Multi-Asset Portfolios with Active and Passive Funds:
A Robust Optimization Framework

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Abstract

This paper introduces a robust optimization framework for multi-asset portfolios that allocate to both active and passive funds. The approach examines how portfolio allocations shift between active and passive funds depending on tactical and thematic investment views, as well as the emphasis placed on active fund alpha in the optimization process. Robust portfolio optimization is used to manage uncertainty in investment views and fund alpha expectations, while explicitly accounting for the ongoing charges of all funds. The framework addresses practical implementation challenges, including the differences between the set of indices used for expressing tactical and thematic views, those used to define the reference strategic asset allocation, and the benchmarks of the investable funds. It also accounts for the tracking error and alignment of funds relative to their respective benchmarks. Detailed guidance is provided for implementation of the framework, and an analytical appendix explains the intuition behind most observed allocations. The result is a transparent and scalable solution for constructing multi-asset portfolios that reflect investment views with real-world constraints.

Keywords: Robust Portfolio Optimization, Multi-Asset Portfolios, Active Funds, Passive Funds, Tactical Asset Allocation, Thematic Views, Fund Alpha, Asset Allocation, Robo Advisory

JEL Classification: D29, G11, L29.

1. INTRODUCTION

The growing digitalization of asset management and the rise of robo-advisors have intensified the need for scalable, robust frameworks that support mass customization of asset allocation portfolios. Beketov et al. (2018) report that most robo-advisors rely on simple mean–variance optimization (MVO) or its variants, such as Black–Litterman (40%), pre-defined grid portfolios (27%), or constant weights (14%), while the remainder employ alternative techniques like liability-driven investment, full-scale optimization, risk parity, constant proportion portfolio insurance or mean reversion trading.

However, traditional mean–variance approaches are widely acknowledged as inadequate for delivering robust and transparent solutions, particularly when asset managers must implement nuanced investment views across numerous mandates with diverse constraints and risk profiles. Even in strategic asset allocation, weight constraints are critical to regularize solutions and ensure financial viability. The Black–Litterman model (1992) introduces implicit regularization via benchmarks or tracking error constraints, making it somewhat more robust. Yet, it remains an MVO variant and inherits its fundamental limitations (Bourgeron et al., (2018)).

Robust portfolio optimization (RPO), as introduced by Lobo et al. (1998) and based on the framework of Ben-Tal and Nemirovski (1998), is designed to address uncertainty in portfolio inputs such as mean returns, variances, and covariances. Unlike traditional optimization, which assumes these inputs are known precisely, robust optimization seeks portfolio allocations that remain effective even when inputs deviate from their estimates. This is typically achieved through worst-case scenario analysis, making the optimization problem more tractable and practical for real-world applications. Tütüncü and König (2004) advanced this approach by formulating a two-step maximum–minimum problem, simplifying robust optimization for long-only portfolios to solving an MVO problem using the worst-case estimates.

Further developments by Ceria and Stubbs (2006) introduced a quadratic form for estimation errors and distinguished between the covariance of asset returns and the covariance of estimation errors, proposing additional constraints to make robust optimization less conservative. Scherer (2006) showed that when the covariance of estimation errors is proportional to the asset return covariance, robust optimization yields portfolios that are weighted averages of the MVO and minimum variance portfolios, converging to the latter as estimation error increases. Fabozzi et al. (2007) contributed an absolute error formulation, though without detailing its differences from other approaches.

Heckel et al. (2016) demonstrated that robust portfolio optimization, when uncertainty is limited to expected returns, behaves predictably across different levels of uncertainty. With low uncertainty, the robust portfolio closely matches the traditional mean-variance portfolio. As uncertainty increases, the robust portfolio shifts toward risk-based allocations, such as minimum variance, inverse variance, equal risk budget, or equal weighting, depending on the specific formulation. At intermediate uncertainty, the robust portfolio is well-approximated by a weighted average of the mean-variance and the relevant risk-based portfolio, especially in quadratic formulations. These findings hold even when portfolio constraints are present.

Yin et al. (2020) proposed a practical framework for implementing RPO in real-world investment settings specifically adapted to the optimization of strategic asset allocation (SAA). They recommended the use of a quadratic uncertainty set and a diagonal uncertainty matrix based on asset variances as the most adequate RPO formulation to reduce the sensitivity of the proposed allocation to changes in expected returns, while preserving the volatility inputs which can be forecast with much higher accuracy (see, e.g., Perchet et al. (2014) or Perchet et al. (2016)) than expected returns. The authors provided clear guidelines for selecting the form and level of uncertainty, advocating for calibrating the uncertainty parameter as half the average Sharpe ratio of the investment universe. Their empirical examples demonstrate that this approach leads to well-diversified portfolios that are robust to estimation errors.

Issaoui et al. (2021) adapted Yin et al. (2020)'s robust portfolio optimization framework to construct highly customized tactical asset allocation (TAA) portfolios from a single set of tactical views. The process begins with risk budgeting to translate qualitative views into target overweight and underweight allocations, independent of the strategic benchmark and constraints. These active allocations are then used to engineer implied returns in a reverse RPO step. Implied returns of a portfolio are the expected returns that if used in the portfolio optimisation model, would result in the observed portfolio weights being optimal. The implied returns are then used in RPO to build consistent and robust portfolios even under complex constraints and multiple benchmarks. By incorporating a factor-based risk model, the framework ensures transparency and enables scalable, automated customization for institutional and robo-advisory applications.

In turn, Somefun et al. (2022) extended the framework proposed by Yin et al. (2020) to the context of SAA portfolios using a core-satellite structure, where diversification asset classes such as thematic investments are allocated to the satellite. The framework accounts for both the exposures of thematic investments to conventional risk factors and the additional alpha and risk specific to themes.

Our contribution builds on the frameworks of Issaoui et al. (2021) and Somefun et al. (2022) by introducing an RPO approach for TAA portfolios, tailored to the operational realities of asset managers, asset owners, distributors and robo-advisors. Specifically, we address four key challenges: i) integrating tactical and thematic investment views, ii) managing uncertainty in expected returns and in expected alpha from actively managed fund, iii) incorporating ongoing fund charges and tracking error constraints, and iv) reconciling inconsistencies between indices used for expressing views, strategic allocation, and fund benchmarks.

Our framework allows tactical views to be expressed through traditional benchmark indices across core asset classes, reflecting common industry practice. We tackle the practical difficulty of implementing these views with both active and passive funds, tackling the mismatch between the indices used for strategic and tactical asset allocation and the actual investable universe of selected funds. The proposed approach constructs robust portfolios that embed tactical views while accounting for ongoing fund charges, expected alpha from active and thematic funds, and tracking errors relative to benchmarks. Crucially, we also address tracking error arising from discrepancies between fund benchmarks, strategic allocation indices, and the indices used for TAA.

A key feature of the output from this framework is its compliance with practical investment constraints. The portfolios are optimized to respect limits such as maximum tracking error, long-only requirements, and caps on exposures to specific assets or asset classes. Another important aspect of the output is its robustness to estimation errors. By leveraging RPO, the framework generates portfolios that are much less sensitive to small changes in tactical views than those produced by traditional MVO. This results in more stable and intuitive allocations, particularly in environments characterized by high asset correlations or significant uncertainty in the views.

Transparency is also a central outcome of the proposed framework. The output can include a detailed decomposition of portfolio risk into systematic (factor-based) and idiosyncratic components. The idiosyncratic risk component in portfolio construction can be further decomposed into two sources: i) one arising from the mismatch between the benchmarks of the funds used for implementation and the traditional indices employed for TAA, and ii) another stemming from the alpha generated by active funds or any residual mismatch between passive funds and their benchmarks. This distinction allows for a more precise attribution of idiosyncratic risk, highlighting both structural differences in benchmark selection and the impact of fund-specific performance characteristics.

Finally, we introduce a parameter into the framework that allows the tracking error of the final TAA portfolio to be adjusted so that it can capture more of the expected returns generated from tactical asset allocation views or, alternatively, more of the alpha expected from investing in the active funds used in the allocation. Changing this parameter allows for the composition of active versus passive funds to change, something we shall discuss in more detail.

This paper provides a clear, step-by-step description of the robust optimization framework for multi-asset portfolios that include both active and passive funds. We begin by explaining how fund exposures to their benchmarks are measured, and how benchmark exposures to the core indices underlying TAA views are estimated. Next, we map direct fund exposures to core indices and introduce the risk model, which distinguishes among systematic, thematic, and specific sources of risk. We then discuss how uncertainty in investment views is defined. This is followed by an outline of the construction of the SAA portfolio and the translation of tactical and thematic views into portfolio allocations. Finally, we bring these components together in the robust portfolio construction process. This structure ensures that each methodological step builds on the previous one, providing a coherent and comprehensive foundation for the framework.

In the section thereafter, we provide a detailed example illustrating the implementation of the proposed framework, using a set of TAA views for illustration and a representative list of active and passive funds. We investigate how varying the parameter that controls the preference for capturing TAA expected returns versus the alpha from active funds affects the final portfolio. This example demonstrates the practical impact of this parameter on portfolio construction and allocation outcomes.

Finally, we devote a section to discussing the results, drawing on a simplified analytical framework presented in the appendices to help explain and interpret the findings from our examples.

2. METHODOLOGY

We propose a framework for constructing TAA portfolios that build upon a predefined SAA portfolio. The goal is to design an investable portfolio composed of both active and passive funds, integrating multiple layers of investment views and constraints in a systematic manner.

The framework assumes a set of tactical views expressed on core benchmark indices representing major asset classes, alongside a pre-selected universe of active and passive funds. Passive funds are expected to closely replicate their respective benchmarks, minimizing tracking error. Active funds are selected for their potential to generate alpha relative to their benchmarks, reflecting differentiated investment skills. Thematic funds are included based on their expected outperformance versus exposure to traditional benchmarks.

The TAA views are expressed for a set of traditional benchmark core indices that span major asset classes and reflect standard exposures commonly used by institutional investors. However, these indices may differ from those used as benchmarks for the selected funds and from those used in the SAA portfolio. Moreover, the actual exposures of the funds, particularly active ones, may not align perfectly with their stated benchmarks. Several funds – such as thematic funds, global equity and fixed income funds, or aggregate fixed income funds – tend to exhibit multiple exposures, which makes their integration into the final portfolio highly complex and impractical without a robust risk modelling approach like the one we propose here.

To reconcile these differences, we first estimate each fund's exposure to its stated benchmark, followed by assessing the benchmark's exposure to the core indices used in the TAA framework. Any indices used in the SAA that are not part of the core index set are also added to the list of benchmarks requiring exposure estimation. This layered mapping ensures alignment between fund-level implementation and the strategic and tactical allocation processes.

Beyond this multilayering, we introduced a risk model based on principal component analysis (PCA) to capture systematic and idiosyncratic risks, including currency exposures, establishing the individual independent sources of risk in the universe of available core indices.

In the reminder of the paper, we shall use *fc* for fund relative to core asset, *fb* for fund relative to benchmark and *bc* for benchmark relative to core asset.

2.1. Exposures of funds to their benchmarks

Here we describe the methodology used to calculate the exposures of both active and passive funds to their respective benchmarks. Our example is constructed for an investor in euro and thus all returns in other currencies are converted into euro for consistency in the regressions below. For fixed income and commodities, the funds selected are hedged into euro and so, no calculations were required.

We use weekly excess returns. At a given time t these are calculated for all funds i and their respective benchmarks as:

$$XR_f^i(t) = TR_f^i(t) - Cash(t) - OCR_f^i \quad (1)$$

$$XR_b^i\,(t) = \; TR_b^i\,(t) - Cash(t) \tag{2}$$

where *TR* and *XR* denote total and excess return respectively, *Cash* is the money market rate in the Eurozone, and *OCR* are the ongoing charges ratio for each fund.

We regress the weekly excess returns of the fund, $XR_f^i\,(t)$, on the weekly excess returns of its benchmark, $XR_b^i(t)$:

$$XR_f^i\,(t) = \; \alpha_{fb}^i + \beta_{fb}^i\, XR_b^i(t) + \varepsilon_{fb}^i(t) \tag{3}$$

Where α_{fb}^i is the intercept of the regression. This regression is estimated using ordinary least squares (OLS) over the five-year period ending in April 2025, with the additional constraint that all betas are non-negative.

We define the information ratio of the fund relative to its exposure to the benchmark as:

$$IR_{\alpha_{fb}^i} = {}^{\alpha_{fb}^i}\!\big/_{\sigma_{fb}^i} \tag{4}$$

Where the specific volatility of the fund, σ_{fb}^i, is the annualized volatility of the residuals $\varepsilon_{fb}^i(t)$.

For passive funds that replicate their benchmark, we assume that the expected future alpha is zero. For active funds, we assume that these were selected because the investor expects them to generate positive alpha in the future. Instead of using the historical alpha estimated from the regression, we set $IR_{\alpha_{fb}^i}$ equal to 0.5 for all the selected active funds and 0 for all passive funds. This parameter could have been chosen differently and could have differed across the selected funds. The actual alpha for each fund will be corrected by its ongoing charges ratio (OCR).

$$\alpha_{fb}^i = \left(IR_{\alpha_{fb}^i} * \sigma_{fb}^i - OCR_f^i \right) \tag{5}$$

2.2. Exposures of benchmarks to core indices

The exposures of fund benchmarks and SAA benchmark indices to the core assets is estimated from regressions of the weekly excess returns of the benchmarks of each fund *i*, $XR_b^i\,(t)$, on the weekly excess returns, $\boldsymbol{XR}_c(t)$, of all the core assets used in the TAA. These are calculated from the total returns of the core assets and cash returns using an equation like (2). The list of core assets is made of 17 traditional core asset classes and three currency pairs against the euro:

$$XR_b^i(t) = \; \alpha_{bc}^i + \boldsymbol{\beta}_{bc}^{i\;\mathrm{T}}\, \boldsymbol{XR}_c(t) + \varepsilon_{bc}^i(t) \tag{6}$$

where $\boldsymbol{\beta}_{bc}^i$ is the vector with the exposures of the benchmark of fund *i* to all the core assets, α_{bc}^i is the intercept of this regression and $\varepsilon_{bc}^i(t)$ are the regression residuals. We define σ_{bc}^i as the annualized volatility of the residuals $\varepsilon_{bc}^i(t)$. By default, all α_{bc}^i are set to zero for the benchmarks of both active and passive funds, a choice which is supported by empirical evidence for most benchmark indices used in our example. However, for thematic benchmark indices, these residuals can be substantial and may even be expected to generate a positive alpha. We will return to this point later and illustrate how thematic views can be incorporated by adjusting these alphas. For clarity, throughout the remainder of the paper, we refer to the volatility of these residuals, obtained from the regression of benchmark returns on core asset returns, as thematic volatility.

In practice, this regression is done in two steps. First, we select the core assets more likely to be of relevance. We start by retaining only those core assets that match the asset class of a given benchmark (e.g., equities, fixed income) and then we apply a LASSO regression to further reduce the selection of core assets, retaining only those with coefficients above an empirically chosen threshold of 0.01. Details of the LASSO regression can be found in Appendix 5.1.

For commodities and listed real estate, no LASSO selection is performed and the respective core asset index for each is retained.

The regression above is then performed for each benchmark by using only the core assets selected for each fund benchmark in this way.

2.3. Exposures of funds to core indices

Using (6) in (3) while setting the α_{bc}^i to zero, we find:

$$XR_f^i(t) = \alpha_{fb}^i + \beta_{fb}^i \, \boldsymbol{\beta}_{bc}^{i\,\mathrm{T}} \, \boldsymbol{XR}_c(t) + \varepsilon_{fc}^i(t) \tag{7}$$

Where:

$$\varepsilon_{fc}^i(t) = \varepsilon_{fb}^i(t) + \beta_{fb}^i \, \varepsilon_{bc}^i(t) \tag{8}$$

We can then define the fund exposures to core assets as:

$$\boldsymbol{\beta}_{fc}^i = \beta_{fb}^i \, \boldsymbol{\beta}_{bc}^{i\,\mathrm{T}} \tag{9}$$

And, assuming that the residuals are uncorrelated, we can define the tracking error of the fund relative to core assets as:

$$\sigma_{fc}^i = \sqrt{\left(\sigma_{fb}^i\right)^2 + \left(\beta_{fb}^i \, \sigma_{bc}^i\right)^2} \tag{10}$$

This tracking error quantifies the residual risk of the fund after accounting for exposures to both its benchmark and the underlying core assets, which includes three currency pairs.

$\boldsymbol{\beta}_{fc}^i$ includes the betas for the three currency pairs. For an equity fund that is not currency-hedged, the beta measuring the foreign currency exposures is calculated by summing the betas that capture the exposure of its benchmark to core assets priced in those same currencies. For the funds hedged into euro, these currency betas are zero.

2.4. Risk Model

Following Bass, Gladstone, and Ang (2017), we apply Principal Component Analysis (PCA) to the correlation matrix of weekly excess returns for 17 major global asset class indices and three currency pairs (see Appendix 5.2), using data from end of April 2005 to end of April 2025. PCA decomposes the correlation matrix into eigenvectors (interpreted as long–short portfolios) and eigenvalues (which indicate their relative importance), effectively reducing dimensionality and redundancy. The resulting PCA correlation matrix is $\mathbf{C}_{core} = \boldsymbol{V}\boldsymbol{\Lambda}\boldsymbol{V}^{\mathrm{T}}$, where $\boldsymbol{V}$ is the $n_c \times n_c$ orthogonal matrix whose columns are the eigenvectors of $\mathbf{C}_{core}$ and $\boldsymbol{\Lambda}$ is the diagonal matrix of eigenvalues $\lambda_1, \lambda_2, \ldots, \lambda_{n_c}$ sorted in descending order. The variance–covariance matrix for the 17 core assets and three currencies (n_c=20) is then constructed as $\boldsymbol{\Sigma}_{core} = \boldsymbol{\Sigma}_{diag}\mathbf{C}_{core}\boldsymbol{\Sigma}_{diag}$, where $\boldsymbol{\Sigma}_{diag}$ is the diagonal matrix of core asset and currency volatilities estimated over the same period. Additional details are provided in Appendix 5.2.

Our approach extends Somefun et al. (2022) by incorporating currency pairs to account for non-hedged currency risk from investing in funds in foreign currency selected for implementation. The first two factors can also be easily interpreted as market risk and duration risk. However, adding the three currency pairs makes the interpretation of the additional four factors more difficult. Roncalli (2014) found that retaining only the few factors which explain most of the correlation structure is an effective way to de-noise the covariance matrix and improve the portfolio optimization stability.

When applied to the core assets we retain only the first six eigenvalues, which explain 83% of the variance, and set the diagonal matrix of $\boldsymbol{\Sigma}_{core}$ to the respective variance of each asset in the list of core assets. The volatility of each core asset is estimated from the same set of weekly returns in local currencies.

$$\boldsymbol{\Sigma}_{core} = \begin{bmatrix} var_{core}^{1} & \cdots & cov_{core}^{1,n_c} \\ \vdots & \ddots & \vdots \\ cov_{core}^{n_c,1} & \cdots & var_{core}^{n_c} \end{bmatrix} \tag{11}$$

With $var_{core}^{i} = cov_{core}^{i,i} = \left(\sigma_{core}^{i}\right)^2$ for core asset *i*.

The final variance–covariance matrix $\boldsymbol{\Sigma}$ for core assets, benchmarks and funds will have size $n \times n$ with $n = n_c + n_b + n_{f,}$ where n_b is the number of unique benchmarks and other indices which are not in the list of core assets and n_f is the number of funds selected. This matrix can be written as the sum:

$$\boldsymbol{\Sigma} = \boldsymbol{\Sigma}_{systematic} + \boldsymbol{\Sigma}_{thematic} + \boldsymbol{\Sigma}_{fund\ specific} \tag{12}$$

and requires the betas of the funds, $\boldsymbol{\beta}_{fc}^{i}$, and benchmarks, $\boldsymbol{\beta}_{bc}^{i}$, relative to core assets as well as the specific variance of funds, $var_{fc}^{i} = \left(\sigma_{fc}^{i}\right)^2$, and of benchmarks, $var_{bc}^{i} = \left(\sigma_{bc}^{i}\right)^2$, relative to core assets.

