan. Origins of monetary policy shifts: A new approach to regime switching in dsge models. *Journal of Economic Dynamics and Control*, 133(C), 2021.

M. Chauvet. An econometric characterization of business cycle dynamics with factor structure and regime switches. *International Economic Review*, 39(4):969–996, 1998.

R. Clarida, J. Gali, and M. Gertler. The science of monetary policy: A new keynesian perspective. *Journal of Economic Literature*, 37(4):1661–1707, December 1999. doi: 10.1257/jel.37.4.1661.

J. H. Cochrane. Portfolio Advice for a Multifactor World. *NBER Working Papers*, (7170), June 1999.

C. Doz, L. Ferrara, and P.-A. Pionnier. Business cycle dynamics after the great recession: An extended markov-switching dynamic factor model. *PSE Working Papers*, 2020.

J. Fleming, C. Kirby, and B. Ostdiek. The economic value of volatility timing using "realized" volatility. *Journal of Financial Economics*, 67(3):473–509, 2003.

J. F. Geweke. Contemporary bayesian econometrics and statistics. *Wiley series in probability and statistics*, 2005.

J. Hamilton. Rational-expectations econometric analysis of changes in regime: An investigation of the term structure of interest rates. *Journal of Economic Dynamics and Control*, 12(2-3): 385–423, 1988.

J. Hamilton and L. Gang. Stock market volatility and the business cycle. *Journal of Applied Econometrics*, 11(5):573–93, 1996.

J. D. Hamilton. A new approach to the economic analysis of nonstationary time series and the business cycle. *Econometrica: Journal of the Econometric Society*, page 357–384, 1989.

J. D. Hamilton. Time series analysis. *Princeton University Press*, 1994.

M. Hamilton and A. de Longis. Dynamic asset allocation through the business cycle: A macro regime approach. *OppenheimerFunds*, 2015.

M. Jarociński and P. Karadi. Deconstructing monetary policy surprises—the role of information shocks. *American Economic Journal: Macroeconomics*, 12(2):1–43, April 2020.

G. R. Jensen and J. M. Mercer. New evidence on optimal asset allocation. *The Financial Review*, 38(3):435–454, 2003.

C. Kim and C. Nelson. Business cycle turning points, a new coincident index and tests of duration dependence based on a dynamic factor model with regime switching. *The Review of Economics and Statistics*, 80(2):188–201, 1998.

C. J. Kim and C. R. Nelson. *State space models with regime switching.* The MIT Press, Cambridge, Massachusetts, 1999.

C.-J. Kim, J. C. Morley, and C. Nelson. Is There a Positive Relationship between Stock Market Volatility and the Equity Premium? (0023), Feb. 2000.

M. J. Kim and D. Kwon. Dynamic asset allocation strategy: an economic regime approach. *Journal of Asset Management*, 24(2):136–147, 2023.

M. Kollar and C. Schmieder. Macro-based asset allocation: An empirical analysis. EIB Working Papers 2019/11, European Investment Bank (EIB), Nov. 2019.

M. Kritzman, S. Page, and D. Turkington. Regime shifts: Implications for dynamic strategies. *Financial Analysts Journal*, 68(3):22–39, 2012.

M. L. Leibowitz, E. H. Sorensen, R. D. Arnott, and H. N. Hanson. A total differential approach to equity duration. *Financial Analysts Journal*, 45(5):30–37, 1989.

D. Leiva-Leon, G. Pérez-Quirós, and E. Rots. Real-time weakness of the global economy: A first assessment of the coronavirus crisis. *Working Paper Series, European Central Bank*, 2381, 2020.

J. Liu, J. Maheu, and Y. Song. Identification and forecasting of bull and bear markets using multivariate returns. Technical report, 04 2024.

J. Maheu and T. McCurdy. Identifying bull and bear markets in stock returns. *Journal of Business & Economic Statistics*, 18(1):100–112, 2000.

J. Maheu, T. McCurdy, and Y. Song. Components of bull and bear markets: Bull corrections and bear rallies. *Journal of Business & Economic Statistics*, 30(3):391–403, 2012.

H. Markowitz. Portfolio selection. *The Journal of Finance*, 7(1):77–91, 1952.

E. Nakamura and J. Steinsson. High-frequency identification of monetary non-neutrality: The information effect. *The Quarterly Journal of Economics*, 133(3):1283–1330, 2018.

A. Orphanides. Historical monetary policy analysis and the Taylor rule. *Journal of Monetary Economics*, 50(5):983–1022, July 2003.

A. Perruchoud. Estimating a taylor rule with markov switching regimes for switzerland. *Swiss Journal of Economics and Statistics (SJES)*, 145(II):187–220, 2009.

R. Rigobon and B. Sack. The impact of monetary policy on asset prices. *Journal of Monetary Economics*, 51(8):1553–1575, 2004.

W. F. Sharpe. The sharpe ratio. pages 169–178, 1994.

J. J. Siegel. Does it pay stock investors to forecast the business cycle? *The Journal of Portfolio Management*, 18(1):27–34, 1991.

C. Sims and T. Zha. Were there regime switches in u.s. monetary policy? FRB Atlanta Working Paper 2004-14, Federal Reserve Bank of Atlanta, 2004.

A. L. Smith and V. J. Valcarcel. The financial market effects of unwinding the federal reserve's balance sheet. *Journal of Economic Dynamics and Control*, 146:104582, 2023.

J. Taylor. Housing and monetary policy. *NBER Working Papers*, (13682), 2007.

J. B. Taylor. Discretion versus policy rules in practice. *Carnegie-Rochester Conference Series on Public Policy*, 39:195–214, 1993.

W. Thorbecke. On Stock Market Returns and Monetary Policy. *Journal of Finance*, 52(2):635–654, June 1997.

M. Woodford. Interest and prices: Foundations of a theory of monetary policy. *Princeton University Press*, page 462–468, 2003.

Appendices

A. Priors

A.1. Monetary policy stance identification

This section describes the priors used for the distributions of the parameter vector $\boldsymbol{\vartheta}^{(MP)}$.
λ_1 is set to one for identification purposes. For all $j = 2, \ldots, m$, we use the following prior to sample λ_j the j-th element of the factor loading matrix $\mathbf{\Lambda}$ in (1)

$$\lambda_j \sim \mathcal{N}(a_j, A_j) \tag{6}$$

where hyperparameters are set to $a_j = 0$ and $A_j = 0.1$. To sample the parameters linked to the residuals $u_{j,t}$ in (2), we use the following priors, for $l = 1$,

$$\begin{aligned} \psi_{j,l} \sim &\quad \mathcal{N}(\pi, \Pi) \quad \pi = 0, \Pi = 0.1 \\ \sigma^2_{e,j} \sim &\quad IG(\nu_i, Z_i) \quad \nu_i = 10, Z_i = 2 \end{aligned} \tag{7}$$

where IG denotes the inverse-gamma distribution. Each row of $\mathbf{P}$ follows a Dirichlet distribution.

$$\begin{aligned} \{p_{1,1}, p_{1,2}, p_{1,3}, p_{1,4}\} &\sim Dir(2000, .5, 150, .5) \\ \{p_{2,1}, p_{2,2}, p_{2,3}, p_{2,4}\} &\sim Dir(.5, 2000, .5, 150) \\ \{p_{3,1}, p_{3,2}, p_{3,3}, p_{3,4}\} &\sim Dir(150, 15, 2000, 100) \\ \{p_{4,1}, p_{4,2}, p_{4,3}, p_{4,4}\} &\sim Dir(.5, 150, 100, 2000) \end{aligned}$$

The prior for the Markov-switching intercept in equation (3) is given by :

$$\boldsymbol{\mu} = (\mu_1, \mu_2, \mu_3, \mu_4)' \sim \mathcal{N}(\alpha^*, A^*) \tag{8}$$

with $\alpha^* = (-.7, .2, -.2, .3)'$ and $A^* = \mathbf{I}_4$.