<u>Systematic variance</u>–<u>covariance matrix</u>: The systematic variance covariance, $\boldsymbol{\Sigma}_{systematic}$, is based on $\boldsymbol{\Sigma}_{core}$ and on $\boldsymbol{\beta}_{all}$, a $n \times n_c$ matrix where the columns have the vectors of exposures *i*) of core assets to themselves (each vector is 1 on the row for the respective core asset and zero otherwise), *ii*) of the benchmarks to the core assets, $\boldsymbol{\beta}_{bc}^{i}$, and *iii*) of the funds to the core assets, $\boldsymbol{\beta}_{fc}^{i}$:

$$\boldsymbol{\Sigma}_{systematic} = \boldsymbol{\beta}_{all}\boldsymbol{\Sigma}_{core}\boldsymbol{\beta}_{all}^{\mathrm{T}} \tag{13}$$

With:

$$
\boldsymbol{\beta}_{all} = \begin{array}{c} \text{Core assets} \\ \begin{bmatrix} 1 & \dots & 0 \\ \vdots & 1 & \vdots \\ 0 & \dots & 1 \\ \beta_{bc}^{1,1} & \dots & \beta_{bc}^{1,n_c} \\ \vdots & \dots & \vdots \\ \beta_{bc}^{n_b,1} & \dots & \beta_{bc}^{n_b,n_c} \\ \beta_{fc}^{1,1} & \dots & \beta_{fc}^{1,n_c} \\ \vdots & \dots & \vdots \\ \beta_{fc}^{n_f,1} & \dots & \beta_{fc}^{n_f,n_c} \end{bmatrix} \end{array} \begin{array}{l} \text{Core assets} \\ \\ \text{Benchmarks} \\ \\ \text{Funds} \end{array} \tag{14}
$$

And thus with:

$$
\boldsymbol{\Sigma}_{systematic} = \begin{array}{c} \begin{array}{ccc} \text{Core assets} & \text{Benchmarks} & \text{Funds} \end{array} \\ \begin{bmatrix} \boldsymbol{\Sigma}_{core} & \boldsymbol{\Sigma}_{systematic}^{core,bench} & \boldsymbol{\Sigma}_{systematic}^{core,fund} \\ {\boldsymbol{\Sigma}_{systematic}^{core,bench}}^{\mathrm{T}} & \boldsymbol{\Sigma}_{systematic}^{bench,bench} & \boldsymbol{\Sigma}_{systematic}^{bench,fund} \\ {\boldsymbol{\Sigma}_{systematic}^{core,fund}}^{\mathrm{T}} & {\boldsymbol{\Sigma}_{systematic}^{bench,fund}}^{\mathrm{T}} & \boldsymbol{\Sigma}_{systematic}^{fund,fund} \end{bmatrix} \end{array} \begin{array}{l} \text{Core assets} \\ \text{Benchmarks} \\ \text{Funds} \end{array} \tag{15}
$$

<u>Thematic variance</u>–<u>covariance matrix</u>: We assume that i) the thematic risks of benchmark indices are not correlated with each other, and ii) the thematic risks of benchmark indices are not correlated with the risk of core assets.

However, it is important to consider the correlations of the thematic risk of each fund with thematic risks of its benchmark index, determined by the beta β_{fb}^{i}.

$$
\boldsymbol{\Sigma}_{thematic} = \begin{array}{c} \begin{array}{ccc} \text{Core assets} & \text{Benchmarks} & \text{Funds} \end{array} \\ \begin{bmatrix} 0 & \dots & 0 & 0 & \dots & 0 & 0 & \dots & 0 \\ \vdots & \ddots & \vdots & \vdots & 0 & \vdots & \vdots & 0 & \vdots \\ 0 & \dots & 0 & 0 & \dots & 0 & 0 & \dots & 0 \\ 0 & \dots & 0 & var_{bc}^{1} & \dots & 0 & \beta_{fb}^{1} var_{bc}^{1} & \dots & 0 \\ \vdots & 0 & \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ 0 & \dots & 0 & 0 & \dots & var_{bc}^{n_b} & 0 & \dots & \beta_{fb}^{n_b} var_{bc}^{n_b} \\ 0 & \dots & 0 & \beta_{fb}^{1} var_{bc}^{1} & \dots & 0 & \left(\beta_{fb}^{1}\right)^2 var_{bc}^{1} & \dots & 0 \\ \vdots & 0 & \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ 0 & \dots & 0 & 0 & \dots & \beta_{fb}^{n_b} var_{bc}^{n_b} & 0 & \dots & \left(\beta_{fb}^{n_f}\right)^2 var_{bc}^{n_f} \end{bmatrix} \end{array} \begin{array}{l} \text{Core assets} \\ \\ \text{Benchmarks} \\ \\ \text{Funds} \end{array} \tag{16}
$$

Additionally, for any two funds, *i* and *j*, with the same benchmark, the off diagonal would be $\beta_{fb}^{i}\beta_{fb}^{j}var_{bc}$ to consider that they have the same benchmark.

Fund specific variance–covariance matrix: We assume that the specific risks of individual funds are uncorrelated with each other, and ii) also uncorrelated with the risks of core assets. Thus, this matrix is based only on the specific variance of funds relative to their benchmarks, $var_{fb}^{i} = \left(\sigma_{fb}^{i}\right)^2$:

$$\begin{array}{cc} & \begin{array}{ccc}\text{Core assets} & \text{Benchmarks} & \text{Funds}\end{array} \\ \boldsymbol{\Sigma}_{fund\ specific} = & \begin{bmatrix} 0 & \dots & 0 & 0 & \dots & 0 & 0 & \dots & 0 \\ \vdots & \ddots & \vdots & \vdots & 0 & \vdots & \vdots & 0 & \vdots \\ 0 & \dots & 0 & 0 & \dots & 0 & 0 & \dots & 0 \\ 0 & \dots & 0 & 0 & \dots & 0 & 0 & \dots & 0 \\ \vdots & 0 & \vdots & \vdots & \ddots & \vdots & \vdots & 0 & \vdots \\ 0 & \dots & 0 & 0 & \dots & 0 & 0 & \dots & 0 \\ 0 & \dots & 0 & 0 & \dots & 0 & var_{fb}^{1} & \dots & 0 \\ \vdots & 0 & \vdots & \vdots & 0 & \vdots & \vdots & \ddots & \vdots \\ 0 & \dots & 0 & 0 & \dots & 0 & 0 & \dots & var_{fb}^{n_f} \end{bmatrix} \begin{array}{l}\text{Core assets} \\ \\ \text{Benchmarks} \\ \\ \text{Funds}\end{array} \end{array} \quad (17)$$

2.5. Uncertainty Matrix

The uncertainty matrix $\boldsymbol{\Omega}$ is a $n \times n$ matrix associated with the uncertainty in the expected returns and is an important input for the robust optimization process that will be detailed below. Our choice for this matrix is:

$$\boldsymbol{\Omega} = \boldsymbol{\Omega}_{systematic} + \boldsymbol{\Sigma}_{thematic} + \boldsymbol{\Sigma}_{fund\ specific} \quad (18)$$

where:

$$\boldsymbol{\Omega}_{systematic} = \boldsymbol{\beta}_{all} diag(\boldsymbol{\Sigma}_{core}) \boldsymbol{\beta}_{all}^{\mathrm{T}} \quad (19)$$

is consistent with the choice of Yin et al. (2022) to use a diagonal uncertainty matrix with the variances of the assets for which we express our tactical views. However, here, while this remains a suitable choice for core assets, we can no longer ignore the correlations between the returns of funds and benchmarks with the core assets returns. These correlations are critical for translating tactical views on core assets and thematic views on benchmarks into effective fund allocations, and this is why the two additional terms $\boldsymbol{\Sigma}_{thematic}$ and $\boldsymbol{\Sigma}_{fund\ specific}$ in equation (18) are required. This feature is one of the most important in this framework when compared to that proposed by Yin et al. (2022), and it is a good example of an application of robust portfolio optimization where the uncertainty matrix of expected returns should not be diagonal. The final form of the uncertainty matrix is then:

$$\begin{array}{cc} & \begin{array}{ccc}\text{Core assets} & \text{Benchmarks} & \text{Funds}\end{array} \\ \boldsymbol{\Omega}_{systemic} = & \begin{bmatrix} \mathbf{diag}(\boldsymbol{\Sigma}_{core}) & \boldsymbol{\Omega}_{systemic}^{core,bench} & \boldsymbol{\Omega}_{systemic}^{core,fund} \\ {\boldsymbol{\Omega}_{systemic}^{core,bench}}^{\mathrm{T}} & \boldsymbol{\Omega}_{systemic}^{bench,bench} & \boldsymbol{\Omega}_{systemic}^{bench,fund} \\ {\boldsymbol{\Omega}_{systemic}^{core,fund}}^{\mathrm{T}} & {\boldsymbol{\Omega}_{systemic}^{bench,fund}}^{\mathrm{T}} & \boldsymbol{\Omega}_{systemic}^{fund,fund} \end{bmatrix} \begin{array}{l}\text{Core assets} \\ \text{Benchmarks} \\ \text{Funds}\end{array} \end{array} \quad (20)$$

2.6. Strategic Asset Allocation portfolio

The SAA portfolio plays a critical role in the final allocation, as the tracking error is measured against this portfolio and constraints on maximum active weights are defined relative to it. Table A1 in Appendix 5.2 presents our selected SAA portfolio. How this portfolio was constructed is beyond the scope of this paper; however, we followed the methodology proposed by Somefun et al. (2021).

While the choice of SAA portfolio influences the results, the framework proposed here is not dependent on this specific selection and different SAA portfolios could have been used. The key point is that the SAA is derived from indices that are not directly investable. Some of these indices are part of the core indices used for TAA views, while others are not. The portfolio optimization process is then used to determine the optimal portfolio benchmarked against this SAA, reflecting tactical and thematic views while investing solely in the selected active and passive funds and meeting all imposed portfolio constraints.

2.7. TAA and thematic views

The set of tactical views created from the views of an investment committee for the direction and conviction of expected returns for each core asset index is grouped into a vector of scores $\boldsymbol{S}_{directional} = (S^1_{directional}, \cdots, S^n_{directional})^{\mathrm{T}}$. While $\boldsymbol{S}_{directional}$ spans all assets (core assets, benchmarks and funds), in our example, only core asset indices are expected to have directional scores different from zero.

Thematic views are grouped into a vector of scores $\boldsymbol{S}_{thematic} = (S^1_{thematic}, \cdots, S^n_{thematic})^{\mathrm{T}}$ capturing only the thematic risk of the given index. While $\boldsymbol{S}_{thematic}$ spans all assets (core assets, benchmarks and funds), only thematic benchmark indices may have non-zero thematic scores.

The unconstrained active long-short portfolio $\boldsymbol{w}_{active}$ is constructed as the sum of two distinct active portfolios: one derived from directional views and the other from the thematic views:

$$\boldsymbol{w}_{active} = \boldsymbol{w}^{directional}_{active} + \boldsymbol{w}^{thematic}_{active} \tag{21}$$

Where:

$$\boldsymbol{w}^{directional}_{active} = \boldsymbol{S}_{directional} * (RB\ \boldsymbol{\sigma}^{-1}) \tag{22}$$

with $\boldsymbol{\sigma} = (\sigma_1, \cdots, \sigma_n)^{\mathrm{T}}$ the vector of asset volatilities and RB the risk budget so that each directional view $RB\ S_i = w^i_{active}\ \sigma_i$ captures the tracking error of the portfolio that is allocated to the view on the asset in question, based on the convictions and directions.

The second term, for thematic views, is given by:

$$\boldsymbol{w}^{thematic}_{active} = \boldsymbol{\beta}_{thematic} * \boldsymbol{S}_{thematic} * (RB\ \boldsymbol{\sigma}^{-1}_{thematic}) \tag{23}$$

where $\boldsymbol{\sigma}_{thematic} = \left(\sigma^1_{bc}, \cdots, \sigma^{n_b}_{bc}\right)^{\mathrm{T}}$ is the vector of asset thematic volatilities, the volatility of the residuals from equation (6). Hence, each thematic score is normalized by the thematic volatility of the corresponding index. The $n \times n$ matrix $\boldsymbol{\beta}_{thematic}$ is defined as:

$$\boldsymbol{\beta}_{thematic} = \begin{array}{c} \text{Core assets} \quad\quad \text{Benchmarks} \quad\quad \text{Funds} \\ \begin{bmatrix} 0 & \dots & 0 & -\beta_{bc}^{1,1} & \dots & -\beta_{bc}^{1,n_b} & 0 & \dots & 0 \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ 0 & \dots & 0 & -\beta_{bc}^{n_c,1} & \dots & -\beta_{bc}^{n_c,n_b} & 0 & \dots & 0 \\ 0 & \dots & 0 & 1 & \dots & 0 & 0 & \dots & 0 \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ 0 & \dots & 0 & 0 & \dots & 1 & 0 & \dots & 0 \\ 0 & \dots & 0 & 0 & \dots & 0 & 0 & \dots & 0 \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ 0 & \dots & 0 & 0 & \dots & 0 & 0 & \dots & 0 \end{bmatrix} \end{array} \begin{array}{l} \text{Core assets} \\ \\ \text{Benchmarks} \\ \\ \text{Funds} \end{array} \quad (24)$$

with the $\beta_{bc}^{j,i}$ in each column also estimated from equation (6).

The rationale behind constructing these thematic long-short portfolios, denoted as $\boldsymbol{w}_{active}^{thematic}$ is to allow for views on the expected alpha of the unique idiosyncratic risk component of a given thematic benchmark relative to its broader systematic risk exposures to core asset classes. This is achieved by decomposing the thematic index into its systematic (betas) and thematic risks using the regression in (6). When the manager holds a positive view on the theme, the portfolio takes a long position in the thematic index and a short position in its corresponding beta exposures to core assets. Conversely, a negative view is expressed by shorting the thematic index and going long on its systematic risk exposure.

2.8. Robust portfolio construction with implied active returns

The final constrained optimal tactical portfolio is built using the method proposed by Issaoui et al. (2021), which relies on robust portfolio optimization. It starts from the implied returns generated by the unconstrained tactical active allocation reflecting tactical and thematic views, $\boldsymbol{w}_{active}$. However, in our approach, we adjust these implied returns to also account for the expected alpha of the active funds.

The robust portfolio optimization to find the optimal constrained set of active weights $\boldsymbol{w}_{active}^{constrained} = \boldsymbol{w}_{\mathrm{TAA}}^{constrained} - \boldsymbol{w}_{\mathrm{SAA}}$ can be found by solving for $\boldsymbol{w}_{\mathrm{TAA}}$:

$$\boldsymbol{w}_{active}^{constrained} = \underset{\boldsymbol{w}_{\mathrm{TAA}}}{argmax} \left(\bar{\boldsymbol{\mu}}^{\mathrm{T}}(\boldsymbol{w}_{\mathrm{TAA}} - \boldsymbol{w}_{\mathrm{SAA}}) - \lambda(\boldsymbol{w}_{\mathrm{TAA}} - \boldsymbol{w}_{\mathrm{SAA}})^{\mathrm{T}}\boldsymbol{\Sigma}(\boldsymbol{w}_{\mathrm{TAA}} - \boldsymbol{w}_{\mathrm{SAA}}) \right.$$
$$\left. -\kappa\sqrt{(\boldsymbol{w}_{\mathrm{TAA}} - \boldsymbol{w}_{\mathrm{SAA}})^{\mathrm{T}}\boldsymbol{\Omega}(\boldsymbol{w}_{\mathrm{TAA}} - \boldsymbol{w}_{\mathrm{SAA}})} \right) \quad (25)$$

Under the desired constraints and with λ the investor risk aversion, κ the investor aversion to the uncertainty in expected returns, $\boldsymbol{\Sigma}$ the variance co-variance matrix defined in (12), $\boldsymbol{\Omega}$ the uncertainty matrix defined in (18). Here, both $\boldsymbol{w}_{\mathrm{SAA}}$ and $\boldsymbol{w}_{\mathrm{TAA}}$ are vectors with *n* rows. The SAA vector will obviously have weights set to zero for all funds, benchmarks and core assets not used in the SAA portfolio. In turn, the tactical portfolio will have all weights constrained to be zero expect for the allocation to active and passive funds. The implied returns $\bar{\boldsymbol{\mu}}$ for all core assets, benchmarks and funds used as inputs in (25) are calculated from the unconstrained active portfolio based on tactical and thematic views, $\boldsymbol{w}_{active}$ in (21) from:

$$\bar{\boldsymbol{\mu}} = 2\lambda \boldsymbol{\Sigma} \boldsymbol{w}_{active} + \kappa \frac{\boldsymbol{\Omega} \boldsymbol{w}_{active}}{\sqrt{\boldsymbol{w}_{active}{}^{\mathrm{T}} \boldsymbol{\Omega} \boldsymbol{w}_{active}}} + \frac{1}{\gamma} \boldsymbol{\alpha}_{fb} \tag{26}$$

With the last term added to accommodate the expected alphas for each active fund relative to the exposure to their respective benchmark, $\alpha_{fb}^{i} = \left(IR_{\alpha_{fb}^{i}} * \sigma_{fb}^{i} - OCR_{f}^{i} \right)$, including the ongoing charges ratio of each fund. This alpha is set to zero for all other assets. γ is a scaling factor used to calibrate the conviction on the expected active fund alphas. For very large values of γ, the conviction on the fund alphas will fall to zero. We set the investor risk aversion to uncertainty to

$$\kappa = 0.23 * min \left(1, \sum_i \left| \boldsymbol{S}_{dir}^{i} \right| + \sum_j \left| \boldsymbol{S}_{thematic}^{j} \right| \right) \tag{27}$$

diluting the aversion to uncertainty from the views, directional or thematic, if the views become increasingly small, so that the term in γ with the alphas from active funds dominates the allocation when there is lack of conviction in tactical and thematic views. On the other hand, κ will not exceed 0.23, slightly above half the average long-term average Sharpe ratio observed for most traditional assets, as proposed by Yin et al. (2020).

The risk aversion term is set to:

$$\lambda = \frac{1}{2} * \frac{0.4}{RB} \tag{28}$$

Substituting equation (28) into (26), while neglecting the second and third terms, and assuming a single directional view on one asset as defined by equation (22), we find that the implied return from the view is given by $\bar{\mu} = 0.4\,\sigma\, S_{directional}$. This implies that a $S_{directional}$=100% corresponds to an expected return of 0.4 per unit of the asset's volatility, aligned with the average long-term Sharpe ratio observed for most traditional asset classes. This choice of λ allows us to give a sensible meaning to the actual figures behind the implied returns.

3. RESULTS

In this section, we illustrate the implementation of our robust optimization framework through a series of examples, with particular emphasis on the intuition behind portfolio allocations.

All data sources and calculation details are documented in the Appendix. The examples begin with a strategic asset allocation (SAA) portfolio outlined in Table A1, which includes the betas of SAA indices against core assets derived from equation (6), as well as thematic volatilities relative to the same core asset combination. Table A2 extends this by presenting the betas and thematic volatilities of fund benchmarks relative to core assets, also obtained via equation (6).

Table A3 details the characteristics of the funds used to construct the TAA portfolios, while Table A4 provides the betas of funds against their respective benchmarks obtained from equation (3), the betas of funds against the core assets obtained from equation (7), and specific volatilities of the funds obtained from equation (8). Table A4 also includes each fund's ongoing charges ratio (OCR), expected information ratio, and final alpha from equation (5).

The factors from the PCA risk model for core assets are summarized in Table A5. The TAA allocations to funds for all the examples considered below is also in the Appendix, in Table A6.

Here, we begin by analyzing the impact of fund alphas on the final TAA portfolio under basic constraints (no shorting, no leverage, and full investment in funds only) without tactical or thematic views. We then introduce a sustainability constraint, reflecting current market practices requiring minimum exposure to sustainable investments. Subsequent examples explore the effect of adding views: First, a tactical view on US equities, followed by a tactical view on EUR sovereign bonds, and finally a thematic view on disruptive technology.

3.1. No tactical or thematic views

We start with the example with no tactical views and no thematic views. In this case, $\boldsymbol{w}_{active}$ will be zero.

In the limit of a vanishingly small $\boldsymbol{w}_{active}$, the term in $\boldsymbol{\Omega}\boldsymbol{w}_{active}/\sqrt{\boldsymbol{w}_{active}{}^{\mathrm{T}}\boldsymbol{\Omega}\boldsymbol{w}_{active}}$ in (27) will converge to a directional unit vector and, because of our choice of κ in (22), vanishing with vanishing active and thematic views, the implied returns in (26) will converge to $\bar{\boldsymbol{\mu}} = \boldsymbol{\alpha}_{fb}/\gamma$, dominated by the alpha of the funds.

In this case, as we decrease our confidence in the fund alphas by increasing γ, the solution to the optimization in (25) for the fully invested $\boldsymbol{w}_{\mathrm{TAA}}$ portfolio with the typical no short and no leverage constraints will converge towards the portfolio with the smallest possible ex-ante tracking error relative to portfolio $\boldsymbol{w}_{\mathrm{SAA}}$ that allocates only to funds:

$$\boldsymbol{w}_{active}^{constrained} = \underset{\boldsymbol{w}_{\mathrm{TAA}}}{argmin}\left((\boldsymbol{w}_{\mathrm{TAA}} - \boldsymbol{w}_{\mathrm{SAA}})^{\mathrm{T}}\boldsymbol{\Sigma}(\boldsymbol{w}_{\mathrm{TAA}} - \boldsymbol{w}_{\mathrm{SAA}})\right) \tag{29}$$

In turn, for a given sufficiently small γ, with (27), the optimization problem in (24) is only determined by the two terms:

$$\boldsymbol{w}_{active}^{constrained} = \underset{\boldsymbol{w}_{\mathrm{TAA}}}{argmax}\left(\frac{\boldsymbol{\alpha}_{fb}{}^{\mathrm{T}}}{\gamma}(\boldsymbol{w}_{\mathrm{TAA}} - \boldsymbol{w}_{\mathrm{SAA}}) - \frac{0.4}{2RB}(\boldsymbol{w}_{\mathrm{TAA}} - \boldsymbol{w}_{\mathrm{SAA}})^{\mathrm{T}}\boldsymbol{\Sigma}(\boldsymbol{w}_{\mathrm{TAA}} - \boldsymbol{w}_{\mathrm{SAA}})\right) \tag{30}$$

and it becomes invariant to the choice of the ratio RB/γ. For a given risk budget RB, reducing γ should favor allocations towards funds with positive alpha, while an increase in γ should reduce their importance relative to the combination of funds that best tracks the SAA portfolio.

The data provided in Tables A1, A2, A4 and A5 is required to implement the portfolio optimization in equation (30). Under the assumption of no tactical or thematic views, the optimization was conducted by increasing the investor's appetite for alpha from active funds. This was achieved by increasing the ratio RB/γ. The only constraints imposed were no short positions, no leverage, and the requirement that the optimal portfolio be fully and exclusively invested in the funds listed in Table A4. The SAA used as a reference is detailed in Table A1. The resulting portfolio allocations to funds is presented in Table A6. To facilitate interpretation and discussion, we analyze the outcomes by aggregating the selected funds along multiple dimensions, highlighting the impact of fund alpha appetite on the tactical portfolio and alignment with the SAA.

At $RB/\gamma = 0$ (i.e. in the limit of maximum aversion to active fund alpha ($\gamma \to \infty$) or the limit of zero risk budget ($RB \to 0$)) the optimal solution corresponds to the portfolio with the minimum tracking error relative to the SAA portfolio while remaining fully invested exclusively in the funds listed in Table A3. Indeed, in the Appendix 5.4.1, we show analytically that, for as long as the long only and the no leverage constrained are fulfilled, the solution to equation (30) yields portfolios that can be expressed as the sum of the fund allocation that minimizes the tracking error relative to the SAA, plus a long-short component that scales with RB/γ, tilting the final allocation towards funds with the highest alpha. Moreover, in Appendix 5.4.2, we show that the square of the tracking error can be written as the sum of the square of the tracking error of the portfolio that minimizes tracking error plus the square of the variance of the long-short component, with the tilts towards high alpha funds increasing with the appetite for active alpha (increasing with $(RB/\gamma)^2$).

As shown in Table 1, with $RB/\gamma = 0$, the optimized portfolio achieves an ex-ante volatility of 7.35%, marginally higher than the volatility of the SAA portfolio of 7.31%. Its tracking error is just 0.49% relative to the SAA, indicating strong alignment with the reference allocation. However, the portfolio comprises both active and passive funds, suggesting that passive funds alone were insufficient to replicate the SAA with minimal tracking error. Even though in this case the alpha signals from active funds play no role in determining the allocation, their inclusion remains important from a risk management standpoint. Active funds help reduce tracking error while adhering to constraints: no shorting, no leverage, and full investment in the funds listed in Table A3 only. Even if active fund alpha plays no role in determining the minimum tracking error, given our fund alpha expectations in Table A4, this portfolio can be expected to marginally outperform the SAA portfolio by 5 basis points (bp) after accounting for fund costs.

Table 1: TAA portfolios as a function of appetite for active alpha with no tactical or thematic views.

	Optimal Tactical Portfolios as a function of RB/γ				
RB/γ	2.00%	0.40%	0.13%	0.04%	0.00%
Portfolio Active Fund Net Alpha	1.41%	0.95%	0.47%	0.18%	0.05%
Tactical & Thematic Implied Excess Returns	-	-	-	-	-
Information ratio	0.80	0.99	0.77	0.35	0.11
Portfolio Volatility	7.73%	7.42%	7.36%	7.35%	7.35%
Portfolio Tracking Error	1.76%	0.97%	0.61%	0.50%	0.49%
Active Fundamental	72.2%	67.0%	58.1%	50.2%	48.0%
Active Quant	27.8%	32.3%	20.9%	12.7%	7.1%
Passive Index	0.0%	0.7%	21.0%	37.1%	44.8%
Equity	50.8%	44.0%	42.2%	41.4%	41.1%
Fixed Income	39.5%	45.8%	47.8%	48.6%	48.9%
Commodities	7.8%	6.3%	6.1%	6.1%	6.1%
Real Estate	1.9%	3.9%	3.9%	3.8%	3.8%

Note: For illustration purpose only. Past performance is not indicative of future performance. Not investment advice.

Source: Based on the results in Table A6 aggregated by asset class, fund approach and philosophy. Authors' calculations on 1st September 2025

As confidence in active fund alpha increases, reflected by a higher RB/γ ratio, the optimized portfolio tilts in favor of active funds. This adjustment captures the additional source of positive, uncorrelated returns, enhancing the portfolio's expected performance. The contribution of net active alpha to the TAA portfolio's excess return over the SAA grows with RB/γ.

At sufficiently high levels of RB/γ, the share of passive funds in the optimized portfolio falls to zero, and beyond that level we can expect the portfolio to tilt in favor of the active funds with the highest alpha. This explains why the allocation to equities starts increasing beyond a certain level: equity active funds have higher alpha than fixed income active funds.

We also see that the allocation to active quant funds in Table 1 declines at the highest RB/γ values. As shown in Table A6, this arises mainly due to a drop in the allocation to fixed income quant funds, which have smaller net active alpha (see Table A4).

These intuitions are supported by a deeper analysis of the optimization problem presented in equation (30). In Appendix 5.4.3, we demonstrate analytically, using a simplified example with two funds, that increasing the appetite for active alpha naturally leads to a greater allocation to the fund offering the highest net active alpha. This explains why, in a scenario involving one active equity fund and one active fixed income fund, it is reasonable to expect a tilt towards equity, given its typically higher net active alpha. Likewise, when choosing between a passive fund and an active fund with positive net active alpha, the allocation increasingly favors the active fund as the appetite for active alpha grows.

Table 2: TAA portfolios as a function of appetite for active alpha with no tactical or thematic views.

	Optimal Tactical Portfolios as function of RB/γ				
RB/γ	2.00%	0.40%	0.13%	0.04%	0.00%
Equity Europe Active	12%	16%	14%	12%	11%
Equity USA Active	8.5%	9.5%	5.5%	1.9%	0.3%
Equity Japan Active	12.7%	7.7%	4.6%	1.5%	0.3%
Equity Emerging Active	9.8%	5.0%	1.7%	0.6%	0.1%
Equity World Active	8.1%	5.4%	2.7%	1.0%	0.3%
Bond EUR Sovereign Active	-	10.2%	7.4%	3.4%	1.7%
Bond EUR Investment Grade Active	-	4.6%	3.1%	1.2%	0.6%
Bond EUR Aggregate Active	-	-	5.0%	11.2%	14.1%
Bond EUR High Yield Active	-	-	0.1%	0.1%	0.1%
Bond USA Investment Grade Active	-	6.6%	5.8%	5.6%	3.7%
Bond USA High Yield Active	-	1.9%	3.2%	3.4%	3.5%
Bond Global Agg Active	32.3%	17.0%	15.5%	14.2%	13.5%
Bond Emerging HC Active	7.2%	5.5%	2.7%	0.4%	-
Commodities Active	7.8%	6.3%	3.9%	2.9%	2.5%
Real Estate Europe Active	1.9%	3.9%	3.8%	3.4%	3.1%
Equity Europe Passive	-	-	1.9%	3.5%	4.1%
Equity USA Passive	-	-	4.5%	8.4%	10.0%
Equity Japan Passive	-	-	2.2%	5.1%	6.3%
Equity Emerging Passive	-	0.7%	4.3%	5.4%	5.9%
Equity World Passive	-	-	0.9%	2.0%	2.4%
Bond EUR Sovereign Passive	-	-	1.8%	2.6%	2.9%
Bond EUR Investment Grade Passive	-	-	-	-	-
Bond USA Investment Grade Passive	-	-	-	-	1.8%
Bond Emerging HC Passive	-	-	3.1%	6.4%	7.1%
Commodities Passive	-	-	2.3%	3.2%	3.7%
Real Estate Europe Passive	-	-	0.1%	0.5%	0.6%

Note: For illustration purpose only. Not investment advice.

Source: Based on the results in Table A6 aggregated by asset class, fund approach, coverage areas, and for bonds, by investment styles. Authors' calculations on 1st September 2025.

In Table 2, we regroup the portfolios with no tactical and no thematic views from Table A6 by fund type, fund philosophy, coverage areas, and investment styles. This aggregation reveals that, as confidence in the expected alpha of active funds increases, the optimal fund allocation diverges significantly from the original minimum tracking error portfolio. In this specific case, several funds that were nearly absent from the portfolio with the smallest tracking error relative to the SAA become dominant at high levels of confidence in active fund alpha. For fixed income, the allocation eventually becomes concentrated entirely in just two active bond funds – those with the highest expected alpha in Table A4.

However, these changes in the allocation to funds arise from increasing exposure to high alpha active funds without compromising the exposures to core assets in the SAA. As shown in Table 3, the exposures to core assets of these optimized portfolios, constructed under varying levels of confidence in active fund alphas, remain closely aligned with the exposures of the SAA. This indicates that the optimizer effectively combines available funds to maximize expected active fund alpha while preserving the overall risk profile of the reference allocation. The rise in tracking error is thus driven almost entirely by the inclusion of uncorrelated alpha sources from active funds, despite substantial changes in the underlying fund composition.

Table 3: Betas of TAA portfolios relative to core assets as a function of appetite for active alpha with no tactical or thematic views.

	Portfolio Beta exposures of to Core Assets					
	Optimal Tactical Portfolios as a function of *RB* / γ					SAA Portfolio
RB / γ	2.00%	0.40%	0.13%	0.04%	0.00%	
Equity Europe EMU	0.05	0.07	0.08	0.08	0.08	0.08
Equity Europe EMU SC	0.01	0.02	0.02	0.02	0.02	0.01
Equity Europe UK	0.04	0.06	0.06	0.06	0.06	0.06
Equity North America USA	0.14	0.12	0.12	0.12	0.12	0.12
Equity North America USA SC	-	0.01	-	-	-	-
Equity Pacific Japan	0.12	0.08	0.07	0.07	0.07	0.07
Equity Emerging Global	0.09	0.05	0.06	0.06	0.06	0.06
Bond EUR Sovereign	0.07	0.14	0.16	0.17	0.17	0.13
Bond EUR Investment Grade	0.05	0.08	0.09	0.09	0.09	0.05
Bond EUR High Yield	-	-	-	-	-	0.01
Bond USD Sovereign	0.14	0.07	0.06	0.05	0.05	0.13
Bond USD Investment Grade	0.03	0.08	0.07	0.07	0.06	0.05
Bond USD High Yield	-	0.02	0.03	0.03	0.03	0.04
Bond EMD HC Sov Global	0.11	0.08	0.07	0.07	0.07	0.07
Bond EMD LC Sov Global	-	-	-	-	-	-
Diversification Real Estate Pan-Europe	0.02	0.04	0.04	0.04	0.04	0.04
Diversification Commodity Global	0.07	0.05	0.05	0.05	0.05	0.05
Currency USD	0.23	0.18	0.18	0.18	0.18	0.18
Currency GBP	0.04	0.06	0.06	0.06	0.06	0.06
Currency JPY	0.12	0.08	0.07	0.07	0.07	0.07

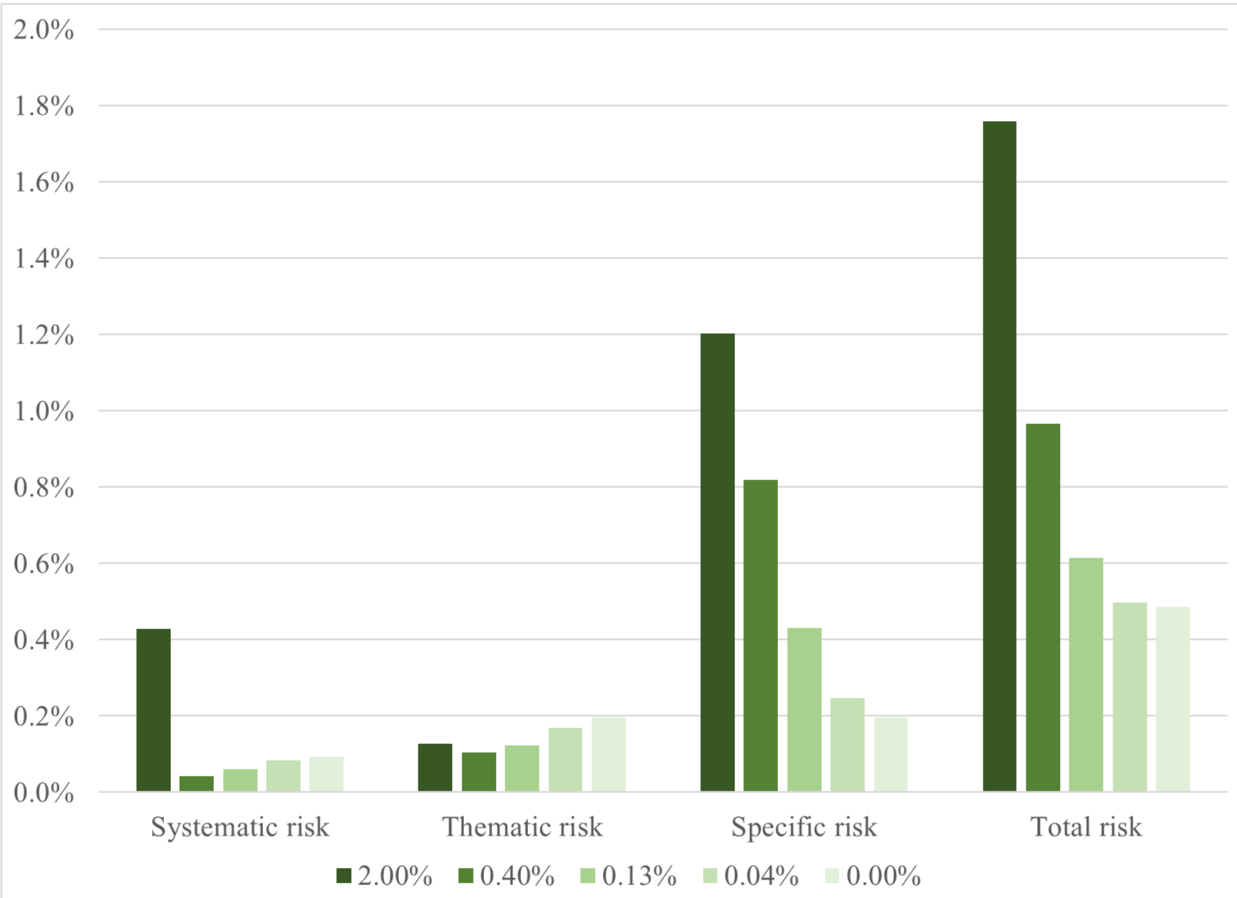
Note: For illustration purpose only.

Source: BNP Paribas Asset Management, MSCI, Russell, Bloomberg, J.P. Morgan, FTSE EPRA. Authors' calculations on 1st September 2025.

Finally, Figure 1 illustrates the contributions of systematic, thematic, and specific risk to portfolio tracking error, based on the risk model described in Section 2.4. The results indicate that as the appetite for active alpha increases with *RB/γ*, the rise in tracking error is driven primarily by higher specific risk, resulting from greater allocations to active funds, as desired. Contributions from thematic and systematic risk remain negligible for most of the range. Only at the highest levels of active alpha appetite does the allocation introduce additional systematic risk, as this becomes the only way to satisfy the increased demand for active exposure. Conversely, when there is no appetite for active alpha (*RB/γ* = 0.00%), tracking error arises mainly from residual specific and thematic risk that cannot be further reduced given the available fund selection, while the contribution from systematic risk remains minimal.

Based on the results in this section, we chose to work with RB/γ = 0.13%, i.e., with a risk budget RB = 2% and with $\gamma = 15$, for the remainder of the paper. This choice represents a reasonable trade-off between capturing meaningful active fund alpha net contributions and avoiding excessive concentration in a narrow subset of higher alpha funds. At this level, the portfolio achieves a diversified allocation across active strategies while remaining well-aligned with the SAA exposures.

Figure 1: Contributions of systematic, thematic, and specific risk to the tracking error of the TAA portfolios as a function of RB/γ.



Note: For illustration purpose only.

Source: BNP Paribas Asset Management, MSCI, Russell, Bloomberg, J.P. Morgan, FTSE EPRA. Authors' calculations on 1st September 2025.

3.2. No tactical or thematic views and additional constraints

We now repeat the exercise from the previous section, this time introducing a constraint on the portfolio's allocation – that of sustainable investments. This constraint should push the portfolios to increasingly favor funds with the highest sustainable investment allocations, even if we did not explicitly express a positive thematic view in favor of funds with higher allocations to sustainable investments.

In Table 4, we regroup the results for *minimum sustainable investment (SI) constraint* in Table A6. The results indicate that, for the set of funds used in the example, portfolio allocations appear largely unchanged up to a 30% minimum allocation to sustainable investments. However, once the minimum allocation to SI exceeds 40%, allocations start changing and the tracking error begins to rise. At the 75% threshold, the tracking error increases significantly as the allocation becomes dominated by active fundamental funds and the allocation to fixed income increases.

Table 4: TAA portfolios with minimum allocation to a sustainable investment constraint and no tactical or thematic views.

	Optimal Tactical Portfolios with Minimum allocation to Sustainable Investments				
Minimum SI	0%	30%	40%	50%	75%
Portfolio Active Fund Net Alpha	0.47%	0.47%	0.51%	0.52%	0.33%
Tactical & Thematic Implied Excess Returns	-	-	-	-	-
Information ratio	0.77	0.77	0.77	0.67	0.12
Portfolio Volatility	7.36%	7.36%	7.37%	7.38%	6.92%
Portfolio Tracking Error	0.61%	0.61%	0.66%	0.77%	2.66%
Active Fundamental	58.1%	58.1%	65.1%	70.6%	95.1%
Active Quant	20.9%	20.9%	19.0%	17.7%	4.9%
Passive Index	21.0%	21.0%	15.9%	11.7%	0.0%
Equity	42.2%	42.2%	42.8%	43.5%	40.0%
Fixed Income	47.8%	47.8%	47.2%	46.5%	54.9%
Commodities	6.1%	6.1%	6.1%	6.1%	0.0%
Real Estate	3.9%	3.9%	3.9%	4.0%	5.2%

Note: For illustration purpose only. Past performance is not indicative of future performance. Not investment advice.

Source: Based on the results in Table A6 aggregated by asset class and fund approach and philosophy. Authors' calculations on 1st September 2025.

Table 5: TAA portfolios with minimum allocation to a sustainable investment constraint and no tactical or thematic views.