The prior for the factor variance in equation (3) is given by, for $i = 1, ..., 4$:

$$\sigma^2_i \sim \quad IG(\nu^*_i, Z^*_i) \quad \nu^*_i = 1, Z^*_i = 20$$

A.2. Business Cycle turning point detection

This section describes the priors used for the distributions of the parameter vector $\boldsymbol{\vartheta}^{(BC)}$.
λ_1 is set to one for identification purposes. For all $j = 2, \ldots, m$, we use the following prior to sample λ_j the j-th element of the factor loading matrix $\mathbf{\Lambda}$ in (1)

$$\lambda_j \sim \mathcal{N}(a_j, A_j) \tag{9}$$

where hyperparameters are set to $a_j = 0$ and $A_j = 0.1$. To sample the parameters linked to the residuals $u_{j,t}$ in (2), we use the following priors, for $l = 1, 2$,

$$\begin{aligned} \psi_{j,l} \sim &\quad \mathcal{N}(\pi, \Pi) \quad \pi = 0, \Pi = 0.1 \\ \sigma^2_{e,j} \sim &\quad IG(\nu_i, Z_i) \quad \nu_i = 10, Z_i = 2 \end{aligned} \tag{10}$$

where IG denotes the inverse-gamma distribution. Additionally, independent beta distributions can be used as conjugate prior for each transition probability

$$\pi(q, p) \propto q^{u_{00}}(1-q)^{u_{01}} p^{u_{11}}(1-p)^{u_{10}} \tag{11}$$

As in Doz et al. (2020), we put an informative prior and set $u_{00} = 470, u_{01} = 9, u_{10} = 9, u_{11} = 90$ in order to take into account the relative persistence of each of the regimes as observed on macroeconomic data. The prior for the Markov-switching intercept in equation (3) is given by :

$$\boldsymbol{\mu} = (\mu_0, \mu_1)' \sim \mathcal{N}(\alpha^*, A^*) \tag{12}$$

with $\alpha^* = (4, -2)'$ and $A^* = \text{diag}(0.02, 0.02)$. We acknowledge that, in the spirit of Leiva-Leon et al. (2020), relatively tight priors are used for identification purposes. The informativeness brought by the first moment is indeed needed to discriminate between the regimes over the parameters space. The prior for the autoregressive parameter ϕ in equation (3) is given by

$$\phi \sim \mathcal{N}(\alpha, A) \tag{13}$$

where $\alpha = 0$, $A = 0.1$. In the case of a MS-DFM-ARCH, we use the following prior for the vector $\boldsymbol{\theta}^{(\text{ARCH})} = (\omega, \alpha)$

$$\log \boldsymbol{\theta}^{(\text{ARCH})} \sim \mathcal{N}(\theta_0^{(\text{ARCH})}, V_\theta)\mathbb{1}(\alpha < 1).$$

$\boldsymbol{\theta}^{(\text{ARCH})}$ thus follows a truncated log-normal distribution with the stationarity restriction that $\alpha < 1$. We set the hyperparameters to $\theta_0^{(\text{ARCH})} = \log(1, 0.5)$ and $V_\theta = \text{diag}(1, 1)$.

A.3. Market sentiment

This section describes the priors used for the distributions of the parameter vector $\boldsymbol{\vartheta}^{(MP)}$. λ_1 is set to one for identification purposes. For all $j = 2, \ldots, m$, we use the following prior to sample λ_j the j-th element of the factor loading matrix $\boldsymbol{\Lambda}$ in (1)

$$\lambda_j \sim \mathcal{N}(a_j, A_j) \tag{14}$$

where hyperparameters are set to $a_j = 0$ and $A_j = 0.1$. To sample the parameters linked to the residuals $u_{j,t}$ in (2), we use the following priors, for $l = 1$,

$$\begin{aligned} \psi_{j,l} \sim &\quad \mathcal{N}(\pi, \Pi) \quad \pi = 0, \Pi = 0.1 \\ \sigma^2_{e,j} \sim &\quad IG(\nu_i, Z_i) \quad \nu_i = 10, Z_i = 2 \end{aligned} \tag{15}$$

where IG denotes the inverse-gamma distribution. Each row of $\mathbf{P}$ follows a Dirichlet distribution.

$$\{p_{1,1}, p_{1,2}, p_{1,4}\} \sim Dir(8, 1.5, 0.5)$$
$$\{p_{2,1}, p_{2,2}, p_{2,4}\} \sim Dir(1.5, 8, 0.5)$$
$$\{p_{3,1}, p_{3,3}, p_{3,4}\} \sim Dir(0.5, 8, 1.5)$$
$$\{p_{4,1}, p_{4,3}, p_{4,4}\} \sim Dir(0.5, 1.5, 8)$$

The prior for the Markov-switching intercept in equation (3) is given by :

$$\boldsymbol{\mu} = (\mu_1, \mu_2, \mu_3, \mu_4)' \sim \mathcal{N}(\alpha^*, A^*) \tag{16}$$

with $\alpha^* = (-.7, .2, -.2, .3)'$ and $A^* = \mathbf{I}_4$.

The prior for the factor variance in equation (3) is given by, for $i = 1, ..., 4$:

$$\sigma_i^2 \sim \quad IG(\nu_i^*, Z_i^*) \quad \nu_i^* = .5, Z_i^* = 20$$

B. Bayesian Estimation

Let $\boldsymbol{z}^{(T)} = \{\boldsymbol{z}_1, ..., \boldsymbol{z}_T\}$ the unobserved state, $\boldsymbol{y}^{(T)} = \{\boldsymbol{y}_1, ..., \boldsymbol{y}_T\}$ the observed data and $S^{(T)} = \{S_1, ..., S_T\}$ the first order Markov-Chain. We describe the Gibbs sampler steps based on Kim and Nelson (1999) and follow their notations. The Gibbs sampler consists of iterating between the three following steps sequentially.

B.1. Generation of the state vector

The joint distribution of $\boldsymbol{z}^{(T)}$, given $\boldsymbol{y}^{(T)}$, $S^{(T)}$ and $\boldsymbol{\vartheta}^{(\cdot)}$ can be defined as

$$p(\boldsymbol{z}^{(T)} \mid \boldsymbol{y}^{(T)}, S^{(T)}, \boldsymbol{\vartheta}^{(\cdot)}) = p(\boldsymbol{z}_T \mid \boldsymbol{y}^{(T)}, S^{(T)}, \boldsymbol{\vartheta}^{(\cdot)}) \prod_{t=1}^{T-1} p(\boldsymbol{z}_t \mid \boldsymbol{y}^{(t)}, S^{(t)}, \boldsymbol{\vartheta}^{(\cdot)}, \boldsymbol{z}_{t+1})$$

which boils down to generating $\boldsymbol{z}_t$ for $t = T, T-1, ..., 1$ from

$$\begin{aligned} &\boldsymbol{z}_T \mid \boldsymbol{y}^{(T)}, S^{(T)}, \boldsymbol{\vartheta}^{(\cdot)} \sim \mathcal{N}(\boldsymbol{z}_{T|T}, \boldsymbol{V}_{T|T}) \\ &\boldsymbol{z}_t \mid \boldsymbol{y}^{(t)}, S^{(t)}, \boldsymbol{z}_{t+1}, \boldsymbol{\vartheta}^{(\cdot)} \sim \mathcal{N}(\boldsymbol{z}_{t|t,\boldsymbol{z}_{t+1}}, \boldsymbol{V}_{t|t,\boldsymbol{z}_{t+1}}) \end{aligned} \tag{17}$$

where $\boldsymbol{z}_{t|t} = E(\boldsymbol{z}_t \mid \boldsymbol{y}^{(t)})$ and $\boldsymbol{V}_{t|t} = Var(\boldsymbol{z}_t \mid \boldsymbol{y}^{(t)})$ for $t = 1, ..., T$. In equation (17), $\boldsymbol{z}_T \mid \boldsymbol{y}^{(T)}, S^{(T)}, \boldsymbol{\vartheta}^{(\cdot)}$ can be generated using the Multi-move Gibbs sampling introduced by Carter and Kohn (1994) as follows

1. We use the Kalman filter to obtain $\boldsymbol{z}_{t|t}$ and $\boldsymbol{V}_{t|t}$ for $t = 1, ..., T$. The last iteration of the filter gives $\boldsymbol{z}_{T|T}$ and $\boldsymbol{V}_{T|T}$ which are then used to generate $\boldsymbol{z}_T$.