	Optimal Tactical Portfolios with Minimum allocation to Sustainable Investments				
Minimum SI	0%	30%	40%	50%	75%
Equity Europe Active	14%	14%	14%	15%	16%
Equity USA Active	5.4%	5.4%	7.0%	7.7%	-
Equity Japan Active	4.6%	4.6%	4.6%	4.6%	-
Equity Emerging Active	1.7%	1.7%	1.7%	1.7%	-
Equity World Active	2.7%	2.7%	3.5%	4.5%	24.4%
Bond EUR Sovereign Active	7.5%	7.5%	7.1%	-	-
Bond EUR Investment Grade Active	3.1%	3.1%	0.8%	-	-
Bond EUR Aggregate Active	5.0%	5.0%	-	-	-
Bond EUR High Yield Active	-	-	0.3%	0.4%	-
Bond USA Investment Grade Active	5.8%	5.8%	4.7%	2.8%	-
Bond USA High Yield Active	3.2%	3.2%	2.8%	1.5%	-
Bond Global Agg Active	15.5%	15.5%	26.6%	37.9%	54.9%
Bond Emerging HC Active	2.7%	2.7%	3.0%	3.9%	-
Commodities Active	3.9%	3.9%	3.9%	3.9%	-
Real Estate Europe Active	3.8%	3.8%	3.9%	4.0%	5.2%
Equity Europe Passive	1.9%	1.9%	1.5%	1.3%	-
Equity USA Passive	4.5%	4.5%	1.2%	-	-
Equity Japan Passive	2.2%	2.2%	1.8%	1.6%	-
Equity Emerging Passive	4.3%	4.3%	4.1%	4.0%	-
Equity World Passive	0.9%	0.9%	2.9%	2.7%	-
Bond EUR Sovereign Passive	1.7%	1.7%	-	-	-
Bond EUR Investment Grade Passive	-	-	-	-	-
Bond USA Investment Grade Passive	-	-	-	-	-
Bond Emerging HC Passive	3.1%	3.1%	2.1%	-	-
Commodities Passive	2.3%	2.3%	2.2%	2.2%	-
Real Estate Europe Passive	0.1%	0.1%	-	-	-

Note: For illustration purpose only. Not investment advice.

Source: Based on the results in Table A6 aggregated by asset class, fund approach, coverage areas, and for bonds, by investment styles. Authors' calculations on 1st September 2025.

Using an alternative grouping, Table 5 shows that when a minimum of 75% allocation to sustainable investments is imposed, the portfolio becomes heavily concentrated in Bond Global Aggregate Active, which includes the Bonds Global Green Active Fundamental Art 9 fund, and Equity World Active, which includes the Equity World Water Active Fundamental Art 9 fund – see Table A6 for allocation with funds. The remaining allocation is directed to Equity Europe Active, spread across two active funds. So, it is the fact that only two funds listed in Table A3 exceed the 75% sustainability threshold which explains the resulting lack of fund diversification.

Moreover, the concentration required to meet a 75% minimum allocation to sustainable investments reduces the allocation to higher-alpha active funds, which explains the fall in the expected information ratio. In the absence of an explicit positive thematic view favoring funds with larger sustainable allocations, and given the current fund universe, such a high minimum threshold increases exposure to a single fixed-income fund with relatively low net active alpha, contributing to the drop in the information ratio.

As shown in Table 6, despite the reduction in fund diversification caused by the increasing allocation to funds with high sustainable investment exposure, the portfolios maintain a reasonable alignment with the core asset class exposures in the SAA up to a minimum 50% allocation. The optimizer successfully finds a reasonable compromise of fund allocation, preserving alignment with the SAA portfolio.

Table 6: Betas of TAA portfolios relative to core assets as a function of minimum allocation to a sustainable investment constraint and no tactical or thematic views

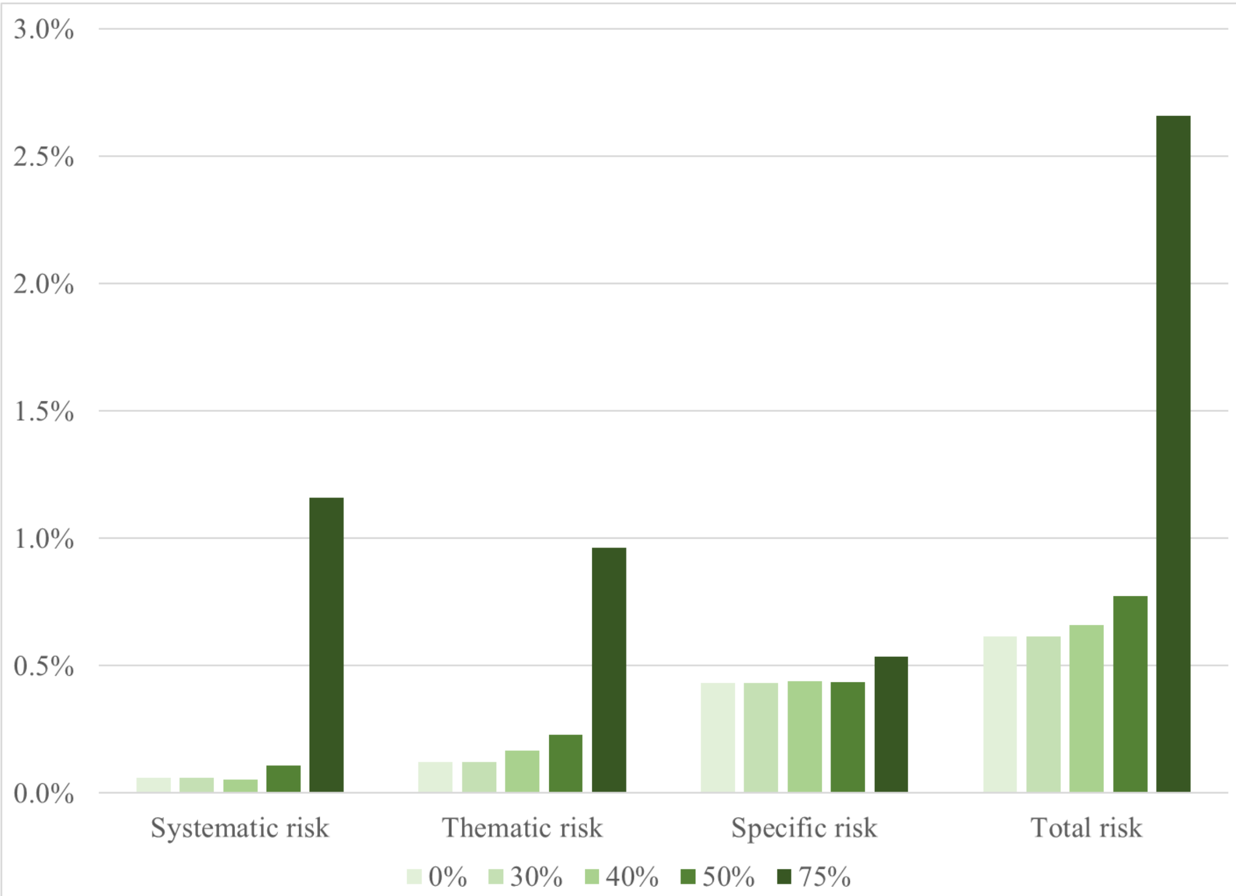
	Portfolio Beta exposures of to Core Assets					
	SAA Portfolio	Optimal Tactical Portfolios with Minimum allocation to Sustainable Investments				
Minimum SI		0%	30%	40%	50%	75%
Equity Europe EMU	0.08	0.08	0.08	0.08	0.08	0.07
Equity Europe EMU SC	0.01	0.02	0.02	0.02	0.02	0.04
Equity Europe UK	0.06	0.06	0.06	0.06	0.07	0.09
Equity USA	0.12	0.12	0.12	0.11	0.11	0.06
Equity USA SC	-	0.00	0.00	0.00	0.01	0.05
Equity Pacific Japan	0.07	0.07	0.07	0.07	0.07	0.03
Equity Emerging Global	0.06	0.06	0.06	0.06	0.06	0.01
Bond EUR Sovereign	0.13	0.16	0.16	0.14	0.11	0.16
Bond EUR Investment Grade	0.05	0.09	0.09	0.11	0.17	0.26
Bond EUR High Yield	0.01	0.00	0.00	0.00	0.00	0.00
Bond USD Sovereign	0.13	0.06	0.06	0.08	0.10	0.15
Bond USD Investment Grade	0.05	0.07	0.07	0.06	0.05	0.03
Bond USD High Yield	0.04	0.03	0.03	0.03	0.01	0.00
Bond EMD HC Sov Global	0.07	0.07	0.07	0.06	0.05	0.00
Bond EMD LC Sov Global	-	0.00	0.00	0.00	0.00	0.00
Diversification Real Estate Pan-Europe	0.04	0.04	0.04	0.04	0.04	0.05
Diversification Commodity Global	0.05	0.05	0.05	0.05	0.05	0.00
Currency USD	0.18	0.18	0.18	0.18	0.17	0.12
Currency GBP	0.06	0.06	0.06	0.06	0.07	0.09
Currency JPY	0.07	0.07	0.07	0.07	0.07	0.03

Note: For illustration purpose only.

Source: BNP Paribas Asset Management, MSCI, Russell, Bloomberg, J.P. Morgan, FTSE EPRA. Authors' calculations on 1st September 2025.

Figure 2 shows the contributions of systematic, thematic, and specific risk to portfolio tracking error of the portfolios discussed in this section. For the given fund selection, the results indicate that when the required minimum allocation to sustainable investments is below 50%, tracking error remains broadly aligned with that of an unconstrained portfolio. In this range, tracking error is driven mainly by specific risk, consistent with the targeted appetite for active alpha. However, at much higher minimum allocations, e.g. at 75%, the portfolio is forced to introduce exposures to systematic and thematic risk. Given the current investment views in this example, these risks are not expected to contribute to performance. Therefore, under these conditions and with this fund universe, imposing a 75% minimum allocation to sustainable investments is suboptimal.

Figure 2: Contributions of systematic, thematic, and specific risk to the tracking error of the TAA portfolios as a function of the minimum sustainable investment (SI) constraint.



Note: For illustration purpose only.

Source: BNP Paribas Asset Management, MSCI, Russell, Bloomberg, J.P. Morgan, FTSE EPRA. Authors' calculations on 1st September 2025.

3.4. Tactical and thematic views

We now examine the impact of incorporating tactical and thematic views into the portfolio. These views are analyzed separately. First, we introduce a directional view on Equity USA, testing five different values for $S_{directional}$ in equation (22), ranging from very negative (-100%) to very positive (+100%). We then repeat the exercise for a directional view on Bond EUR Sovereign, with $S_{directional}$ varying also from -100% to +100%. Finally, we assess a thematic view on Disruptive Tech, where $S_{thematic}$ in equation (23) also ranges from -100% to +100%. The thematic view requires constructing the matrix in equation (24) by using the exposures of the Equity Global All Countries Disruptive Tech index in Table A4.

The portfolios are constructed by solving equation (25), using the implied returns from equation (26), the active allocations constructed from equations (23) and (24) and the different $S_{directional}$ and $S_{thematic}$ signals. As noted earlier, we set RB = 2% and γ = 15. The TAA portfolios are fully invested and subject to the no short and no leverage constraints. Table A6 in the appendix presents the optimal TAA portfolios together with the underlying fund allocations. Below, we analyze the results by examining various aggregations of the portfolios shown in Table A6.

Table 7 summarizes several key results for these portfolios. First, it shows the net alpha contribution from active fund investments, calculated by multiplying the TAA portfolio weights from Table A6 by the net alphas of each fund listed in Table A4. This approach reflects the use of γ to change the impact of active fund alphas in the portfolio optimization, without altering our underlying views on those alphas. For a similar reason, Table 7 also presents the expected excess return of the TAA portfolio relative to the SAA arising from the tactical and thematic views, calculated by multiplying the active weights of portfolios TAA relative to SAA by the first term of the implied returns from equation (26), i.e., excluding the second term on κ and third term on $1/\gamma$.

Table 7: TAA portfolios with either tactical or thematic views.

	Optimal Tactical Portfolios with views														
	Equity USA view ($S_{directional}$)					Bond EUR Sovereign view ($S_{directional}$)					Disruptive Tech Thematic view ($S_{thematic}$)				
S	-100%	-50%	0%	50%	100%	-100%	-50%	0%	50%	100%	-100%	-50%	0%	50%	100%
Portfolio Active Fund Net Alpha	0.29%	0.37%	0.47%	0.35%	0.31%	0.44%	0.42%	0.47%	0.24%	0.16%	0.22%	0.28%	0.47%	0.51%	0.50%
Tactical & Thematic Implied Excess Returns	0.72%	0.20%	-	0.21%	0.78%	0.19%	0.06%	-	0.11%	0.32%	-	-	-	0.11%	0.31%
Total Excess Return	1.01%	0.56%	0.47%	0.56%	1.10%	0.63%	0.52%	0.47%	0.44%	0.48%	0.22%	0.28%	0.47%	0.67%	0.81%
Information ratio	0.52	0.48	0.77	0.47	0.53	0.59	0.63	0.77	0.54	0.41	0.41	0.51	0.77	0.72	0.67
Portfolio Volatility	6.19%	6.71%	7.36%	8.11%	8.85%	7.72%	7.55%	7.36%	7.28%	7.17%	7.41%	7.38%	7.36%	7.45%	7.54%
Portfolio Tracking Error	1.94%	1.18%	0.61%	1.19%	2.05%	1.06%	0.83%	0.61%	0.81%	1.16%	0.54%	0.55%	0.61%	0.93%	1.22%
Active Fundamental	68.9%	65.8%	58.1%	51.8%	46.2%	51.8%	53.5%	58.1%	38.2%	28.0%	52.2%	55.5%	58.1%	68.2%	69.9%
Active Quant	10.7%	13.9%	20.9%	13.4%	12.1%	19.3%	18.6%	20.9%	6.2%	3.1%	10.1%	11.8%	20.9%	12.9%	9.7%
Passive Index	20.4%	20.3%	21.0%	34.7%	41.7%	28.9%	27.9%	21.0%	55.6%	68.9%	37.7%	32.8%	21.0%	18.9%	20.4%
Equity	29.8%	35.6%	42.2%	48.2%	53.8%	44.2%	43.6%	42.2%	39.6%	38.0%	41.8%	41.9%	42.2%	40.3%	39.4%
Fixed Income	60.0%	54.2%	47.8%	42.0%	36.4%	44.9%	46.0%	47.8%	50.8%	53.0%	48.2%	48.1%	47.8%	49.7%	50.7%
Commodities	6.7%	6.3%	6.1%	5.7%	5.4%	7.8%	7.1%	6.1%	4.9%	3.9%	6.0%	6.0%	6.1%	6.1%	6.1%
Real Estate	3.5%	3.8%	3.9%	4.1%	4.4%	3.0%	3.3%	3.9%	4.7%	5.1%	4.0%	4.0%	3.9%	3.9%	3.8%

Note: For illustration purpose only. Past performance is not indicative of future performance. Not investment advice.

Source: Based on the results in Table A6 aggregated by asset class and fund approach and philosophy. Authors' calculations on 1st September 2025.

As the tactical view on the core asset Equity USA shifts from negative to positive, the portfolio's allocation to equities increases significantly, as expected, offset by a corresponding decrease in fixed income exposure. Conversely, for Bond EUR Sovereign, a more positive view results in higher allocations to fixed income, with a reduction in equity exposure.

For the tactical views on Equity USA and Bond EUR Sovereign, the results in Table 7 show that strong directional views lead to an increase in expected excess returns and to an increase in tracking error, regardless of whether the view is positive or negative.

Another important observation is that the allocation to passive funds tends to increase as the tactical view on equities or sovereign bonds becomes more positive.

For the thematic view on Disruptive Tech, only positive views create positive excess returns and increase tracking error. Here, the share of passive decreases as we become more bullish on Disruptive Tech.

The results in Table 8 help clarify what is going on. A positive tactical view on Equity USA is primarily implemented through increased allocations to US passive equity funds, while the share of US active equity funds remains relatively stable. This stability persists until the tactical view becomes sufficiently negative, at which point all US equities are removed from the portfolio. As the view on Equity USA turns negative, the declining allocation to US equities is offset by a shift towards the most attractive form of fixed income, specifically, the two Aggregate Fixed Income Active funds, which are the least volatile.

A strong positive tactical view on Bond EUR Sovereign is primarily implemented through increased allocations to passive sovereign bond funds, accompanied by a broad reduction in active fixed income exposure, except for a few select positions. As the tactical view turns negative, the portfolio shifts towards greater allocations to active fixed income funds, particularly concentrating in Bond Global Aggregate Active and Bond Euro Investment Grade funds. This is because these two funds have the least volatility. This reallocation occurs with minimal changes to equity exposures, both active and passive, highlighting the preference for reallocating within fixed income rather than adjusting equity positions in response to sovereign bond views.

Table 8: TAA portfolios with either tactical or thematic views.

	Optimal Tactical Portfolios with views														
	Equity US view ($S_{directional}$)					Bond EUR Sovereign view ($S_{directional}$)					Disruptive Tech Thematic view ($S_{thematic}$)				
S	-100%	-50%	0%	50%	100%	-100%	-50%	0%	50%	100%	-100%	-50%	0%	50%	100%
Equity Europe Active	14%	14%	14%	13%	12%	14%	14%	14%	12%	10%	12%	13%	14%	13%	13%
Equity USA Active	-	3.7%	5.5%	3.7%	3.4%	4.0%	4.2%	5.5%	2.7%	1.5%	2.0%	2.9%	5.5%	3.2%	-
Equity Japan Active	3.6%	3.6%	4.6%	3.0%	2.6%	3.6%	3.7%	4.6%	2.2%	1.0%	1.7%	2.5%	4.6%	3.0%	2.6%
Equity Emerging Active	1.5%	1.4%	1.7%	1.1%	0.9%	1.6%	1.6%	1.7%	0.7%	0.1%	0.7%	0.9%	1.7%	1.2%	1.0%
Equity World Active	0.3%	2.0%	2.7%	1.7%	1.6%	2.5%	2.1%	2.7%	1.3%	1.7%	0.7%	0.9%	2.7%	9.5%	12.3%
Bond EUR Sovereign Active	1.7%	3.7%	7.4%	3.8%	1.6%	-	-	7.4%	1.1%	-	3.7%	4.6%	7.4%	5.6%	5.3%
Bond EUR Investment Grade Active	-	-	3.1%	-	-	11.7%	7.2%	3.1%	-	-	-	-	3.1%	-	-
Bond EUR Aggregate Active	18.1%	13.1%	5.0%	-	-	-	-	5.0%	-	-	1.7%	3.2%	5.0%	6.7%	6.4%
Bond EUR High Yield Active	1.0%	1.0%	0.1%	0.6%	0.6%	2.1%	1.4%	0.1%	0.4%	-	1.2%	1.0%	0.1%	0.8%	0.9%
Bond USA Investment Grade Active	3.8%	3.8%	5.8%	4.9%	4.6%	3.2%	3.6%	5.8%	-	-	3.9%	4.1%	5.8%	4.2%	4.1%
Bond USA High Yield Active	4.1%	4.0%	3.2%	3.1%	2.8%	5.6%	4.7%	3.2%	2.3%	1.0%	3.8%	3.6%	3.2%	3.5%	3.6%
Bond Global Aggregate Active	25.0%	22.4%	15.5%	19.4%	15.4%	16.4%	23.2%	15.5%	8.6%	1.2%	23.8%	22.9%	15.5%	22.5%	23.7%
Bond Emerging HC Active	-	-	2.7%	4.0%	5.4%	-	-	2.7%	5.9%	6.5%	0.6%	1.1%	2.7%	0.9%	0.2%
Commodities Active	3.5%	3.5%	3.9%	3.4%	3.3%	3.4%	3.4%	3.9%	3.2%	2.9%	2.9%	3.1%	3.9%	3.3%	3.2%
Real Estate Europe Active	3.3%	3.6%	3.8%	3.9%	4.1%	2.7%	3.0%	3.8%	4.3%	4.7%	3.5%	3.6%	3.8%	3.6%	3.5%
Equity Europe Passive	2.7%	2.9%	1.9%	2.2%	2.3%	2.9%	2.8%	1.9%	2.4%	2.5%	3.4%	3.0%	1.9%	2.5%	2.6%
Equity USA Passive	-	-	4.5%	13.1%	18.5%	7.8%	7.6%	4.5%	5.8%	5.2%	8.8%	8.0%	4.5%	-	-
Equity Japan Passive	3.4%	3.4%	2.2%	3.7%	4.1%	3.1%	3.1%	2.2%	4.6%	5.6%	5.1%	4.4%	2.2%	4.0%	4.5%
Equity Emerging Passive	4.4%	4.6%	4.3%	4.8%	5.0%	4.4%	4.4%	4.3%	5.2%	5.6%	5.3%	5.1%	4.3%	3.9%	3.4%
Equity World Passive	-	-	0.9%	2.1%	3.3%	-	-	0.9%	3.0%	4.3%	1.8%	1.5%	0.9%	-	-
Bond EUR Sovereign Passive	-	-	1.8%	4.5%	5.7%	-	-	1.8%	27.3%	38.4%	3.9%	2.5%	1.8%	0.2%	0.5%
Bond EUR Investment Grade Passive	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
Bond USA Investment Grade Passive	-	-	-	-	0.4%	-	-	-	5.2%	5.8%	-	-	-	-	-
Bond Emerging HC Passive	6.3%	6.2%	3.1%	1.6%	-	5.9%	6.0%	3.1%	-	-	5.8%	5.1%	3.1%	5.3%	6.1%
Commodities Passive	3.3%	2.8%	2.3%	2.4%	2.1%	4.4%	3.7%	2.3%	1.7%	1.0%	3.1%	2.8%	2.3%	2.8%	2.9%
Real Estate Europe Passive	0.2%	0.2%	0.1%	0.3%	0.3%	0.3%	0.2%	0.1%	0.4%	0.4%	0.5%	0.4%	0.1%	0.3%	0.3%

Note: For illustration purpose only. Not investment advice.

Source: Based on the results in Table A6 aggregated by asset class, fund approach, coverage areas, and for bonds, by investment styles. Authors' calculations on 1st September 2025.

The thematic view on Disruptive Tech primarily results in a reallocation from US passive equities, and to a lesser extent from US active equities, towards Equity World Active. This shift reflects the inclusion of the Equity World Disruptive Tech Active Fundamental Art 8 fund within the Equity World Active category, allowing the portfolio to express the thematic view while maintaining diversification. The adjustment highlights how thematic convictions can be implemented through broader active global equity exposures when dedicated passive instruments are limited or unavailable.

Table 9 presents the portfolio regrouped by asset class and style, allowing for a clearer interpretation of thematic impacts. A positive view on Disruptive Tech leads to a noticeable increase in allocation to this theme, primarily funded by reductions across other equity styles. In contrast, implementing a negative view proves more challenging, as the baseline allocation to Disruptive Tech in the portfolio without thematic conviction is already minimal and quickly drops to zero. This explains the absence of positive excess returns in Table 7 for portfolios reflecting a negative view on Disruptive Tech.

Finally, we examine the portfolios through their beta exposures to core asset classes, focusing on those targeted by tactical views, namely Equity USA and Bond EUR Sovereign. Table 10 presents the results, and despite the significant changes in fund allocations shown in Table A6, the actual exposures to core assets behave as expected. Most exposures remain closely aligned with those in the SAA portfolio, while exposures to the core assets influenced by tactical views adjust accordingly. This confirms that the optimization process effectively translates tactical signals into meaningful shifts in asset class exposure without disrupting the overall portfolio structure.

Table 9: TAA portfolios with thematic views on Disruptive Tech.

	Optimal Tactical Portfolios with views				
	Disruptive Tech Thematic view ($S_{thematic}$)				
$S_{thematic}$	-100%	-50%	0%	50%	100%
Equity Mid-large	35%	34%	32%	24%	22%
Equity Dividend	5.6%	5.4%	5.1%	5.4%	5.5%
Equity Value	0.4%	0.6%	1.2%	0.7%	-
Equity Growth	0.4%	0.6%	1.0%	0.5%	-
Equity Water	0.2%	0.2%	0.6%	0.1%	-
Equity Disruptive Tech	-	-	0.9%	8.5%	12.3%
Equity Low Risk	0.5%	0.7%	1.2%	0.8%	-
Bonds Sovereign	7.6%	7.1%	9.2%	5.8%	5.8%
Bonds Investment Grade	3.9%	4.1%	8.9%	4.2%	4.1%
Bonds High Yield	4.9%	4.7%	3.3%	4.4%	4.5%
Bonds Aggregate	10.5%	12.0%	12.6%	15.9%	16.1%
Bonds HC	6.3%	6.2%	5.8%	6.1%	6.3%
Bonds Green	15.0%	14.1%	7.9%	13.3%	14.0%
Commodities	6.0%	6.0%	6.1%	6.1%	6.1%
Real Estate	4.0%	4.0%	3.9%	3.9%	3.8%

Note: For illustration purpose only. Not investment advice.

Source: Based on the results in Table A6, aggregated by asset class and investment styles. Authors' calculations on 1st September 2025.

For Equity USA, a strongly negative directional view leads to a complete removal of exposure to this core asset class, accompanied by an increased allocation to Bond EUR Sovereign. Conversely, under the most aggressive positive view on Equity USA, the portfolio significantly overweights its exposure relative to the SAA, with a corresponding reduction in Bond EUR Sovereign exposure. These shifts are also reflected in changes to USD currency exposure, which adjusts in line with the evolving allocation to US equities.

Table 10: Betas of TAA portfolios relative to core assets in portfolios with either tactical or thematic views

	Beta exposures of to Core Assets															
	SAA	Equity US view ($S_{directional}$)					Bond EUR Sovereign view ($S_{directional}$)					Disruptive Tech Thematic view ($S_{thematic}$)				
S	Portfolio	-100%	-50%	0%	50%	100%	-100%	-50%	0%	50%	100%	-100%	-50%	0%	50%	100%
Equity Europe EMU	0.08	0.08	0.08	0.08	0.07	0.07	0.08	0.08	0.08	0.07	0.07	0.08	0.08	0.08	0.07	0.07
Equity Europe EMU SC	0.01	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.03
Equity Europe UK	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.05	0.05	0.06	0.06	0.06	0.06	0.05
Equity North America USA	0.12	-	0.04	0.12	0.19	0.25	0.12	0.12	0.12	0.11	0.11	0.12	0.12	0.12	0.12	0.12
Equity North America USA SC	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
Equity Pacific Japan	0.07	0.07	0.07	0.07	0.07	0.07	0.07	0.07	0.07	0.07	0.07	0.07	0.07	0.07	0.07	0.07
Equity Emerging Global	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06
Bond EUR Sovereign	0.13	0.20	0.18	0.16	0.13	0.11	0.04	0.06	0.16	0.30	0.38	0.15	0.15	0.16	0.16	0.16
Bond EUR Investment Grade	0.05	0.13	0.11	0.09	0.07	0.06	0.14	0.14	0.09	0.03	0.01	0.09	0.09	0.09	0.09	0.10
Bond EUR High Yield	0.01	0.01	0.01	-	0.01	0.01	0.02	0.01	-	-	-	0.01	0.01	-	0.01	0.01
Bond USD Sovereign	0.13	0.09	0.08	0.06	0.06	0.05	0.07	0.08	0.06	0.03	-	0.08	0.08	0.06	0.08	0.08
Bond USD Investment Grade	0.05	0.06	0.05	0.07	0.06	0.06	0.05	0.05	0.07	0.06	0.06	0.06	0.06	0.07	0.06	0.06
Bond USD High Yield	0.04	0.04	0.04	0.03	0.03	0.03	0.05	0.04	0.03	0.02	0.01	0.04	0.03	0.03	0.03	0.03
Bond EMD HC Sov Global	0.07	0.07	0.07	0.07	0.07	0.07	0.06	0.07	0.07	0.08	0.09	0.07	0.07	0.07	0.07	0.07
Bond EMD LC Sov Global	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
Diversification Real Estate Pan-Europe	0.04	0.03	0.04	0.04	0.04	0.04	0.03	0.03	0.04	0.05	0.05	0.04	0.04	0.04	0.04	0.04
Diversification Commodity Global	0.05	0.06	0.05	0.05	0.05	0.04	0.06	0.06	0.05	0.04	0.03	0.05	0.05	0.05	0.05	0.05
Currency USD	0.18	0.06	0.11	0.18	0.25	0.31	0.19	0.18	0.18	0.18	0.17	0.18	0.18	0.18	0.18	0.18
Currency GBP	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.05	0.05	0.06	0.06	0.06	0.06	0.05
Currency JPY	0.07	0.07	0.07	0.07	0.07	0.07	0.07	0.07	0.07	0.07	0.07	0.07	0.07	0.07	0.07	0.07

Note: For illustration purpose only.

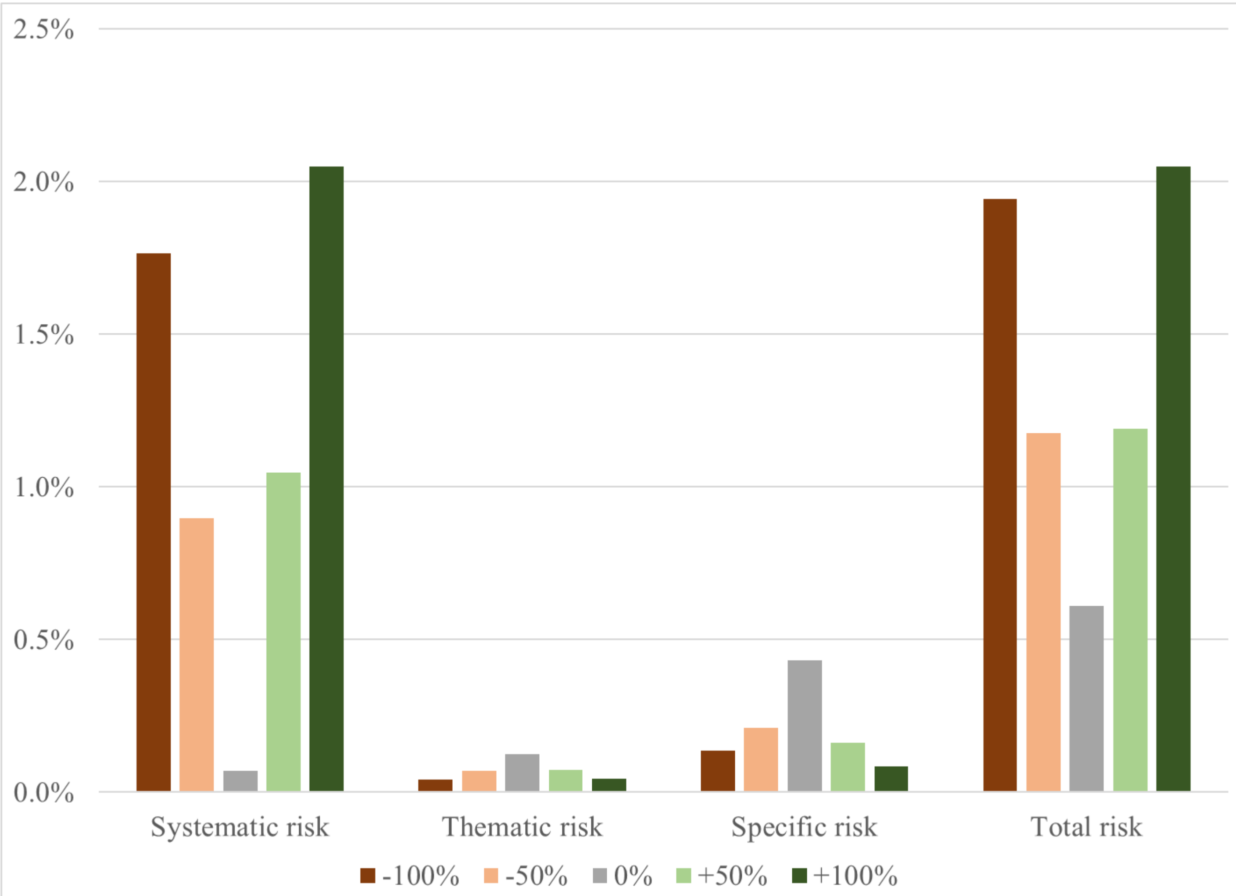
Source: BNP Paribas Asset Management, MSCI, Russell, Bloomberg, J.P. Morgan, FTSE EPRA. Authors' calculations on 1st September 2025.

For tactical views on Bond EUR Sovereign, exposure to this core asset class decreases as the view turns negative, with increased allocations to Bond EUR Investment Grade serving as a compensating adjustment. Under a strongly positive view, exposure to Bond EUR Sovereign rises significantly, primarily at the expense of Bond EUR Investment Grade, and to a lesser extent, Bond USD Sovereign and Bond USD High Yield.

For the thematic view on Disruptive Tech, all excess returns are generated with minimal impact on exposures to core asset classes, which remain closely aligned with those in the SAA portfolio. This stability is reassuring, as it confirms that thematic tilts can be implemented without compromising the portfolio's overall strategic balance.

In Appendix 5.4.4, we present a simplified example that explains many of the results observed in this section. We focus on a portfolio invested in a single asset class only, equities, on which there is a tactical view. The portfolio can invest in a passive equity fund and in an active equity fund offering positive net active alpha. We show that, as the tactical view on equities becomes more positive, the overall allocation to the passive equity fund increases for as long as the active fund exhibits a higher tracking error relative to the equity core index than the passive fund. Conversely, when views turn negative, the portfolio has a preference to reduce the exposure to the passive fund. To simplify, we ignored the uncertainty term in the robust optimization, assumed that both funds have broadly similar beta to the core asset index and that the no leverage and no short constraints were fulfilled. We also provide a more a general example: that the tracking error of the TAA portfolio relative to the SAA portfolio increases as views become more extreme, whether positive or negative. The tracking error behaves in a monotonic fashion, reaching a minimum when the view on equities is neutral. By deriving the analytical expression for this tracking error, we demonstrate in Appendix 5.4.5 that it is a positive function of the positive expected return implied by the tactical view, regardless of the tactical direction of the view.

Figure 3: Contributions of systematic, thematic, and specific risk to the tracking error of the TAA portfolios as a function of the signal $S_{directional}$ for the US Equity tactical view.



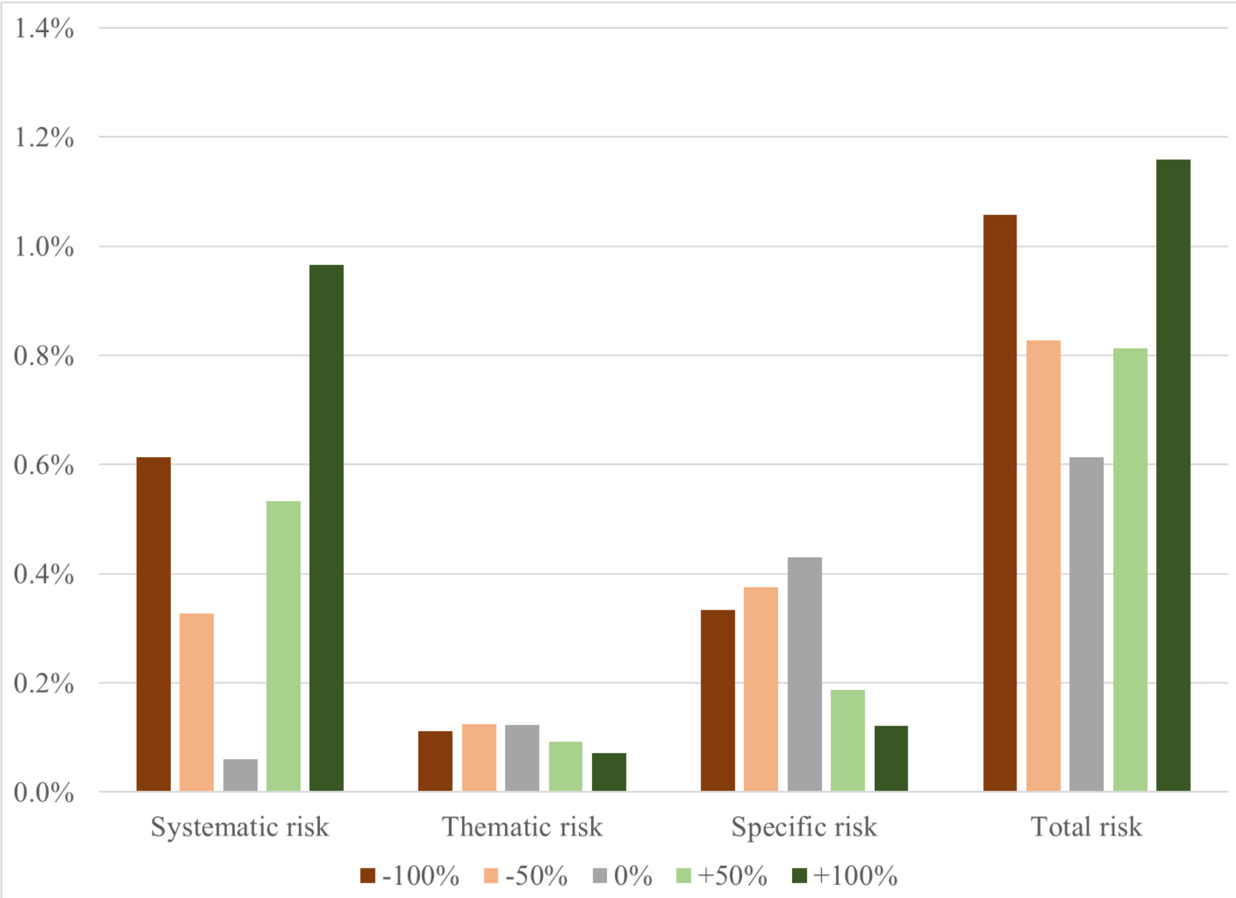
Note: For illustration purpose only.

Source: BNP Paribas Asset Management, MSCI, Russell, Bloomberg, J.P. Morgan, FTSE EPRA. Authors' calculations on 1st September 2025.

Figures 3, 4, and 5 show the contributions of systematic, thematic, and specific risk to the tracking error of the portfolios discussed in this section, reflecting tactical directional views on US Equity, EUR Sovereign Bonds, and a thematic view on Disruptive Tech, respectively.

Figures 3 and 4 show a clear increase in portfolio tracking error as tactical views on US Equity or EUR Sovereign Bonds become more positive or negative. This increase is driven primarily by a higher contribution from systematic risk. When the tactical view is neutral on these asset classes, systematic risk becomes negligible, and tracking error is instead dominated by specific risk. In both cases, thematic risk remains consistently small. These results align with expectations for portfolios incorporating tactical views and are supported by the analytical discussions in Appendices 5.4.4 and 5.4.5.

Figure 4: Contributions of systematic, thematic, and specific risk to the tracking error of the TAA portfolios as a function of the signal $S_{directional}$ for the Bond EUR Sovereign tactical view.

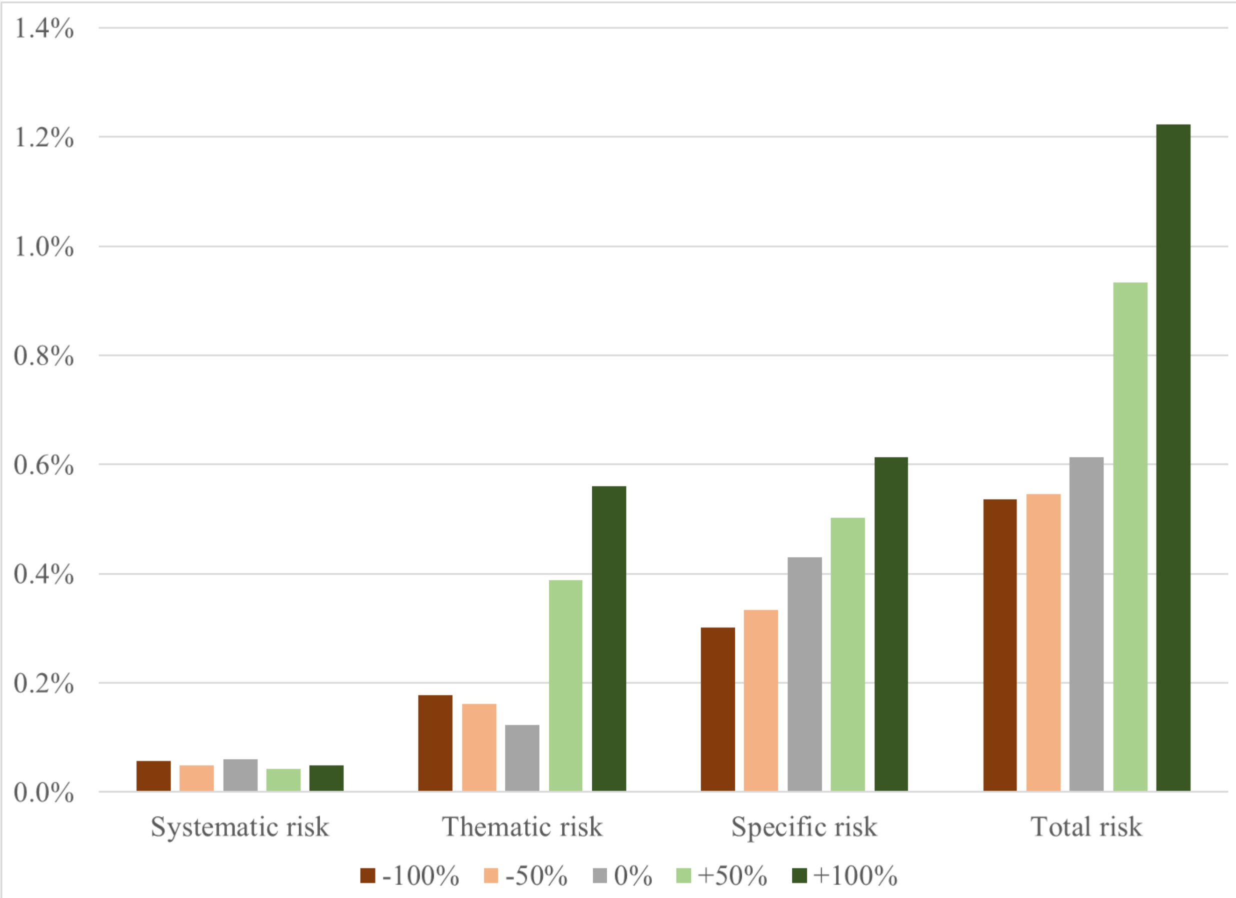


Note: For illustration purpose only.