2. For $t = T-1, T-2, ..., 1$, $\boldsymbol{z}_{t|t}$ and $\boldsymbol{V}_{t|t}$, $\boldsymbol{z}_{t+1}$ can be considered as an incremental vector

of observations in the system. The distribution $p(\boldsymbol{z}_t \mid \boldsymbol{y}^{(T)}, S^{(t)}, \boldsymbol{\vartheta}^{(\cdot)}, \boldsymbol{z}_{t+1})$ is then deduced from the Kalman smoother. From equation (4), updating equation are then given by

$$\begin{aligned}\boldsymbol{z}_{t|t,\boldsymbol{z}_{t+1}} &= \boldsymbol{z}_{t|t} + \boldsymbol{V}_{t|t}\boldsymbol{\Xi}\tilde{\varsigma}_t/R_t \\ \boldsymbol{V}_{t|t,\boldsymbol{z}_{t+1}} &= \boldsymbol{V}_{t|t} - \boldsymbol{V}_{t|t}\boldsymbol{\Xi}'\boldsymbol{\Xi}\boldsymbol{V}'_{t|t}/R_t\end{aligned}$$

where $\tilde{\varsigma}_t = \boldsymbol{z}_{t+1} - \boldsymbol{\delta}_{S_{t+1}} - \boldsymbol{\Xi}\boldsymbol{z}_{t|t}$ and $R_t = \boldsymbol{\Xi}\boldsymbol{V}_{t|t}\boldsymbol{\Xi}' + \sigma^2_{t+1}$.

B.2. Generation of the Markov Chain

Once $\boldsymbol{z}^{(T)}$ has been simulated, given $\boldsymbol{\vartheta}^{(\cdot)}$, the Markov Chain $S^{(T)}$ can be generated from the following distribution

$$\begin{aligned}p(S^{(T)} \mid \boldsymbol{y}^{(T)}, \boldsymbol{z}^{(T)}, \boldsymbol{\vartheta}^{(\cdot)}) &= p(S_T \mid \boldsymbol{y}^{(T)}, \boldsymbol{z}^{(T)}, \boldsymbol{\vartheta}^{(\cdot)}) \prod_{t=1}^{T-1} p(S_t \mid \boldsymbol{y}^{(t)}, \boldsymbol{z}^{(t)}, S_{t+1}, \boldsymbol{\vartheta}^{(\cdot)}) \\ &= p(S_T \mid \boldsymbol{z}^{(T)}, \boldsymbol{\vartheta}^{(\cdot)}) \prod_{t=1}^{T-1} p(S_t \mid \boldsymbol{z}^{(t)}, S_{t+1}, \boldsymbol{\vartheta}^{(\cdot)})\end{aligned}$$

as the distribution of $S^{(T)}$ is orthogonal to $\boldsymbol{y}^{(T)}$ given $\boldsymbol{z}^{(T)}$. We can thus obtain conditional draws for $S^{(T)}$ as follows

1. We use the Hamilton (1989) filter on (3) to generate $p(S_t \mid \boldsymbol{z}^{(t)}, \boldsymbol{\vartheta}^{(\cdot)})$ for $t = 1, 2, ..., T$ and save them. The last iteration gives $p(S_T \mid \boldsymbol{z}^{(T)}, \boldsymbol{\vartheta}^{(\cdot)})$ from which we get S_T.

2. To draw S_t given $\boldsymbol{z}^{(T)}$ and S_{t+1}, for $t = T-1, T-2, ..., 1$ the following result is used

$$p(S_t \mid \boldsymbol{z}^{(t)}, S_{t+1}, \boldsymbol{\vartheta}^{(\cdot)}) = \frac{p(S_{t+1} \mid S_t) p(S_t \mid \boldsymbol{z}^{(t)}, \boldsymbol{\vartheta}^{(\cdot)})}{p(S_{t+1} \mid \boldsymbol{z}^{(t)}, \boldsymbol{\vartheta}^{(\cdot)})} \propto p(S_{t+1} \mid S_t) p(S_t \mid Z^{(t)}, \boldsymbol{\vartheta}^{(\cdot)})$$

   where $p(S_{t+1} \mid S_t)$ is the transition probability in $\boldsymbol{\vartheta}^{(\cdot)}$ and $p(S_t \mid \boldsymbol{z}^{(t)}, \boldsymbol{\vartheta}^{(\cdot)})$ is obtained form the values saved in the previous step.

3. The last step consists in drawing from

$$Pr(S_t = 1 \mid \boldsymbol{z}^{(t)}, S_{t+1}, \boldsymbol{\vartheta}^{(\cdot)}) = \frac{p(S_{t+1} \mid S_t = 1) p(S_t = 1 \mid \boldsymbol{z}^{(t)}, \boldsymbol{\vartheta}^{(\cdot)})}{\sum_{j=0}^{1} p(S_{t+1} \mid S_t = j) p(S_t = j \mid \boldsymbol{z}^{(t)}, \boldsymbol{\vartheta}^{(\cdot)})}$$

   where S_t is drawn from a uniform distribution $S_t \sim \mathcal{U}(0,1)$. If the generated number is smaller than $Pr(S_t = 1 \mid S_{t+1}, \boldsymbol{z}^{(t)}, \boldsymbol{\vartheta}^{(\cdot)})$, $S_t = 1$, otherwise $S_t = 0$.

B.3. Generation of the parameters vector

B.3.1. Monetary policy stance identification

To generate $\mathbf{P}$, we follow Geweke (2005). We denote the sum of transitions from the state $S_{t-1} = i$ to $S_t = j$ by n_{ij}. Each row of $\mathbf{P}$ are drawn from posteriors:

$$Dir(2000 + n_{11}, .5 + n_{12}, 150 + n_{13}, .5 + n_{14})$$
$$Dir(.5 + n_{21}, 2000 + n_{22}, .5 + n_{23}, 150 + n_{24})$$
$$Dir(150 + n_{31}, 15 + n_{32}, 2000 + n_{33}, 100 + n_{34})$$
$$Dir(.5 + n_{41}, 150 + n_{42}, 100 + n_{43}, 2000 + n_{44})$$

Given $\boldsymbol{y}^{(T)}$ and $f^{(T)}$, we can rewrite equation-by-equation equation (1) with

$$y^*_{j,t} = \lambda_j f^*_{j,t} + e_{j,t}$$

for $j = 2, \ldots, m$, where $y^*_{j,t}$ and $f^*_{j,t}$ are the j-the respective components of

$$\begin{aligned} \boldsymbol{y}^*_t = &\quad \boldsymbol{y}_t - \bar{\boldsymbol{\psi}}_1 \circ \boldsymbol{y}_{t-1} \\ \boldsymbol{f}^*_t = &\quad \boldsymbol{e}_m f_t - \bar{\boldsymbol{\psi}}_1 f_{t-1} \end{aligned} \tag{18}$$

with $\boldsymbol{e}_m$ denoting a vector of 1 of length m qnd $\bar{\boldsymbol{\psi}}_l = (\psi_{1,l}, \ldots, \psi_{m,l})$, $l = 1$ being the order of the AR specification in equation (2). From (14) and (20), we obtain conditional draws for λ_j from the posterior distribution

$$\mathcal{N}\left[\left(A_j^{-1} + \sigma_{e,j}^{-2} f_j^{*(T)'} f_j^{*(T)}\right)^{-1}\left(A_j a_j + \sigma_{e,j}^{-2} f_j^{*(T)'} y_j^{*(T)}\right), \left(A_j^{-1} + \sigma_{e,j}^{-2} f_j^{*(T)'} f_j^{*(T)}\right)^{-1}\right].$$