Source: BNP Paribas Asset Management, MSCI, Russell, Bloomberg, J.P. Morgan, FTSE EPRA. Authors' calculations on 1st September 2025.

Figure 5 illustrates the impact of changing a thematic view on Disruptive Tech on the contributions of systematic, thematic, and specific risk to the tracking error of the TAA portfolios. As expected, changes in this view have almost no effect on systematic risk, which remains negligible throughout. When the view becomes more positive, the tracking error rises due to higher contributions from both thematic and specific risk. The increase in specific risk occurs because the only funds in the selected universe with exposure to the Disruptive Tech theme are active funds, which carry specific risk. Conversely, when the view turns negative, tracking error changes only marginally as thematic risk increases only slightly, while specific risk declines. This limited impact is due to the long-only constraint: portfolios can remove funds exposed to Disruptive Tech but cannot short sell them, restricting the ability to fully capitalize on this negative thematic view.

Figure 5: Contributions of systematic, thematic, and specific risk to the tracking error of the TAA portfolios as a function of the $S_{thematic}$ for the Disruptive Tech thematic view.



Note: For illustration purpose only.

Source: BNP Paribas Asset Management, MSCI, Russell, Bloomberg, J.P. Morgan, FTSE EPRA. Authors' calculations on 1st September 2025.

4. CONCLUSIONS

This paper introduces a robust optimization framework for constructing multi-asset portfolios that allocate to both active and passive funds, addressing the practical challenges faced by asset managers, asset owners, distributors and robo-advisors. By explicitly accounting for tactical and thematic investment views, expected alpha from active funds, ongoing fund charges, tracking error constraints, and the fact that investment views can be expressed using a set of benchmark indices that may differ from those used in the SAA portfolio and for benchmarking passive and active funds, the framework enables the creation of TAA portfolios that reflect – in a transparent and scalable way – the different sets of investment views.

The framework provides the means to trade off net active alpha from active funds against the expected return from tactical views. This will result in different allocations to passive and active funds as a function of the appetite for active alpha. As confidence in active fund alpha increases, the portfolio tilts towards funds with higher expected net alpha, while strong tactical views are primarily implemented through increased allocations to passive funds of the underlying asset class. The analytical examples in appendices and empirical results confirm that tracking error rises monotonically as tactical views become more extreme, irrespectively of direction of view, with the lowest tracking error occurring at neutral views.

Overall, the proposed framework provides a practical and robust solution for implementing multi-asset portfolios that reflect investment views with real-world constraints. It offers asset managers, institutional investors and robo-advisors a transparent methodology for balancing strategic exposures, active alpha, and tactical convictions, paving the way for scalable and customizable portfolio solutions in an increasingly digitalized investment landscape.

5. APPENDIX

5.1 Exposures of benchmarks and funds to the core assets

The LASSO regressions (Tibshiran (1996)) used to decide which core assets should be retained in the regressions (5) of benchmarks against core assets are based on the following equation:

$$\widehat{\boldsymbol{\beta}}_{LASSO} = \min_{\boldsymbol{\beta}}(\boldsymbol{XR}_b^i\,(t) - \left(\boldsymbol{\beta}_{bc}^i\right)^{\mathrm{T}}\boldsymbol{XR}_c(t))^{\mathrm{T}}(\boldsymbol{XR}_b^i\,(t) + {\boldsymbol{\beta}_{bc}^i}^{\mathrm{T}}\boldsymbol{XR}_c(t)) \;+\lambda\left\|\boldsymbol{\beta}_{bc}^i\right\|_{\ell_1} \quad \text{(A1)}$$

which minimizes the residual sum of squares with an ℓ_1 penalty on the coefficients, and the regularization parameter λ tuned via cross-validation over a grid of 100 values spanning from λ_{max}, the smallest value that zeroes all coefficients, to $\lambda_{min}= \lambda_{max}/10^5$. The final λ is selected using the one-standard-error rule to balance sparsity and predictive performance, as proposed in Hastie et al. (2015).

In Table A1, we show the assets in the SAA portfolio with their corresponding weights and benchmark indices. The thematic volatility and betas are obtained from the regression in equation (6) after selecting the core assets for the regression using the LASSO approach described above. In Table A2, we show results equivalent to those in Table A1 but now for the benchmarks of funds with characteristics in Table A3.

Table A1: SAA Indices: SAA weights, thematic volatilities and beta against Core Assets

							Beta of SAA assets to Core Assets																			
SAA Assets	Tickers	Index provider	Currency hedging into EUR	SAA portfolio allocation	Thematic Volatility of Benchmark	R square of regression SAA indices relative Core Assets	Equity Europe EMU	Equity Europe EMU SC	Equity Europe UK	Equity USA	Equity USA SC	Equity Japan	Equity Emerging Global	Bond EUR Sovereign	Bond EUR Investment Grade	Bond EUR High Yield	Bond USD Sovereign	Bond USD Investment Grade	Bond USD High Yield	Bond EMD HC Sov Global	Bond EMD LC Sov Global	Diversification Real Estate Pan-Europe	Diversification Commodity Global	Currency USD	Currency GBP	Currency JPY
Equity Europe	M7EU Index	1	No	16%	1.8%	0.99	0.49	0.09	0.37																0.37	
Equity USA	NDDUUS Index	1	No	12%	0.0%	1.00				1.00														1.00		
Equity Japan	NDDLJN Index	1	No	7%	0.0%	1.00						1.00														1.00
Equity Emerging Global	NDUEEGF Index	1	No	6%	0.0%	1.00							1.00											1.00		
Bond EUR Aggregate	LBEATREU Index	2	Yes	20%	0.3%	1.00								0.65	0.27		0.03									
Bond USD Aggregate	LBUSTRUU Index	2	Yes	20%	0.9%	0.96											0.61	0.24		0.06						
Bond High Yield Global	LG30TREH Index	2	-	6%	1.0%	0.98										0.21			0.60	0.24						
Bond EMD HC Sov Global	JPGCCOMP Index	3	Yes	4%	0.0%	1.00														1.00						
Diversification Real Estate Pan-Europe	TRNHUE Index	4	Yes	4%	0.0%	1.00																1.00				
Diversification Commodity Global	BCOMXALT Index	2	Yes	5%	0.0%	1.00																	1.00			

Note: Regression estimation from end of April 2005 through end of April 2025 using weekly EUR returns and returns hedged into EUR for international fixed income and commodities. For illustration purpose only.

Source: 1) MSCI, 2) Bloomberg, 3) J.P. Morgan, 4) FTSE EPRA. Authors' calculations on 1st September 2025.

Table A2: Fund Benchmarks: Thematic volatilities and beta against Core Assets.

						Beta of Benchmarks to Core Assets																			
Benchmarks	Tickers	Index provider	Currency hedging into EUR	Thematic Volatility of Benchmark	R square of regression Benchmark relative to Core Assets	Equity Europe EMU	Equity Europe EMU SC	Equity Europe UK	Equity USA	Equity USA SC	Equity Japan	Equity Emerging Global	Bond EUR Sovereign	Bond EUR Investment Grade	Bond EUR High Yield	Bond USD Sovereign	Bond USD Investment Grade	Bond USD High Yield	Bond EMD HC Sov Global	Bond EMD LC Sov Global	Diversification Real Estate Pan-Europe	Diversification Commodity Global	Currency USD	Currency GBP	Currency JPY
Equity Europe	M7EU Index	1	No	1.8%	0.99	0.49	0.09	0.37																0.37	
Equity Europe ESG	MXEUEFMT Index	1	No	2.0%	0.99	0.49	0.10	0.36																0.36	
Equity USA S&P500	SPTR500N Index	2	No	0.6%	1.00				0.99														0.99		
Equity USA Value	M1US000V Index	1	No	5.1%	0.90				0.78	0.13													0.91		
Equity USA Growth	RLG Index	3	No	4.5%	0.94				1.05														1.05		
Equity Japan	NDDLJN Index	1	No	0.0%	1.00						1.00														1.00
Equity Japan ESG	MXJPMTNY Index	1	No	0.5%	1.00						1.00														1.00
Equity Emerging Global	NDUEEGF Index	1	No	0.0%	1.00							1.00											1.00		
Equity Emerging Global ESG	MXEMEFMT Index	1	No	0.6%	1.00							1.00											1.00		
Equity Global Developed ESG	MXWOEMNU Index	1	No	1.6%	0.99	0.11	0.04	0.10	0.58	0.01	0.11	0.05											0.64	0.10	0.11
Equity Global All Countries Water	EOWR Index	4	No	6.3%	0.84	0.01	0.12	0.17	0.26	0.20	0.12	0.06											0.51	0.17	0.12
Equity Global All Countries Disruptive Tech	MXACIDSR Index	1	No	5.5%	0.90	0.00	0.08		0.88			0.11											0.99		
Equity Global Developed Low Risk	M7CXLAA Index	1	No	5.4%	0.80			0.07	0.49		0.15												0.49	0.07	0.15
Bond EUR Sovereign	LBEATREU Index	5	-	0.0%	1.00								1.00												
Bond EUR Sovereign ESG	GBIEIGTR Index	6	-	0.2%	1.00								0.99												
Bond EUR Investment Grade	LECPTREU Index	5	-	0.0%	1.00									1.00											
Bond EUR Investment Grade SRI	I34235EU Index	5	-	0.3%	0.99								0.01	1.04			0.01								
Bond EUR High Yield	LF88TREU Index	5	-	0.0%	1.00										1.00										
Bond EUR Aggregate	LEATTREU Index	5	-	0.3%	1.00								0.65	0.27		0.03									
Bond USD Investment Grade	LUACTRUU Index	5	Yes	0.0%	1.00												1.00								
Bond USD Investment Grade SRI	I37668US Index	5	Yes	0.1%	1.00												1.01								
Bond USD High Yield	LF89TRUU Index	5	Yes	0.0%	1.00													1.00							
Bond EMD HC Sov Global	JPGCCOMP Index	6	Yes	0.0%	1.00														1.00						
Bond Global Aggregate	LEGATREH Index	5	Yes	0.6%	0.97								0.20	0.13		0.37	0.08		0.03						
Bond Global Aggregate Green	GBGLTREH Index	5	Yes	0.9%	0.96								0.30	0.49		0.28	0.06								
Diversification Commodity Energy & Metals	BNPIC52T Index	7	Yes	4.7%	0.92																	0.79			
Diversification Commodity Global	BCOMXALT Index	5	Yes	0.0%	1.00																	1.00			
Diversification Real Estate EMU	NR0EUE Index	8	-	6.8%	0.91																0.96				
Diversification Real Estate Pan-Europe	TRNHUE Index	8	Yes	0.0%	1.00																1.00				

Note: Regression estimation from end of April 2005 through end of April 2025 using weekly EUR returns and returns hedged into EUR for international fixed income and commodities. *Note:* For illustration purpose only.

Source: 1) MSCI, 2) S&P, 3) Russell, 4) FTSE, 5) Bloomberg, 6) J.P. Morgan, 7) BNP Paribas, 8) FTSE EPRA. Authors' calculations on 1st September 2025.

Table A3: Characteristics of the funds selected for portfolio construction.

Fund Asset Class	Fund Coverage	Fund Style	Fund Approch	Fund Philosophy	SFDR classification Article	Currency Hedging into EUR	Fund Benchmark	Sustainable Investment	Inception Date
Equity	Europe	Mid-large	Active	Fundamental	8	No	Equity Europe	45%	21-Jun-04
Equity	Europe	Mid-large	Active	Quant	8	No	Equity Europe	50%	13-Jul-16
Equity	Europe	Mid-large	Passive	Index	8	No	Equity Europe ESG	40%	12-Jul-16
Equity	Europe	Dividend	Active	Fundamental	8	No	Equity Europe	50%	29-Sep-03
Equity	USA	Mid-large	Active	Fundamental	8	No	Equity USA S&P500	37%	3-Feb-17
Equity	USA	Mid-large	Passive	Index	6	No	Equity USA S&P500	0%	10-Jun-08
Equity	USA	Value	Active	Quant	8	No	Equity USA Value	40%	15-Sep-17
Equity	USA	Growth	Active	Fundamental	8	No	Equity USA Growth	25%	3-Jan-95
Equity	Japan	Mid-large	Active	Fundamental	8	No	Equity Japan	30%	31-Dec-90
Equity	Japan	Mid-large	Active	Quant	8	No	Equity Japan	50%	21-Nov-16
Equity	Japan	Mid-large	Passive	Index	8	No	Equity Japan ESG	40%	2-Aug-23
Equity	Emerging	Mid-large	Active	Fundamental	8	No	Equity Emerging Global	20%	20-Oct-97
Equity	Emerging	Mid-large	Passive	Index	8	No	Equity Emerging Global ESG	20%	3-Sep-12
Equity	World	Water	Active	Fundamental	9	No	Equity Global All Countries Water	85%	29-Sep-17
Equity	World	Disruptive Tech	Active	Fundamental	8	No	Equity Global All Countries Disruptive Tech	30%	17-May-13
Equity	World	Low Risk	Active	Quant	8	No	Equity Global Developed Low Risk	50%	7-Dec-12
Equity	World	Mid-large	Passive	Index	8	No	Equity Global Developed ESG	30%	26-Oct-22
Bonds	EUR	Sovereign	Active	Fundamental	8	-	Bond EUR Sovereign	20%	27-Jun-01
Bonds	EUR	Sovereign	Passive	Index	8	-	Bond EUR Sovereign ESG	0%	31-May-17
Bonds	EUR	Investment Grade	Active	Fundamental	8	-	Bond EUR Investment Grade	40%	25-Jul-01
Bonds	EUR	Investment Grade	Active	Quant	8	-	Bond EUR Investment Grade	50%	24-Jan-18
Bonds	EUR	Investment Grade	Passive	Index	8	-	Bond EUR Investment Grade SRI	30%	15-Jan-19
Bonds	EUR	High Yield	Active	Fundamental	8	-	Bond EUR High Yield	20%	8-Dec-03
Bonds	EUR	Aggregate	Active	Fundamental	8	-	Bond EUR Aggregate	20%	4-Apr-00
Bonds	USD	Investment Grade	Active	Quant	8	Yes	Bond USD Investment Grade	35%	28-Jun-19
Bonds	USD	Investment Grade	Passive	Index	8	Yes	Bond USD Investment Grade SRI	25%	12-Sep-23
Bonds	USD	High Yield	Active	Fundamental	8	Yes	Bond USD High Yield	10%	2-Apr-01
Bonds	Emerging	HC	Active	Fundamental	8	Yes	Bond EMD HC Sov Global	1%	30-Apr-99
Bonds	Emerging	HC	Passive	Index	8	Yes	Bond EMD HC Sov Global	0%	16-May-11
Bonds	Global	Aggregate	Active	Fundamental	8	Yes	Bond Global Aggregate	20%	4-Aug-06
Bonds	Global	Green	Active	Fundamental	9	Yes	Bond Global Aggregate Green	80%	7-Sep-17
Commodities	Global	Commodities	Passive	Index	6	Yes	Diversification Commodity Energy & Metals	0%	25-Apr-16
Commodities	Global	Commodities	Active	Fundamental	6	Yes	Diversification Commodity Global	0%	15-Nov-19
Real Estate	Eurozone	Real Estate	Passive	Index	6	Yes	Diversification Real Estate EMU	0%	10-Jul-13
Real Estate	Europe	Real Estate	Active	Fundamental	8	Yes	Diversification Real Estate Pan-Europe	50%	28-May-15

Notes: The European Regulation 2019/2088 on Sustainable Finance Disclosure Regulation (SFDR) in the financial services sector (SFDR) classifies funds into three main categories based on their sustainability characteristics and objectives: "Article 9" investments, which have a sustainable investment objective, "Article 8" investments, which declare that social and/or environmental criteria are taken into account, "Article 6" investments, which do not have a sustainable investment objective and do not declare that they take ESG criteria into account, which are essentially all other investments that are neither "Article 8" nor "Article 9". For illustration purposes only.

Source: BNP Paribas Asset Management, MSCI, Russell, Bloomberg, J.P. Morgan, FTSE EPRA. Authors' calculations on 1st September 2025.

In Table A4, we show the betas of the funds relative to the core assets calculated from equation (7). These are based on the betas of the benchmarks to core assets in Table A2 and the betas of funds relative to their own benchmark using equation (3). The results from this last regression are also in Table A4, which also includes the OCR of all funds and our expected information ratios for each fund, and the final alpha obtained from equation (5).

Table A4: Funds: Betas against benchmarks, specific volatility against benchmarks, expected information ratios before costs, OCR, expected alpha and beta against core assets.