Given $\boldsymbol{y}^{(T)}$ and $f^{(T)}$, from (1) we can measure $\boldsymbol{u}^{(T)}$ and from equation (2) and the prior distribution (10), for all $j = 1, \ldots, m$, we can draw $\boldsymbol{\psi}_j$ from the posterior distribution

$$\mathcal{N}\left[\left(\boldsymbol{\Pi}_j^{-1} + \sigma_{e,j}^{-2} \boldsymbol{w}_j^{(T)'} \boldsymbol{w}_j^{(T)}\right)^{-1}\left(\boldsymbol{\Pi}_j^{-1} \boldsymbol{\pi}_j + \sigma_{e,j}^{-2} \boldsymbol{w}_j^{(T)'} u_j^{(T)}\right), \left(\boldsymbol{\Pi}_j^{-1} + \sigma_{e,j}^{-2} \boldsymbol{w}_j^{(T)'} \boldsymbol{w}_j^{(T)})^{-1}\right)\right]$$

where $\boldsymbol{w}_{j,t} = (u_{j,t-1}, u_{j,t-2})'$. Similarly, from the generated $\boldsymbol{\psi}_j$ and from (10), we can draw $\sigma^2_{e,j}$ from the posterior distribution

$$IG\left(\nu_j + \frac{T}{2}, Z_j + \frac{\left(u_j^{(T)} - \boldsymbol{\psi}_j' \boldsymbol{w}_j^{(T)}\right)'\left(u_j^{(T)} - \boldsymbol{\psi}_j' \boldsymbol{w}_j^{(T)}\right)}{2}\right).$$

Finally, we turn to the generation of (μ_0, μ_1, ϕ). Rewriting equation (3), we have

$$\frac{f_t}{\sigma_{S_t}} = \frac{\mu_{S_t}}{\sigma_{S_t}} + \eta_t$$

Let us denote G^*_t the left-hand side of the above equation and

$$Q^{*(T)} = \left[\begin{array}{cccc} S^{(T)} = 1 & S^{(T)} = 2 & S^{(T)} = 3 & S^{(T)} = 4 \end{array}\right]$$

From the prior distribution (12), $\boldsymbol{\mu}$ can be drawn from the posterior distribution :

$$\boldsymbol{\mu} \sim \mathcal{N}((A^{*-1} + Q^{*(T)\prime}Q^{*(T)})^{-1}(A^{*-1}\alpha + Q^{*(T)\prime}G^{*(T)}), (A^{*-1} + Q^{*(T)\prime}Q^{*(T)})^{-1}),$$

and only draws verifying the condition $\mu_1 < \mu_3 < 0 < \mu_4 < \mu_2$ are kept. For $i = 1...n$, $\sigma^2_{S_t=i}$ is drawn from the posterior distribution:

$$IG\left(.5 + \frac{\sum_{t=1}^{T}(S_t = i)}{2}, 20 + \frac{(f_{S^{(T)}=i} - \mu_{S_t=i})'(f_{S^{(T)}=i} - \mu_{S_t=i})}{2}\right).$$

and only draws verifying the condition $\sigma_1 > \sigma_3$ and $\sigma_2 > \sigma_4$

B.3.2. Business Cycle turning point detection

We now turn to the generation of draws for the vector of parameters. To do so, we will sequentially draw components of the $\boldsymbol{\vartheta}$ vector as follows.

We obtain conditional draws for the transition probabilities p and q following Albert and Chib (1993). In particular, given $S^{(T)}$ and the initial state, we denote the sum of transitions from the state $S_{t-1} = i$ to $S_t = j$ by n_{ij}, the log-likelihood is given by

$$L(q, p) = q^{n_{00}}(1 - q)^{n_{01}}p^{n_{11}}(1 - p)^{n_{10}}.$$

By combining the likelihood function and the conjugate priors presented in the previous section, from equation (11), we get the conditional distributions of (p, q) as the product of the independent beta distributions from which we generate p and q as

$$\begin{aligned} q \mid S^{(T)} &\sim \text{Beta}(u_{00} + n_{00}, u_{01} + n_{01}) \\ p \mid S^{(T)} &\sim \text{Beta}(u_{11} + n_{11}, u_{10} + n_{10}). \end{aligned}$$

Given $\boldsymbol{y}^{(T)}$ and $f^{(T)}$, we can rewrite equation-by-equation equation (1) with

$$y^*_{j,t} = \lambda_j f^*_{j,t} + e_{j,t}$$

for $j = 2, \ldots, m$, where $y^*_{j,t}$ and $f^*_{j,t}$ are the j-the respective components of

$$\begin{aligned} \boldsymbol{y}^*_t = &\quad \boldsymbol{y}_t - \bar{\boldsymbol{\psi}}_1 \circ \boldsymbol{y}_{t-1} \\ \boldsymbol{f}^*_t = &\quad \boldsymbol{e}_m f_t - \bar{\boldsymbol{\psi}}_1 f_{t-1} \end{aligned} \qquad (19)$$

with $\boldsymbol{e}_m$ denoting a vector of 1 of length m qnd $\bar{\boldsymbol{\psi}}_l = (\psi_{1,l}, \ldots, \psi_{m,l})$, $l = 1$ being the order of the AR specification in equation (2). From (14) and (20), we obtain conditional draws for λ_j from the posterior distribution

$$\mathcal{N}\left[\left(A_j^{-1} + \sigma_{e,j}^{-2} f_j^{*(T)\prime} f_j^{*(T)}\right)^{-1}\left(A_j a_j + \sigma_{e,j}^{-2} f_j^{*(T)\prime} y_j^{*(T)}\right), \left(A_j^{-1} + \sigma_{e,j}^{-2} f_j^{*(T)\prime} f_j^{*(T)}\right)^{-1}\right].$$

Given $\boldsymbol{y}^{(T)}$ and $f^{(T)}$, from (1) we can measure $\boldsymbol{u}^{(T)}$ and from equation (2) and the prior distribution (10), for all $j = 1, \ldots, m$, we can draw $\boldsymbol{\psi}_j$ from the posterior distribution

$$\mathcal{N}\left[\left(\boldsymbol{\Pi}_j^{-1} + \sigma_{e,j}^{-2}\boldsymbol{w}_j^{(T)'}\boldsymbol{w}_j^{(T)}\right)^{-1}\left(\boldsymbol{\Pi}_j^{-1}\boldsymbol{\pi}_j + \sigma_{e,j}^{-2}\boldsymbol{w}_j^{(T)'}u_j^{(T)}\right), \left(\boldsymbol{\Pi}_j^{-1} + \sigma_{e,j}^{-2}\boldsymbol{w}_j^{(T)'}\boldsymbol{w}_j^{(T)})^{-1}\right)\right]$$

where $\boldsymbol{w}_{j,t} = (u_{j,t-1}, u_{j,t-2})'$. Similarly, from the generated $\boldsymbol{\psi}_j$ and from (10), we can draw $\sigma_{e,j}^2$ from the posterior distribution

$$IG\left(\nu_j + \frac{T}{2}, Z_j + \frac{\left(u_j^{(T)} - \boldsymbol{\psi}_j'\boldsymbol{w}_j^{(T)}\right)'\left(u_j^{(T)} - \boldsymbol{\psi}_j'\boldsymbol{w}_j^{(T)}\right)}{2}\right).$$

Finally, we turn to the generation of $(\mu_0, \mu_1, \phi, \boldsymbol{\theta}^{(ARCH)'})$. The parameters are drawn from the three full conditional distributions $p(\boldsymbol{\mu} \mid \boldsymbol{z}^{(T)}, S^{(T)}, \phi, \boldsymbol{\theta}^{(\text{ARCH})})$, $p(\phi \mid \boldsymbol{z}^{(T)}, S^{(T)}, \boldsymbol{\theta}^{(\text{ARCH})})$ and $p(\boldsymbol{\theta}^{(\text{ARCH})} \mid \boldsymbol{z}^{(T)}, S^{(T)}, \phi)$ sequentially. Since μ_{S_t} and ϕ appear in the conditional variance equation, those distributions are non-standard, as noted by Chan and Grant (2016), and Metropolis Hastings algorithms are required. Rewriting equation (3), we have

$$\frac{f_t - \phi f_{t-1}}{\sigma_t} = \frac{\mu_0(1 - S_t) + \mu_1 S_t}{\sigma_t} + \eta_t$$

Let us denote G_t^* the left-hand side of the above equation and Let us denote G_t^* the left-hand side of the above equation and

$$Q^{*(T)} = \left[\begin{array}{cc} 1 - S^{(T)} & S^{(T)} \end{array}\right]$$

From the prior distribution (12), to sample $\boldsymbol{\mu}$, we use a multivariate Gaussian proposal :

$$\mathcal{N}\left[(A^{*-1} + Q^{*(T)\prime}Q^{*(T)})^{-1}(A^{*-1}\alpha + Q^{*(T)\prime}G^{*(T)}), (A^{*-1} + Q^{*(T)\prime}Q^{*(T)})^{-1}\right]$$

and only keep draws verifying $\mu_0 > \mu_1$. Rewriting again equation (3) yields

$$\frac{f_t - \mu_0(1 - S_t) - \mu_1 S_t}{\sigma_t} = \phi\frac{f_{t-1}}{\sigma_t} + \eta_t.$$

Denoting $\tilde{G}_t$ the left-hand side of the above equation and $\tilde{Q}_t$ the right-hand side. To sample ϕ we use a Gaussian proposal with mean $\bar{\phi}$ and variance V_ϕ given by

$$\begin{aligned} \bar{\phi} = &\quad (A^{-1} + \tilde{Q}'\tilde{Q})^{-1}(A^{-1}\alpha + \tilde{Q}'\tilde{G}) \\ V_\phi = &\quad (A^{-1} + \tilde{Q}'\tilde{Q})^{-1}. \end{aligned}$$

Only draws satisfying the stationarity condition $|\phi| < 1$ are kept. Finally to sample $\boldsymbol{\theta}^{(\text{ARCH})}$, we use a Gaussian proposal centered at the mode of $p(\boldsymbol{\theta}^{(\text{ARCH})} \mid \boldsymbol{z}^{(T)}, S_t, \phi)$ with covariance matrix set to be the outer product of the scores.

B.3.3. Market sentiment

To generate $\mathbf{P}$, we follow Geweke (2005). We denote the sum of transitions from the state $S_{t-1} = i$ to $S_t = j$ by n_{ij}. Each row of $\mathbf{P}$ are drawn from posteriors:

$$
\begin{aligned}
&Dir(8 + n_{11}, 1.5 + n_{12}, 0.5 + n_{14}) \\
&Dir(1.5 + n_{21}, 8 + n_{22}, 0.5 + n_{24}) \\
&Dir(0.5 + n_{31}, 8 + n_{33}, 1.5 + n_{34}) \\
&Dir(0.5 + n_{41}, 1.5 + n_{43}, 8 + n_{44})
\end{aligned}
$$

Given $\boldsymbol{y}^{(T)}$ and $f^{(T)}$, we can rewrite equation-by-equation equation (1) with

$$y^*_{j,t} = \lambda_j f^*_{j,t} + e_{j,t}$$

for $j = 2, \ldots, m$, where $y^*_{j,t}$ and $f^*_{j,t}$ are the j-the respective components of

$$
\begin{aligned}
\boldsymbol{y}^*_t &= \boldsymbol{y}_t - \bar{\boldsymbol{\psi}}_1 \circ \boldsymbol{y}_{t-1} \\
\boldsymbol{f}^*_t &= \boldsymbol{e}_m f_t - \bar{\boldsymbol{\psi}}_1 f_{t-1}
\end{aligned}
\tag{20}
$$

with $\boldsymbol{e}_m$ denoting a vector of 1 of length m qnd $\bar{\boldsymbol{\psi}}_l = (\psi_{1,l}, \ldots, \psi_{m,l})$, $l = 1$ being the order of the AR specification in equation (2). From (14) and (20), we obtain conditional draws for λ_j from the posterior distribution

$$\mathcal{N}\left[\left(A_j^{-1} + \sigma_{e,j}^{-2} f_j^{*(T)'} f_j^{*(T)}\right)^{-1} \left(A_j a_j + \sigma_{e,j}^{-2} f_j^{*(T)'} y_j^{*(T)}\right), \left(A_j^{-1} + \sigma_{e,j}^{-2} f_j^{*(T)'} f_j^{*(T)}\right)^{-1}\right].$$

Given $\boldsymbol{y}^{(T)}$ and $f^{(T)}$, from (1) we can measure $\boldsymbol{u}^{(T)}$ and from equation (2) and the prior distribution (10), for all $j = 1, \ldots, m$, we can draw $\boldsymbol{\psi}_j$ from the posterior distribution

$$\mathcal{N}\left[\left(\boldsymbol{\Pi}_j^{-1} + \sigma_{e,j}^{-2} \boldsymbol{w}_j^{(T)'} \boldsymbol{w}_j^{(T)}\right)^{-1} \left(\boldsymbol{\Pi}_j^{-1} \boldsymbol{\pi}_j + \sigma_{e,j}^{-2} \boldsymbol{w}_j^{(T)'} u_j^{(T)}\right), \left(\boldsymbol{\Pi}_j^{-1} + \sigma_{e,j}^{-2} \boldsymbol{w}_j^{(T)'} \boldsymbol{w}_j^{(T)})^{-1}\right)\right]$$

where $\boldsymbol{w}_{j,t} = (u_{j,t-1}, u_{j,t-2})'$. Similarly, from the generated $\boldsymbol{\psi}_j$ and from (10), we can draw $\sigma^2_{e,j}$ from the posterior distribution

$$IG\left(\nu_j + \frac{T}{2}, Z_j + \frac{\left(u_j^{(T)} - \boldsymbol{\psi}_j' \boldsymbol{w}_j^{(T)}\right)' \left(u_j^{(T)} - \boldsymbol{\psi}_j' \boldsymbol{w}_j^{(T)}\right)}{2}\right).$$

Finally, we turn to the generation of (μ_0, μ_1, ϕ). Rewriting equation (3), we have

$$\frac{f_t}{\sigma_{S_t}} = \frac{\mu_{S_t}}{\sigma_{S_t}} + \eta_t$$

Let us denote G_t^* the left-hand side of the above equation and

$$Q^{*(T)} = \begin{bmatrix} S^{(T)} = 1 & S^{(T)} = 2 & S^{(T)} = 3 & S^{(T)} = 4 \end{bmatrix}$$

From the prior distribution (12), $\boldsymbol{\mu}$ can be drawn from the posterior distribution :

$$\boldsymbol{\mu} \sim \mathcal{N}((A^{*-1} + Q^{*(T)\prime}Q^{*(T)})^{-1}(A^{*-1}\alpha + Q^{*(T)\prime}G^{*(T)}), (A^{*-1} + Q^{*(T)\prime}Q^{*(T)})^{-1}),$$

and only draws verifying the condition $\mu_1 < 0, \mu_2 > 0, \mu_3 < 0, \mu_4 > 0$, and the long run conditions $\frac{\pi_1}{\pi_1+\pi_2}\mu_1 + \frac{\pi_2}{\pi_1+\pi_2}\mu_2 > 0$, $\frac{\pi_3}{\pi_3+\pi_4}\mu_3 + \frac{\pi_4}{\pi_3+\pi_4}\mu_4 < 0$. For $i = 1...n$, $\sigma^2_{S_t=i}$ is drawn from the posterior distribution:

$$IG\left(.5 + \frac{\sum_{t=1}^{T}(S_t = i)}{2}, 20 + \frac{(f_{S^{(T)}=i} - \mu_{S_t=i})'(f_{S^{(T)}=i} - \mu_{S_t=i})}{2}\right).$$

For the univariate specification of the market sentiment, the dynamic factor structure presented in equations (1) to (3) is not used. We have $y_t \mid S_t \sim \mathcal{N}(\mu_{S_t}, \sigma^2_{S_t})$. We thus discard the step generating the state vector presented in B.1.