							Beta of Funds to Core Assets																				
Fund Designation	Beta of fund relative to its benchmark	Specific Volatility of fund relative to exposu to its benchmark	R square of regression Fund relative to its Benchmark	Expected information ratio before costs	OCR	Expected alpha	Equity Europe EMU	Equity Europe EMU SC	Equity Europe UK	Equity USA	Equity USA SC	Equity Japan	Equity Emerging Global	Bond EUR Sovereign	Bond EUR Investment Grade	Bond EUR High Yield	Bond USD Sovereign	Bond USD Investment Grade	Bond USD High Yield	Bond EMD HC Sov Global	Bond EMD LC Sov Global	Diversification Real Estate Pan-Europe	Diversification Commodity Global	Currency USD	Currency GBP	Currency JPY
Equity Europe Mid-large Active Fundamental Art 8	1.01	3.5%	0.95	0.5	0.96%	0.8%	0.50	0.09	0.37																0.37	
Equity Europe Mid-large Active Quant Art 8	0.93	3.4%	0.94	0.5	0.61%	1.1%	0.46	0.09	0.34																0.34	
Equity Europe Mid-large Passive Index Art 8	1.00	0.5%	1.00	0.0	0.15%	-0.2%	0.49	0.10	0.36																0.36	
Equity Europe Dividend Active Fundamental Art 8	0.96	2.4%	0.97	0.5	0.96%	0.2%	0.47	0.09	0.35																0.35	
Equity USA Mid-large Active Quant Art 8	0.93	4.1%	0.93	0.5	0.61%	1.5%				0.92														0.92		
Equity USA Mid-large Passive Index Art 6	1.00	0.1%	1.00	0.0	0.13%	-0.1%				0.99														0.99		
Equity USA Value Active Quant Art 8	1.02	4.1%	0.92	0.5	0.61%	1.5%				0.79	0.13													0.92		
Equity USA Growth Active Fundamental Art 8	0.99	3.6%	0.97	0.5	0.96%	0.8%				1.04														1.04		
Equity Japan Mid-large Active Fundamental Art 8	0.91	6.3%	0.83	0.5	0.98%	2.1%						0.91														0.91
Equity Japan Mid-large Active Quant Art 8	0.93	5.8%	0.86	0.5	0.61%	2.3%						0.93														0.93
Equity Japan Mid-large Passive Index Art 8	1.00	0.1%	1.00	0.0	0.16%	-0.2%						1.00														1.00
Equity Emerging Mid-large Active Fundamental Art 8	0.86	8.0%	0.76	0.5	1.11%	2.9%							0.86											0.86		
Equity Emerging Mid-large Passive Index Art 8	1.00	0.3%	1.00	0.0	0.27%	-0.3%							1.00											1.00		
Equity World Water Active Fundamental Art 9	0.98	4.4%	0.92	0.5	1.11%	1.1%	0.01	0.12	0.16	0.26	0.19	0.12	0.05											0.50	0.16	0.12
Equity World Disruptive Tech Active Fundamental Art 8	1.14	6.1%	0.92	0.5	0.96%	2.1%		0.09		1.01			0.12											1.13		
Equity World Low Risk Active Quant Art 8	1.06	5.4%	0.81	0.5	0.97%	1.7%			0.08	0.52		0.16												0.52	0.08	0.16
Equity World Mid-large Passive Index Art 8	1.00	0.1%	1.00	0.0	0.16%	-0.2%	0.11	0.04	0.10	0.58	0.01	0.11	0.05											0.64	0.10	0.11
Bonds EUR Sovereign Active Fundamental Art 8	0.97	0.8%	0.98	0.5	0.43%	0.0%								0.97												
Bonds EUR Sovereign Passive Index Art 8	1.00	0.2%	1.00	0.0	0.15%	-0.2%								0.99												
Bonds EUR Investment Grade Active Fundamental Art 8	1.02	0.7%	0.97	0.5	0.49%	-0.1%									1.02											
Bonds EUR Investment Grade Active Quant Art 8	1.03	0.6%	0.98	0.5	0.31%	0.0%									1.03											
Bonds EUR Investment Grade Passive Index Art 8	1.00	0.1%	1.00	0.0	0.15%	-0.2%								0.01	1.04			0.01								
Bonds EUR High Yield Active Fundamental Art 8	1.02	1.1%	0.96	0.5	0.74%	-0.2%										1.02										
Bonds EUR Aggregate Active Fundamental Art 8	0.99	0.6%	0.99	0.5	0.49%	-0.2%								0.64	0.27		0.03									
Bonds USD Investment Grade Active Quant Art 8	0.98	0.6%	0.99	0.5	0.23%	0.1%												0.98								
Bonds USD Investment Grade Passive Index Art 8	1.01	0.1%	1.00	0.0	0.15%	-0.2%												1.01								
Bonds USD High Yield Active Fundamental Art 8	0.96	1.2%	0.96	0.5	0.74%	-0.1%													0.96							
Bonds Emerging HC Active Fundamental Art 8	1.31	2.8%	0.93	0.5	0.74%	0.7%														1.31						
Bonds Emerging HC Passive Index Art 8	0.99	0.8%	0.99	0.0	0.20%	-0.2%														0.99						
Bonds Global Aggregate Active Fundamental Art 8	1.14	2.5%	0.81	0.5	0.52%	0.7%								0.23	0.14		0.42	0.10		0.04						
Bonds Global Green Active Fundamental Art 9	0.96	0.7%	0.98	0.5	0.43%	-0.1%								0.29	0.47		0.27	0.06								
Commodities Global All Passive Index Art 6	1.00	0.1%	1.00	0.0	0.39%	-0.4%																	0.79			
Commodities Global All Active Fundamental Art 6	0.85	5.8%	0.90	0.5	0.96%	1.9%																	0.85			
Real Estate Eurozone Real Estate Passive Index Art 6	1.00	0.2%	1.00	0.0	0.40%	-0.4%																0.96				
Real Estate Europe Real Estate Active Fundamental Art 8	0.97	3.0%	0.98	0.5	0.96%	0.6%																0.97				

Note: Regression estimation from end of April 2020 through end of April 2025 using weekly EUR returns and returns hedged into EUR for international fixed income and commodities. For illustration purpose only. Past performance is not indicative of future performance. Not investment advice.

Source: BNP Paribas Asset Management, MSCI, Russell, Bloomberg, J.P. Morgan, FTSE EPRA. Authors' calculations on 1st September 2025.

5.2 PCA factors for the core assets

We use the weekly returns of the core assets in local currency and in excess of local cash (EUR, USD, GBP or JPY). The data spans 20 years from the end of April 2005 to the end of April 2025. The risk factors are derived from a PCA applied to reduce the dimensionality of the correlation matrix. The PCA factors represent portfolios allocated to the original assets. We retain the first six factors, which together explain 83% of the total risk. Finally, we reconstruct the correlation matrix using these PCA factors, ensuring that the diagonal elements are set to one.

When compared to the approaches in Issaoui et al. (2021) and Somefun et al. (2022), here we introduce currencies as currency exposure is essential for fund implementation since the portfolio valuation occurs in a specific currency and several of the selected funds face currency risk relative to that currency.

Table A5: Statistical risk factors from risk model

		Statistical factors						
% of variance explained	Total 83%	Factor 1 45%	Factor 2 17%	Factor 3 9%	Factor 4 5%	Factor 5 4%	Factor 6 3%	Index provider
Core Assets	Tickers	Core Asset weights in PCA orthonormal factors						
Equity Europe EMU	NDDLEURO Index	29.0%	-11.0%	-14.0%	-26.0%	-4.0%	-2.0%	1
Equity Europe EMU SC	NCLDEMU Index	30.0%	-8.0%	-12.0%	-17.0%	1.0%	2.0%	1
Equity Europe UK	NDDLUK Index	28.0%	-11.0%	-10.0%	-29.0%	-5.0%	20.0%	1
Equity USA	NDDUUS Index	29.0%	-8.0%	-6.0%	-15.0%	0.0%	10.0%	1
Equity USA SC	RU20INTR Index	28.0%	-9.0%	-7.0%	-16.0%	-2.0%	13.0%	2
Equity Japan	NDDLJN Index	23.0%	-12.0%	-12.0%	2.0%	12.0%	-47.0%	1
Equity Emerging Global	NDUEEGF Index	29.0%	-6.0%	9.0%	3.0%	-21.0%	-3.0%	1
Bond EUR Sovereign	LEATTREU Index	3.0%	43.0%	-5.0%	-28.0%	-14.0%	-21.0%	3
Bond EUR Investment Grade	LECPTREU Index	13.0%	43.0%	-9.0%	7.0%	15.0%	0.0%	3
Bond EUR High Yield	LF88TREU Index	26.0%	11.0%	-8.0%	33.0%	44.0%	12.0%	3
Bond USD Sovereign	LUATTRUU Index	-4.0%	46.0%	17.0%	-16.0%	-21.0%	-12.0%	3
Bond USD Investment Grade	LUACTRUU Index	12.0%	45.0%	10.0%	9.0%	10.0%	2.0%	3
Bond USD High Yield	LF89TRUU Index	27.0%	12.0%	-4.0%	29.0%	33.0%	23.0%	3
Bond EMD HC Sov Global	JPGCCOMP Index	26.0%	19.0%	12.0%	8.0%	1.0%	-4.0%	4
Bond EMD LC Sov Global	JGENVUUG Index	26.0%	5.0%	29.0%	0.0%	-24.0%	-4.0%	4
Diversification Real Estate Pan-Europe	TRNHUE Index	27.0%	3.0%	-16.0%	-15.0%	-17.0%	-7.0%	5
Diversification Commodity Global	BCOMXALT Index	17.0%	-9.0%	17.0%	45.0%	-51.0%	34.0%	3
Currency USD	USDEUR Curncy	-13.0%	3.0%	-60.0%	-4.0%	4.0%	14.0%	3
Currency GBP	GBPEUR Curncy	5.0%	5.0%	-50.0%	46.0%	-38.0%	-40.0%	3
Currency JPY	JPYEUR Curncy	-15.0%	27.0%	-30.0%	-11.0%	-23.0%	53.0%	3

Note: Risk model estimation from end of April 2005 through end of April 2025 using weekly local returns based on a PCA model. For illustration purposes only.

Source: 1) MSCI, 2) Russell, 3) Bloomberg, 4) J.P. Morgan, 5) FTSE EPRA. Authors' calculations on 1st September 2025.

5.3. TAA portfolio allocation to funds

In Table A6, we show the fund allocation for all the examples in the main text.

Table A6: TAA portfolio allocation to funds.

	RB/γ					γ = 15, *RB* = 2% (*RB* / γ = 0.13%)																			
						Minimum SI Constraint					Equity USA Tactical View					Bond EUR Sovereign Tactical View					Disruptive Tech Thematic View				
	2.00%	0.40%	0.13%	0.04%	0.00%	0%	30%	40%	50%	75%	-100%	-50%	0%	50%	100%	-100%	-50%	0%	50%	100%	-100%	-50%	0%	50%	100%
Portfolio Volatility	7.7%	7.4%	7.4%	7.4%	7.4%	7.4%	7.4%	7.4%	7.4%	6.9%	6.2%	6.7%	7.4%	8.1%	8.8%	7.7%	7.5%	7.4%	7.3%	7.2%	7.4%	7.4%	7.4%	7.5%	7.5%
Portfolio Tracking Error	1.8%	1.0%	0.6%	0.5%	0.5%	0.6%	0.6%	0.7%	0.8%	2.7%	1.9%	1.2%	0.6%	1.2%	2.1%	1.1%	0.8%	0.6%	0.8%	1.2%	0.5%	0.5%	0.6%	0.9%	1.2%
Equity Europe Mid-large Active Fundamental Art 8	-	5.3%	3.8%	3.0%	2.6%	3.8%	3.8%	3.7%	3.7%	-	3.1%	3.4%	3.8%	3.6%	3.7%	2.3%	2.8%	3.8%	4.0%	4.4%	3.1%	3.2%	3.8%	3.3%	3.2%
Equity Europe Mid-large Active Quant Art 8	11.7%	8.5%	5.1%	3.6%	2.9%	5.1%	5.1%	5.3%	5.6%	4.9%	5.0%	4.9%	5.1%	4.1%	3.7%	5.4%	5.2%	5.1%	3.3%	2.2%	3.7%	4.0%	5.1%	4.4%	4.2%
Equity Europe Mid-large Passive Index Art 8	-	-	1.9%	3.5%	4.1%	1.9%	1.9%	1.5%	1.3%	-	2.7%	2.9%	1.9%	2.2%	2.3%	2.9%	2.8%	1.9%	2.4%	2.5%	3.4%	3.0%	1.9%	2.5%	2.6%
Equity Europe Dividend Active Fundamental Art 8	-	2.0%	5.1%	5.6%	5.8%	5.1%	5.1%	5.4%	6.1%	10.7%	5.8%	5.7%	5.1%	5.0%	4.8%	6.5%	6.1%	5.1%	4.5%	3.9%	5.6%	5.4%	5.1%	5.4%	5.5%
Equity USA Mid-large Active Quant Art 8	6.1%	6.1%	3.2%	1.1%	0.2%	3.2%	3.2%	4.1%	4.7%	-	-	2.4%	3.2%	2.1%	1.7%	2.9%	2.8%	3.2%	1.3%	0.3%	1.2%	1.7%	3.2%	2.0%	-
Equity USA Mid-large Passive Index Art 6	-	-	4.5%	8.4%	10.0%	4.5%	4.5%	1.2%	-	-	-	-	4.5%	13.1%	18.5%	7.8%	7.6%	4.5%	5.8%	5.2%	8.8%	8.0%	4.5%	-	-
Equity USA Value Active Quant Art 8	2.4%	2.3%	1.2%	0.4%	0.1%	1.2%	1.2%	1.5%	1.7%	-	-	0.9%	1.2%	0.7%	0.6%	1.1%	1.0%	1.2%	0.4%	-	0.4%	0.6%	1.2%	0.7%	-
Equity USA Growth Active Fundamental Art 8	-	1.1%	1.0%	0.4%	0.1%	1.0%	1.0%	1.3%	1.3%	-	-	0.4%	1.0%	0.9%	1.1%	-	0.3%	1.0%	1.0%	1.3%	0.4%	0.6%	1.0%	0.5%	-
Equity Japan Mid-large Active Fundamental Art 8	5.1%	3.4%	2.1%	0.7%	0.1%	2.1%	2.1%	2.0%	1.8%	-	1.7%	1.7%	2.1%	1.3%	1.1%	1.8%	1.8%	2.1%	0.9%	0.3%	0.8%	1.1%	2.1%	1.4%	1.2%
Equity Japan Mid-large Active Quant Art 8	7.6%	4.3%	2.5%	0.8%	0.1%	2.5%	2.5%	2.6%	2.9%	-	1.9%	2.0%	2.5%	1.7%	1.5%	1.9%	2.0%	2.5%	1.3%	0.7%	0.9%	1.3%	2.5%	1.7%	1.4%
Equity Japan Mid-large Passive Index Art 8	-	-	2.2%	5.1%	6.3%	2.2%	2.2%	1.8%	1.6%	-	3.4%	3.4%	2.2%	3.7%	4.1%	3.1%	3.1%	2.2%	4.6%	5.6%	5.1%	4.4%	2.2%	4.0%	4.5%
Equity Emerging Mid-large Active Fundamental Art 8	9.8%	5.0%	1.7%	0.6%	0.1%	1.7%	1.7%	1.7%	1.7%	-	1.5%	1.4%	1.7%	1.1%	0.9%	1.6%	1.6%	1.7%	0.7%	0.1%	0.7%	0.9%	1.7%	1.2%	1.0%
Equity Emerging Mid-large Passive Index Art 8	-	0.7%	4.3%	5.4%	5.9%	4.3%	4.3%	4.1%	4.0%	-	4.4%	4.6%	4.3%	4.8%	5.0%	4.4%	4.4%	4.3%	5.2%	5.6%	5.3%	5.1%	4.3%	3.9%	3.4%
Equity World Water Active Fundamental Art 9	-	1.1%	0.6%	0.3%	0.1%	0.6%	0.6%	1.0%	1.7%	24.4%	-	0.6%	0.6%	0.2%	0.2%	0.6%	0.5%	0.6%	-	-	0.2%	0.2%	0.6%	0.1%	-
Equity World Disruptive Tech Active Fundamental Art 8	3.7%	2.0%	0.9%	0.2%	-	0.9%	0.9%	1.1%	1.1%	-	-	0.3%	0.9%	0.8%	1.0%	-	-	0.9%	1.2%	1.7%	-	-	0.9%	8.5%	12.3%
Equity World Low Vol Active Quant Art 8	4.5%	2.3%	1.2%	0.5%	0.2%	1.2%	1.2%	1.4%	1.6%	-	0.3%	1.1%	1.2%	0.6%	0.4%	1.9%	1.6%	1.2%	-	-	0.5%	0.7%	1.2%	0.8%	-
Equity World Mid-large Passive Index Art 8	-	-	0.9%	2.0%	2.4%	0.9%	0.9%	2.9%	2.7%	-	-	-	0.9%	2.1%	3.3%	-	-	0.9%	3.0%	4.3%	1.8%	1.5%	0.9%	-	-
Bonds EUR Sovereign Active Fundamental Art 8	-	10.2%	7.4%	3.4%	1.7%	7.5%	7.5%	7.1%	-	-	1.7%	3.7%	7.4%	3.8%	1.6%	-	-	7.4%	1.1%	-	3.7%	4.6%	7.4%	5.6%	5.3%
Bonds EUR Sovereign Passive Index Art 8	-	-	1.8%	2.6%	2.9%	1.7%	1.7%	-	-	-	-	-	1.8%	4.5%	5.7%	-	-	-	-	-	-	-	-	-	-
Bonds EUR Investment Grade Active Fundamental Art 8	-	-	-	0.1%	0.5%	-	-	-	-	-	-	-	-	-	-	6.9%	3.2%	-	-	-	-	-	-	-	-
Bonds EUR Investment Grade Active Quant Art 8	-	4.6%	3.1%	1.1%	0.1%	3.1%	3.1%	0.8%	-	-	-	-	3.1%	-	-	4.8%	4.0%	3.1%	-	-	-	-	3.1%	-	-
Bonds EUR Investment Grade Passive Index Art 8	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	1.8%	27.3%	38.4%	3.9%	2.5%	1.8%	0.2%	0.5%
Bonds EUR High Yield Active Fundamental Art 8	-	-	0.1%	0.1%	0.1%	-	-	0.3%	0.4%	-	1.0%	1.0%	0.1%	0.6%	0.6%	2.1%	1.4%	0.1%	0.4%	-	1.2%	1.0%	0.1%	0.8%	0.9%
Bonds EUR Aggregate Active Fundamental Art 8	-	-	5.0%	11.2%	14.1%	5.0%	5.0%	-	-	-	18.1%	13.1%	5.0%	-	-	-	-	5.0%	-	-	1.7%	3.2%	5.0%	6.7%	6.4%
Bonds USD Investment Grade Active Quant Art 8	-	6.6%	5.8%	5.6%	3.7%	5.8%	5.8%	4.7%	2.8%	-	3.8%	3.8%	5.8%	4.9%	4.6%	3.2%	3.6%	5.8%	-	-	3.9%	4.1%	5.8%	4.2%	4.1%
Bonds USD Investment Grade Passive Index Art 8	-	-	-	-	1.8%	-	-	-	-	-	-	-	-	-	0.4%	-	-	-	5.2%	5.8%	-	-	-	-	-
Bonds USD High Yield Active Fundamental Art 8	-	1.9%	3.2%	3.4%	3.5%	3.2%	3.2%	2.8%	1.5%	-	4.1%	4.0%	3.2%	3.1%	2.8%	5.6%	4.7%	3.2%	2.3%	1.0%	3.8%	3.6%	3.2%	3.5%	3.6%
Bonds Emerging HC Active Fundamental Art 8	7.2%	5.5%	2.7%	0.4%	-	2.7%	2.7%	3.0%	3.9%	-	-	-	2.7%	4.0%	5.4%	-	-	2.7%	5.9%	6.5%	0.6%	1.1%	2.7%	0.9%	0.2%
Bonds Emerging HC Passive Index Art 8	-	-	3.1%	6.4%	7.1%	3.1%	3.1%	2.1%	-	-	6.3%	6.2%	3.1%	1.6%	-	5.9%	6.0%	3.1%	-	-	5.8%	5.1%	3.1%	5.3%	6.1%
Bonds Global Aggregate Active Fundamental Art 8	32.3%	14.2%	7.6%	5.0%	4.0%	7.6%	7.6%	6.1%	2.4%	-	12.4%	10.2%	7.6%	6.8%	4.9%	16.4%	14.3%	7.6%	1.9%	-	8.9%	8.8%	7.6%	9.2%	9.7%
Bonds Global Green Active Fundamental Art 9	-	2.7%	7.9%	9.1%	9.5%	7.9%	7.9%	20.5%	35.5%	54.9%	12.6%	12.2%	7.9%	12.5%	10.5%	-	8.8%	7.9%	6.7%	1.2%	15.0%	14.1%	7.9%	13.3%	14.0%
Commodities Global All Passive Index Art 6	-	-	2.3%	3.2%	3.7%	2.3%	2.3%	2.2%	2.2%	-	3.3%	2.8%	2.3%	2.4%	2.1%	4.4%	3.7%	2.3%	1.7%	1.0%	3.1%	2.8%	2.3%	2.8%	2.9%
Commodities Global All Active Fundamental Art 6	7.8%	6.3%	3.9%	2.9%	2.5%	3.9%	3.9%	3.9%	3.9%	-	3.5%	3.5%	3.9%	3.4%	3.3%	3.4%	3.4%	3.9%	3.2%	2.9%	2.9%	3.1%	3.9%	3.3%	3.2%
Real Estate Eurozone Real Estate Passive Index Art 6	-	-	0.1%	0.5%	0.6%	0.1%	0.1%	-	-	-	0.2%	0.2%	0.1%	0.3%	0.3%	0.3%	0.2%	0.1%	0.4%	0.4%	0.5%	0.4%	0.1%	0.3%	0.3%
Real Estate Europe Real Estate Active Fundamental Art 8	1.9%	3.9%	3.8%	3.4%	3.1%	3.8%	3.8%	3.9%	4.0%	5.2%	3.3%	3.6%	3.8%	3.9%	4.1%	2.7%	3.0%	3.8%	4.3%	4.7%	3.5%	3.6%	3.8%	3.6%	3.5%

Note: The full allocation of the TAA portfolios to funds derived throughout the article. The first ten portfolios from the left have no tactical and no thematic views. The alphas from the funds impact the final allocation as a function of the choice of the ratio *RB/γ*. The other portfolios have either a tactical view on Equity USA, a tactical view on Bond EUR Sovereign or a thematic view on Disruptive Tech. For illustration purpose only. Past performance is not indicative of future performance. Not investment advice.

Source: BNP Paribas Asset Management, MSCI, Russell, Bloomberg, J.P. Morgan, FTSE EPRA. Authors' calculations on 1st September 2025.

5.4. Decomposition of the portfolio in the absence of tactical and thematic views

In this appendix, we provide analytical support that, as long as the long-only and no-leverage constraints are satisfied, the solution to equation (30) for fully invested portfolios yields a solution that can be decomposed into two components: (i) the allocation to funds that minimizes tracking error relative to the SAA, where the SAA itself allocates only to a selection of benchmark and core asset indices, and (ii) a long-short component that scales with *RB/γ*, tilting the allocation towards funds with the highest alpha. Furthermore, we show that the squared tracking error of the portfolio invested exclusively in funds, relative to the SAA, is equal to the sum of the squared tracking error of the minimum tracking error portfolio and the squared variance of the long-short component, with the latter weighted by $(RB/\gamma)^2$, increasing as the appetite for active alpha increases.