C. Market sentiment univariate specification in sample and out of sample probabilities

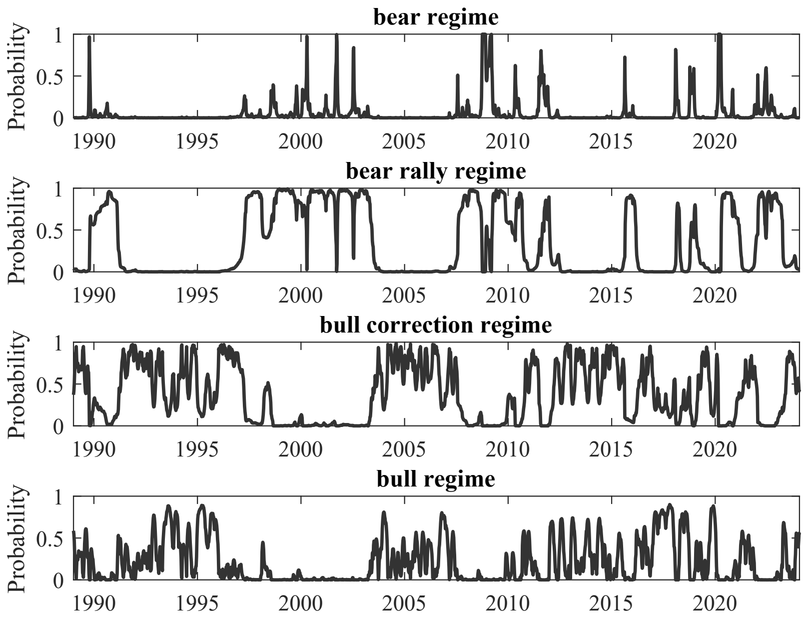

Figure 13: *In sample probabilities from the univariate specification of the market sentiment*

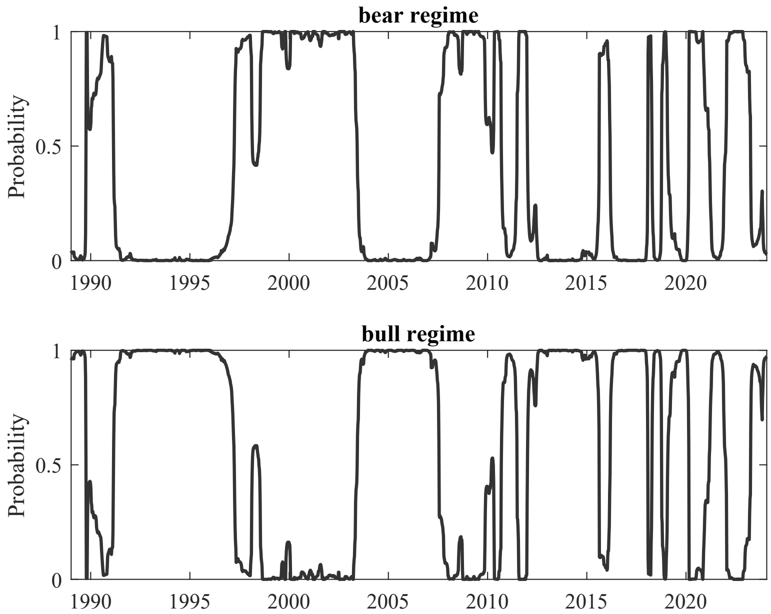

Figure 14: *Bull/bear in sample probabilities from the univariate specification of the market sentiment*

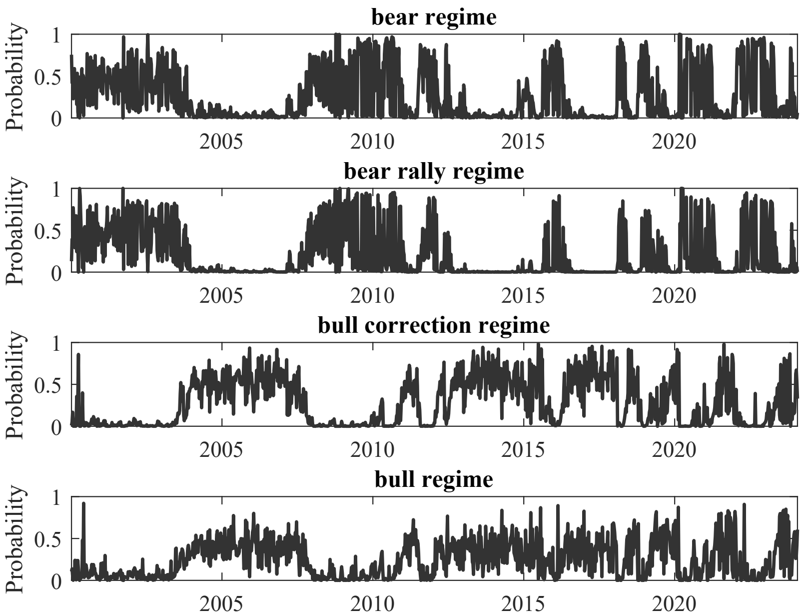

Figure 15: *Real-time probabilities from the univariate specification of the market sentiment*

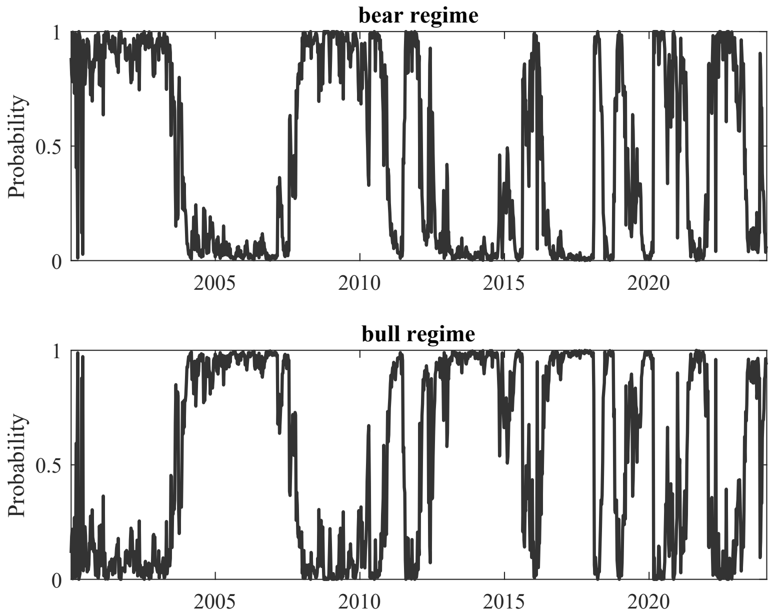

Figure 16: *Real-time bull/bear probabilities from the univariate specification of the market sentiment*

D. Performances of the portfolios

D.1. Rolling window performances

Table 8: 1Y rolling performances for the five groups of competing strategies and benchmarks

	1 Y Return	**1 Y Vol**	**1 Y SR**	**1 Y Max DD**
MS-Sign-Return-Multi	6.1%	8.6%	0.51	6.5%
MS-Bull-Bear-Return-Multi	6.5%	9.7%	0.49	7.8%
MS-Sign-Return-Uni	3.8%	9.2%	0.22	8.2%
MS-Bull-Bear-Return-Uni	**6.7%**	**9.0%**	**0.56**	**7.2%**
MS-Sign-Return-Multi-BC	7.0%	11.4%	0.46	10.0%
MS-Bull-Bear-Return-Multi-BC	8.2%	11.7%	0.55	9.9%
MS-Sign-Return-Uni-BC	6.3%	10.8%	0.43	9.5%
MS-Bull-Bear-Return-Uni-BC	**7.5%**	**10.3%**	**0.56**	**8.6%**
MS-Sign-Return-Multi-MP	4.8%	8.8%	0.35	7.5%
MS-Bull-Bear-Return-Multi-MP	6.9%	9.5%	0.54	8.0%
MS-Sign-Return-Uni-BC	4.1%	7.3%	0.32	6.3%
MS-Bull-Bear-Return-Uni-MP	**6.3%**	**8.0%**	**0.58**	**6.1%**
MP-BC	**8.3%**	**12.0%**	**0.55**	**10.1%**
MS-Sign-Return-Multi-MP-BC	6.5%	9.8%	0.49	8.3%
MS-Bull-Bear-Return-Multi-MP-BC	7.6%	10.2%	0.57	8.6%
MS-Sign-Return-Uni-MP-BC	6.1%	8.7%	0.51	7.1%
MS-Bull-Bear-Return-Uni-MP-BC	**7.3%**	**8.9%**	**0.63**	**7.0%**
S&P500	8.3%	16.3%	0.42	14.3%
60Bond/40Equity	5.6%	9.2%	0.42	8.2%
Cash3m	1.6%			

Table 9: 2Y rolling performances for the five groups of competing strategies and benchmarks

	2 Y Return	**2 Y Vol**	**2 Y SR**	**2 Y Max DD**
MS-Sign-Return-Multi	6.4%	8.5%	0.54	8.4%
MS-Bull-Bear-Return-Multi	7.2%	9.7%	0.56	9.5%
MS-Sign-Return-Uni	4.0%	9.4%	0.23	11.2%
MS-Bull-Bear-Return-Uni	**7.7%**	**9.1%**	**0.65**	**8.2%**
MS-Sign-Return-Multi-BC	7.1%	11.4%	0.47	13.8%
MS-Bull-Bear-Return-Multi-BC	8.2%	11.7%	0.55	13.4%
MS-Sign-Return-Uni-BC	6.4%	10.8%	0.43	13.1%
MS-Bull-Bear-Return-Uni-BC	**7.8%**	**10.3%**	**0.59**	**11.1%**
MS-Sign-Return-Multi-MP	5.1%	8.9%	0.37	10.0%
MS-Bull-Bear-Return-Multi-MP	6.9%	9.4%	0.55	10.2%
MS-Sign-Return-Uni-BC	4.3%	7.3%	0.35	8.3%
MS-Bull-Bear-Return-Uni-MP	**6.9%**	**8.1%**	**0.64**	**7.0%**
MP-BC	**8.4%**	**11.9%**	**0.56**	**13.2%**
MS-Sign-Return-Multi-MP-BC	6.8%	9.8%	0.52	11.1%
MS-Bull-Bear-Return-Multi-MP-BC	7.7%	10.2%	0.59	11.1%
MS-Sign-Return-Uni-MP-BC	6.4%	8.6%	0.55	9.0%
MS-Bull-Bear-Return-Uni-MP-BC	**7.8%**	**8.9%**	**0.68**	**8.6%**
S&P500	8.1%	16.5%	0.39	20.2%
60Bond/40Equity	6.0%	9.3%	0.45	11.2%
Cash3m	1.6%			

Table 10: 5Y rolling performances for the five groups of competing strategies and benchmarks

	5 Y Return	**5 Y Vol**	**5 Y SR**	**5 Y Max DD**
MS-Sign-Return-Multi	5.4%	8.5%	0.42	13.5%
MS-Bull-Bear-Return-Multi	7.7%	9.7%	0.62	11.1%
MS-Sign-Return-Uni	2.1%	9.8%	0.03	19.9%
MS-Bull-Bear-Return-Uni	**8.3%**	**9.1%**	**0.71**	**9.1%**
MS-Sign-Return-Multi-BC	7.6%	11.1%	0.53	19.4%
MS-Bull-Bear-Return-Multi-BC	8.7%	11.2%	0.62	18.3%
MS-Sign-Return-Uni-BC	5.9%	10.8%	0.38	21.5%
MS-Bull-Bear-Return-Uni-BC	**8.4%**	**10.1%**	**0.66**	**14.0%**
MS-Sign-Return-Multi-MP	5.4%	8.4%	0.44	12.5%
MS-Bull-Bear-Return-Multi-MP	7.2%	8.6%	0.63	11.6%
MS-Sign-Return-Uni-BC	3.3%	6.9%	0.22	12.5%
MS-Bull-Bear-Return-Uni-MP	**7.4%**	**8.0%**	**0.71**	**7.7%**
MP-BC	**9.0%**	**11.4%**	**0.64**	**16.2%**
MS-Sign-Return-Multi-MP-BC	7.3%	9.3%	0.60	13.3%
MS-Bull-Bear-Return-Multi-MP-BC	8.2%	9.4%	0.68	13.5%
MS-Sign-Return-Uni-MP-BC	6.2%	8.1%	0.55	12.2%
MS-Bull-Bear-Return-Uni-MP-BC	**8.3%**	**8.5%**	**0.77**	**9.4%**
S\&P500	7.5%	16.6%	0.35	31.4%
60Bond/40Equity	5.6%	9.2%	0.42	17.5%
Cash3m	1.7%			

Table 11: 10Y rolling performances for the five groups of competing strategies and benchmarks

	10 Y Return	**10 Y Vol**	**10 Y SR**	**10 Y Max DD**
MS-Sign-Return-Multi	6.3%	9.3%	0.49	18.7%
MS-Bull-Bear-Return-Multi	7.3%	9.8%	0.57	13.1%
MS-Sign-Return-Uni	1.2%	11.5%	-0.04	36.8%
MS-Bull-Bear-Return-Uni	**8.2%**	**9.1%**	**0.71**	**10.2%**
MS-Sign-Return-Multi-BC	7.7%	11.5%	0.52	27.3%
MS-Bull-Bear-Return-Multi-BC	8.9%	11.8%	0.61	24.3%
MS-Sign-Return-Uni-BC	5.5%	11.6%	0.33	34.0%
MS-Bull-Bear-Return-Uni-BC	**8,4%**	**10,2%**	**0,66**	**18,2%**
MS-Sign-Return-Multi-MP	5,5%	8,6%	0,44	16.6%
MS-Bull-Bear-Return-Multi-MP	7.5%	9.0%	0.63	13.3%
MS-Sign-Return-Uni-BC	3.0%	7.3%	0.17	19.9%
MS-Bull-Bear-Return-Uni-MP	**7.2%**	**7.9%**	**0.69**	**9.0%**
MP-BC	**9.3%**	**11.6%**	**0.66**	**19.6%**
MS-Sign-Return-Multi-MP-BC	7.5%	9.4%	0.61	16.4%
MS-Bull-Bear-Return-Multi-MP-BC	8.5%	9.7%	0.69	16.5%
MS-Sign-Return-Uni-MP-BC	6.3%	8.3%	0.55	16.3%
MS-Bull-Bear-Return-Uni-MP-BC	**8.4%**	**8.5%**	**0.79**	**10.7%**
S\&P500	7.0%	18.1%	0.29	49.9%
60Bond/40Equity	5.6%	10.0%	0.38	29.4%
Cash3m	1.7%			

D.2. Cumulative distributions of rolling window returns for selected strategies

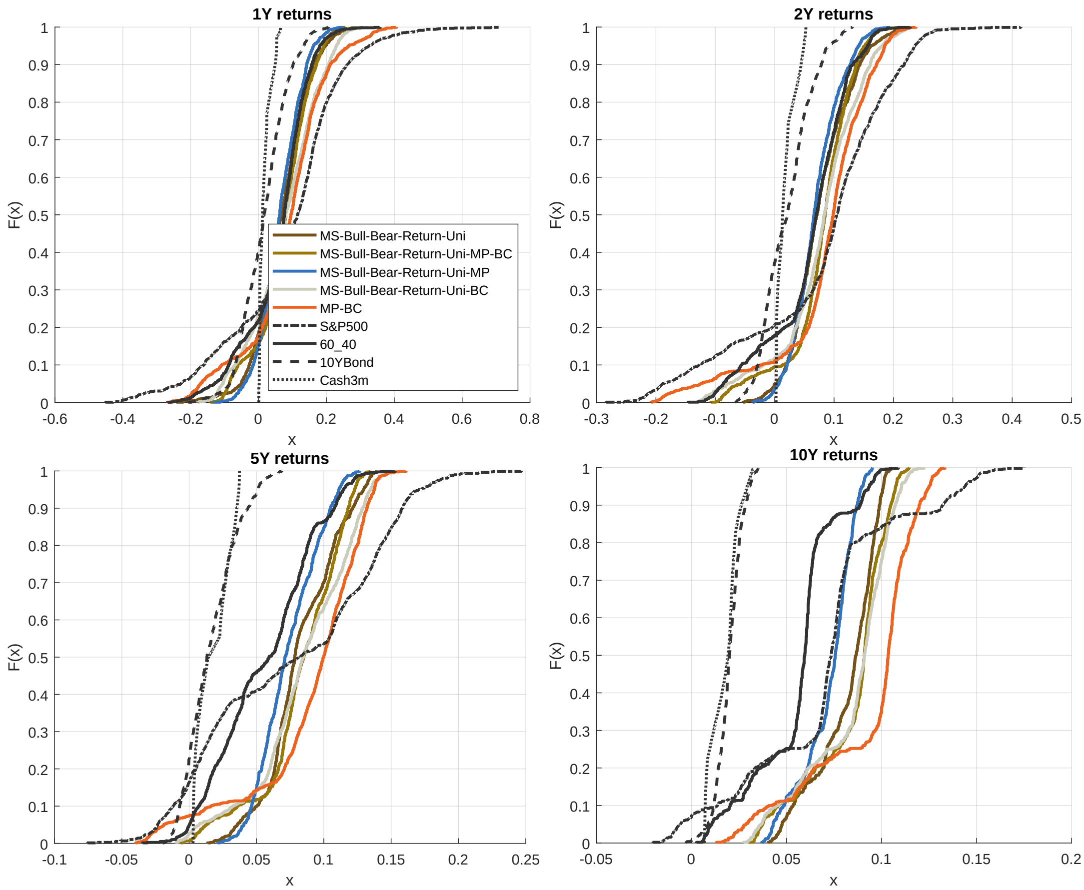

Figure 17: *Cumulative distribution functions of rolling window Returns from 1Y to 10Y holding horizons*

D.3. Yearly returns of the strategies in "normal" and "abnormal" times

Table 12: Annual returns during "normal" times

	2002	2003	2007	2010	2011	2012	2013	2014	2019	2021	Average
MS-Bull-Bear-Return-Uni	7.1%	3.8%	5.7%	6.1%	16.0%	2.6%	30.7%	12.2%	15.4%	12.9%	11.3%
MS-Bull-Bear-Return-Uni-BC	-12.1%	16.6%	4.9%	7.5%	3.1%	7.6%	31.9%	13.8%	22.2%	23.0%	11.9%
MS-Bull-Bear-Return-Uni-MP	8.0%	1.5%	4.9%	5.3%	16.1%	1.9%	22.2%	11.5%	14.9%	12.7%	9.9%
MP-BC	**-21.6%**	**20.8%**	**4.5%**	**9.7%**	**2.4%**	**12.0%**	**24.1%**	**14.7%**	**30.4%**	**28.8%**	**12.6%**
MS-Bull-Bear-Return-Uni-MP-BC	**-7.6%**	**11.0%**	**4.7%**	**7.8%**	**9.4%**	**6.9%**	**23.1%**	**13.1%**	**22.4%**	**20.5%**	**11.1%**
S&P500	-23.0%	26.8%	6.1%	13.4%	2.1%	13.8%	33.4%	15.4%	32.3%	29.3%	15.0%
60/40	-10.4%	15.1%	5.8%	10.5%	7.5%	9.6%	14.3%	12.4%	22.2%	15.3%	10.2%
10YBond	10.2%	-1.9%	4.5%	4.9%	13.6%	2.7%	-9.7%	7.3%	7.8%	-3.4%	3.6%

Table 13: Annual returns during recession periods

	2001	2008	2009	2020	Average
MS-Bull-Bear-Return-Uni	-6.7%	14.0%	-7.7%	-1.0%	-0.3%
MS-Bull-Bear-Return-Uni-BC	-10.0%	-4.4%	16.0%	2.5%	1.0%
MS-Bull-Bear-Return-Uni-MP	-4.6%	10.3%	-6.8%	-2.8%	-1.0%
MP-BC	**-15.7%**	**6.0%**	**15.8%**	**8.0%**	**3.5%**
MS-Bull-Bear-Return-Uni-MP-BC	**-10.0%**	**8.2%**	**4.3%**	**2.8%**	**1.3%**
S&P500	-10.7%	-39.0%	31.5%	16.1%	-0.5%
60/40	-6.6%	-19.1%	13.0%	15.5%	0.7%
10YBond	-2.0%	18.3%	-12.7%	10.5%	3.6%

Table 14: Annual returns during tightening cylces

	2000	2004	2005	2006	2015	2016	2017	2018	2022	Average
MS-Bull-Bear-Return-Uni	1.2%	8.4%	4.4%	15.0%	-5.3%	12.3%	21.0%	-1.6%	-23.6%	3.5%
MS-Bull-Bear-Return-Uni-BC	-0.6%	10.9%	4.6%	15.3%	-1.9%	9.7%	21.1%	-3.4%	-12.8%	4.8%
MS-Bull-Bear-Return-Uni-MP	1.8%	10.1%	4.2%	11.5%	-2.6%	12.5%	19.6%	-1.2%	-10.4%	5.1%
MP-BC	**-2.4%**	**13.2%**	**4.4%**	**11.6%**	**2.2%**	**10.2%**	**19.8%**	**-4.1%**	**-11.2%**	**4.9%**
MS-Bull-Bear-Return-Uni-MP-BC	**0.1%**	**11.7%**	**4.3%**	**11.6%**	**-0.1%**	**11.4%**	**19.7%**	**-2.4%**	**-10.6%**	**5.1%**
S&P500	-7.4%	12.1%	4.8%	15.5%	0.8%	10.6%	21.4%	-5.1%	-17.8%	3.9%
60/40	-0.7%	7.1%	2.2%	7.8%	0.8%	6.1%	12.3%	-3.9%	-17.7%	1.6%
10YBond	8.5%	-0.4%	-1.9%	-3.1%	-0.2%	-1.0%	-0.2%	-3.1%	-18.8%	-2.2%