We start by simplifying equation (25), rewriting this equation with a new set of portfolio vectors $\boldsymbol{w}_{\text{SAA}}^{A}$, $\boldsymbol{w}_{\text{TAA}}^{A}$ and implied returns $\bar{\boldsymbol{\mu}}^{A}$ of size $n_{\text{SAA}} + n_f$, where the rows in $\boldsymbol{w}_{\text{SAA}}$, $\boldsymbol{w}_{\text{TAA}}$ and $\bar{\boldsymbol{\mu}}$ for the core assets and benchmarks that are not in the SAA were removed. Similarly, $\boldsymbol{\Sigma}_A$ and $\boldsymbol{\Omega}_A$ of size $(n_{\text{SAA}} + n_f) \times (n_{\text{SAA}} + n_f)$ are based on $\boldsymbol{\Sigma}$ and $\boldsymbol{\Omega}$ by removing rows and columns for those same assets. Because those assets will always have zero weight in both the SAA and TAA portfolios, equation (25) can be re-written in terms of these new vectors and matrices as:

$$\max_{\boldsymbol{w}_{\text{TAA}}^{A}} \Big((\bar{\boldsymbol{\mu}}^{A})^{\text{T}}(\boldsymbol{w}_{\text{TAA}}^{A} - \boldsymbol{w}_{\text{SAA}}^{A}) - \lambda(\boldsymbol{w}_{\text{TAA}}^{A} - \boldsymbol{w}_{\text{SAA}}^{A})^{\text{T}}\boldsymbol{\Sigma}_A(\boldsymbol{w}_{\text{TAA}}^{A} - \boldsymbol{w}_{\text{SAA}}^{A})$$

$$- \kappa\sqrt{(\boldsymbol{w}_{\text{TAA}}^{A} - \boldsymbol{w}_{\text{SAA}}^{A})^{\text{T}}\boldsymbol{\Omega}_A(\boldsymbol{w}_{\text{TAA}}^{A} - \boldsymbol{w}_{\text{SAA}}^{A})} \Big) \quad \text{(A2)}$$

Under the typical constraints of fully invested portfolio $\sum_i w_{\text{TAA}_i}^{A} = 1$, no short, $w_i \geq 0$, and no leverage $w_{\text{TAA}_i}^{A} \leq 0$ if the asset i is not investable and $w_{\text{TAA}_i}^{A} \leq 1$ otherwise. We could have included additional linear constraints – for example, sustainable investment exposures – or quadratic constraints, e.g., to control the final volatility of the portfolio.

We ignore the uncertainty term in κ for simplicity and, because the weights of SAA indices in the TAA portfolio are set to zero, we can further simplify (A2) in terms of the vector $\boldsymbol{w}_{\text{TAA}}^{f}$ created from $\boldsymbol{w}_{\text{TAA}}^{A}$ by keeping only the rows for funds:

$$\max_{\boldsymbol{w}_{\text{TAA}}^{f}} \left(\boldsymbol{\mu}_f \boldsymbol{w}_{\text{TAA}}^{f} - \lambda {\boldsymbol{w}_{\text{TAA}}^{f}}^{\text{T}} \boldsymbol{\Sigma}_f \boldsymbol{w}_{\text{TAA}}^{f} \right) \quad \text{(A3)}$$

With:

$$\boldsymbol{\mu}_f = \bar{\boldsymbol{\mu}}_f + 2\lambda\, \bar{\boldsymbol{\mu}}_{\text{SAA}}^{f} \quad \text{(A4)}$$

where $\bar{\boldsymbol{\mu}}_f$ is the implied return vector constructed from $\bar{\boldsymbol{\mu}}$ in (26) by keeping only the rows for funds only, and $\bar{\boldsymbol{\mu}}_{\text{SAA}}^{f}$ is the vector of implied returns $\boldsymbol{\Sigma}_A \boldsymbol{w}_{\text{SAA}}^{A}$ derived from the SAA portfolio and where only rows for funds are kept.

The Karush-Kuhn-Tucker conditions for this case, with $\mathbf{1}$ the unitary vector of size n_f, can be written as:

- Stationarity: $2\lambda\boldsymbol{\Sigma}_f\boldsymbol{w}^f_{\text{TAA}} - \boldsymbol{\mu}_f + \boldsymbol{\delta}_{max} - \boldsymbol{\delta}_{min} + \delta_{funding}\mathbf{1} = \mathbf{0}$
- Primal feasibility: $\boldsymbol{w}^f_{\text{TAA}} \leq \mathbf{1}$, $-\boldsymbol{w}^f_{\text{TAA}} \leq \mathbf{0}$ and $\mathbf{1}^{\text{T}}\boldsymbol{w}^f_{\text{TAA}} = 1$
- Dual feasibility: $\boldsymbol{\delta_{max}} \geq \mathbf{0}$, $\boldsymbol{\delta_{min}} \geq \mathbf{0}$
- Complementary slackness: $\delta_{max,i}.\left(w_{\text{TAA},i} - 1\right) = 0$, $\delta_{min,i}.w_{\text{TAA},i} = 0$ for all i

Hence:

$$\begin{aligned}\boldsymbol{w}^f_{\text{TAA}} &= \tfrac{1}{2\lambda}\boldsymbol{\Sigma}_f^{-1}\left(\boldsymbol{\mu}_f + \boldsymbol{\delta}_{max} - \boldsymbol{\delta}_{min} - \delta_{funding}\mathbf{1}\right) \\ &= \tfrac{1}{2\lambda}\boldsymbol{\Sigma}_f^{-1}\left(\bar{\boldsymbol{\mu}}_f + 2\lambda\,\bar{\boldsymbol{\mu}}^f_{\text{SAA}} + \boldsymbol{\delta}_{max} - \boldsymbol{\delta}_{min} - \delta_{funding}\mathbf{1}\right) \qquad \text{(A5)}\end{aligned}$$

For as long as $0 < w_{\text{TAA},i} < 1$ holds, the Lagrangian multipliers $\delta_{max,i} = \delta_{min,i} = 0$ for all i, simplifying (A5):

$$\boldsymbol{w}^f_{\text{TAA}} = \tfrac{1}{2\lambda}\boldsymbol{\Sigma}_f^{-1}\left(\boldsymbol{\mu}_f - \delta_{funding}\mathbf{1}\right) \qquad \text{(A6)}$$

Since $\mathbf{1}^{\text{T}}\boldsymbol{w}^f_{\text{TAA}} = 1$ we have $\delta_{funding} = \frac{\frac{1}{2\lambda}\mathbf{1}^{\text{T}}\boldsymbol{\Sigma}_f^{-1}\boldsymbol{\mu}_f - 1}{\frac{1}{2\lambda}\mathbf{1}^{\text{T}}\boldsymbol{\Sigma}_f^{-1}\mathbf{1}}$.

5.4.1 TAA portfolio with no tactical and no thematic views

Based on the previous results, we now demonstrate that, as long as $0 < w_{\text{TAA},i} < 1$ holds for all funds and in the absence of tactical or thematic views, the optimal TAA portfolio allocation can be expressed as the sum of the fully invested long-only fund portfolio that minimizes tracking error relative to the SAA, and a long-short portfolio of funds representing deviations from this minimum tracking error allocation. The magnitude of this deviation portfolio scales with RB/γ, reflecting the appetite for active alpha.

With the Lagrangian multipliers $\delta_{max,i} = \delta_{min,i} = 0$ for all i:

$$\boldsymbol{w}^f_{\text{TAA}} = \tfrac{1}{2\lambda}\boldsymbol{\Sigma}_f^{-1}(\bar{\boldsymbol{\mu}}_f + 2\lambda\,\bar{\boldsymbol{\mu}}^f_{\text{SAA}} - \delta_{funding}\mathbf{1}) \qquad \text{(A7)}$$

replacing $\delta_{funding}$:

$$\begin{aligned}\boldsymbol{w}^f_{\text{TAA}} &= \tfrac{1}{2\lambda}\boldsymbol{\Sigma}_f^{-1}\left(\bar{\boldsymbol{\mu}}_f + 2\lambda\,\bar{\boldsymbol{\mu}}^f_{\text{SAA}} + \frac{1-\frac{1}{2\lambda}\mathbf{1}^{\text{T}}\boldsymbol{\Sigma}_f^{-1}\left(\bar{\boldsymbol{\mu}}_f + 2\lambda\,\bar{\boldsymbol{\mu}}^f_{\text{SAA}}\right)}{\frac{1}{2\lambda}\mathbf{1}^{\text{T}}\boldsymbol{\Sigma}_f^{-1}\mathbf{1}}\mathbf{1}\right) \\ &= \boldsymbol{\Sigma}_f^{-1}\,\bar{\boldsymbol{\mu}}^f_{\text{SAA}} + \boldsymbol{\Sigma}_f^{-1}\left(\frac{1}{2\lambda}\bar{\boldsymbol{\mu}}_f + \frac{1-\frac{1}{2\lambda}\mathbf{1}^{\text{T}}\boldsymbol{\Sigma}_f^{-1}\bar{\boldsymbol{\mu}}_f - \mathbf{1}^{\text{T}}\boldsymbol{\Sigma}_f^{-1}\bar{\boldsymbol{\mu}}^f_{\text{SAA}}}{\mathbf{1}^{\text{T}}\boldsymbol{\Sigma}_f^{-1}\mathbf{1}}\mathbf{1}\right) \qquad \text{(A8)}\end{aligned}$$

In the case with no tactical or thematic views:

$$\boldsymbol{w}^{f,no\ views}_{\text{TAA}} = \boldsymbol{\Sigma}_f^{-1}\left(\bar{\boldsymbol{\mu}}^f_{\text{SAA}} + \frac{1-\mathbf{1}^{\text{T}}\boldsymbol{\Sigma}_f^{-1}\bar{\boldsymbol{\mu}}^f_{\text{SAA}}}{\mathbf{1}^{\text{T}}\boldsymbol{\Sigma}_f^{-1}\mathbf{1}}\mathbf{1}\right) + \frac{RB}{0.4\,\gamma}\boldsymbol{\Sigma}_f^{-1}\left(\boldsymbol{\alpha}^f_{fb} - \frac{\mathbf{1}^{\text{T}}\boldsymbol{\Sigma}_f^{-1}\boldsymbol{\alpha}^f_{fb}}{\mathbf{1}^{\text{T}}\boldsymbol{\Sigma}_f^{-1}\mathbf{1}}\mathbf{1}\right) \qquad \text{(A9)}$$

And this is the sum of:

$$\boldsymbol{w}_{\text{TAA}}^{f,no\ views} = \boldsymbol{w}_{\min TE}^{f} + \frac{RB}{0.4\,\gamma}\boldsymbol{w}_{alpha}^{f} \tag{A10}$$

with the long-short dollar neutral portfolio of fund tilts:

$$\boldsymbol{w}_{alpha}^{f} = \boldsymbol{\Sigma}_f^{-1}\left(\boldsymbol{\alpha}_{fb}^{f} - \frac{\mathbf{1}^{\mathrm{T}}\boldsymbol{\Sigma}_f^{-1}\boldsymbol{\alpha}_{fb}^{f}}{\mathbf{1}^{\mathrm{T}}\boldsymbol{\Sigma}_f^{-1}\mathbf{1}}\mathbf{1}\right) \tag{A11}$$

relative to the long-only fully invested allocation to funds with the smallest possible tracking error relative to the SAA portfolio:

$$\boldsymbol{w}_{\min TE}^{f} = \boldsymbol{\Sigma}_f^{-1}\left(\bar{\boldsymbol{\mu}}_{\text{SAA}}^{f} + \frac{1-\mathbf{1}^{\mathrm{T}}\boldsymbol{\Sigma}_f^{-1}\bar{\boldsymbol{\mu}}_{\text{SAA}}^{f}}{\mathbf{1}^{\mathrm{T}}\boldsymbol{\Sigma}_f^{-1}\mathbf{1}}\mathbf{1}\right) \tag{A12}$$

5.4.2 Tracking error of TAA portfolio with no tactical and no thematic views

We show that, under the same assumptions as in Appendix 5.4.1, the squared tracking error of the TAA portfolio can be decomposed into the sum of two terms: (i) the squared tracking error of the fully invested long-only portfolio that minimizes tracking error with respect to the SAA, and (ii) the variance of the long-short component, which tilts the allocation towards high-alpha funds. The contribution of this long-short component increases proportionally with $(RB/\,\gamma)^2$.

In the absence of tactical or thematic views, the tracking error of the TAA portfolio is:

$$\left(TE_{\boldsymbol{w}_{\text{TAA}}^{no\ views}}\right)^{\mathbf{2}} = \left(\boldsymbol{w}_{\text{TAA}}^{A,no\ views} - \boldsymbol{w}_{\text{SAA}}^{A}\right)^{\mathrm{T}}\boldsymbol{\Sigma}_A\left(\boldsymbol{w}_{\text{TAA}}^{A,no\ views} - \boldsymbol{w}_{\text{SAA}}^{A}\right) \tag{A13}$$

Using (A10):

$$\left(TE_{\boldsymbol{w}_{\text{TAA}}^{no\ views}}\right)^{\mathbf{2}} = \left(TE_{\boldsymbol{w}_{\min TE}^{f}}\right)^{2} + \left(\frac{RB}{0.4\gamma}\right)^{2}\boldsymbol{w}_{alpha}^{f}{}^{\mathrm{T}}\boldsymbol{\Sigma}_f\boldsymbol{w}_{alpha}^{f}$$
$$+2\left(\frac{RB}{0.4\gamma}\right)\boldsymbol{w}_{alpha}^{A}{}^{\mathrm{T}}\boldsymbol{\Sigma}_A\left(\boldsymbol{w}_{\min TE}^{A} - \boldsymbol{w}_{\text{SAA}}^{A}\right) \tag{A14}$$

with $\boldsymbol{w}_{alpha}^{A}$ constructed from $\boldsymbol{w}_{alpha}^{f}$ by adding the respective rows for SAA assets and filling them with zero. Since these tilts on the SAA assets in $\boldsymbol{w}_{alpha}^{f}$ are zero, and using (A12):

$$\left(TE_{\boldsymbol{w}_{\text{TAA}}^{no\ views}}\right)^{\mathbf{2}} = \left(TE_{\boldsymbol{w}_{\min TE}^{f}}\right)^{2} + \left(\frac{RB}{0.4\gamma}\right)^{2}\boldsymbol{w}_{alpha}^{f}{}^{\mathrm{T}}\boldsymbol{\Sigma}_f\boldsymbol{w}_{alpha}^{f} + 2\left(\frac{RB}{0.4\gamma}\right)\boldsymbol{w}_{alpha}^{f}{}^{\mathrm{T}}\left(\boldsymbol{\Sigma}_f\boldsymbol{w}_{\min TE}^{f} - \bar{\boldsymbol{\mu}}_{\text{SAA}}^{f}\right)$$
$$= \left(TE_{\boldsymbol{w}_{\min TE}^{f}}\right)^{2} + \left(\frac{RB}{0.4\gamma}\right)^{2}\boldsymbol{w}_{alpha}^{f}{}^{\mathrm{T}}\boldsymbol{\Sigma}_f\boldsymbol{w}_{alpha}^{f} + 2\left(\frac{RB}{0.4\gamma}\right)\boldsymbol{w}_{alpha}^{f}{}^{\mathrm{T}}\left(\frac{1-\mathbf{1}^{\mathrm{T}}\boldsymbol{\Sigma}_f^{-1}\,\bar{\boldsymbol{\mu}}_{\text{SAA}}^{f}}{\mathbf{1}^{\mathrm{T}}\boldsymbol{\Sigma}_f^{-1}\mathbf{1}}\mathbf{1}\right) \tag{A15}$$

The sum of weights of both $\boldsymbol{w}_{\mathrm{TAA}}^{no\ views}$ and $\boldsymbol{w}_{\min TE}^{f}$ adds to 100%. Thus, $\boldsymbol{w}_{alpha}^{f}$ is a zero sum long-short portfolio and therefore the last term in the second line of (A15) is zero. The square of the tracking error can then be written as:

$$\left(TE_{\boldsymbol{w}_{\mathrm{TAA}}^{no\ views}}\right)^{\mathbf{2}} = \left(TE_{\boldsymbol{w}_{\min TE}^{f}}\right)^{2} + \left(\frac{RB}{0.4\gamma}\right)^{2} {\boldsymbol{w}_{alpha}^{f}}^{\mathrm{T}} \boldsymbol{\Sigma}_f \boldsymbol{w}_{alpha}^{f} \tag{A16}$$

5.4.3. Impact of RB/γ on the allocation to equities versus fixed income and passive versus active funds in the absence of tactical or thematic views

In this appendix we provide a simplified model to explain the portfolio tilts observed in Section 3.1, in particular why, in the absence of tactical or thematic views, portfolios increase the allocation to equities and to active funds as RB/γ increases.

We consider a universe of two funds, one for equities and one for fixed income. The SAA is allocated in some proportion to the benchmarks of these funds. In this case, we can significantly simplify the notation because the benchmarks of the funds are the core assets used in the SAA. With this in mind:

$$\boldsymbol{\Sigma}_f = \begin{bmatrix} \sigma_{f,EQ}^2 & cov_{EQ,FI}^{f} \\ cov_{EQ,FI}^{f} & \sigma_{f,FI}^2 \end{bmatrix} \tag{A17}$$

where $\sigma_{f,EQ}$ and $\sigma_{f,FI}$ are the volatilities of returns of the equity and fixed income funds and $cov_{EQ,FI}^{f}$ is the covariance of the returns of the funds. The inverse of this variance-covariance matrix is then:

$$\boldsymbol{\Sigma}_f^{-1} = \frac{1}{|\boldsymbol{\Sigma}_f|} \begin{bmatrix} \sigma_{f,FI}^2 & -cov_{EQ,FI}^{f} \\ -cov_{EQ,FI}^{f} & \sigma_{f,EQ}^2 \end{bmatrix} \tag{A18}$$

Where $|\boldsymbol{\Sigma}_f| = \sigma_{f,FI}^2\, \sigma_{f,EQ}^2 - \left(cov_{EQ,FI}^{f}\right)^2$ is the determinant of $\boldsymbol{\Sigma}_f$. In this case, we can also simplify the notation of:

$$\boldsymbol{\alpha}_{fb}^{f} = \begin{bmatrix} \alpha_{EQ} \\ \alpha_{FI} \end{bmatrix} \tag{A19}$$

and:

$$\boldsymbol{w}_{\mathrm{TAA}}^{f,no\ views} = \begin{bmatrix} w_{EQ}^{f} \\ w_{FI}^{f} \end{bmatrix} \tag{A20}$$

so that in the absence of tactical and thematic views, using (A10) and (A11):

$$\begin{bmatrix} w_{EQ}^{f} \\ w_{FI}^{f} \end{bmatrix} = \boldsymbol{w}_{\min TE}^{f} + \frac{RB}{0.4\,\gamma} \boldsymbol{\Sigma}_f^{-1} \left(\boldsymbol{\alpha}_{fb}^{f} - \frac{\mathbf{1}^{\mathrm{T}} \boldsymbol{\Sigma}_f^{-1} \begin{bmatrix} \alpha_{EQ} \\ \alpha_{FI} \end{bmatrix}}{\mathbf{1}^{\mathrm{T}} \boldsymbol{\Sigma}_f^{-1} \mathbf{1}} \mathbf{1} \right) \tag{A21}$$

where $w_{EQ}^{f} + w_{FI}^{f} = 1$.

To assess how the allocation to changes as a function of RB/γ, let us take the derivative of (A21) relative to RB/γ, recalling that $\boldsymbol{w}^{f}_{\min TE}$ is independent of RB/γ:

$$\begin{bmatrix} \partial w^{f}_{EQ}/\partial(RB/\gamma) \\ \partial w^{f}_{FI}/\partial(RB/\gamma) \end{bmatrix} = \frac{1}{0.4}\,\boldsymbol{\Sigma}_f^{-1}\left(\begin{bmatrix} \alpha_{EQ} \\ \alpha_{FI} \end{bmatrix} - \frac{\mathbf{1}^{\mathrm{T}}\boldsymbol{\Sigma}_f^{-1}\begin{bmatrix} \alpha_{EQ} \\ \alpha_{FI} \end{bmatrix}}{\mathbf{1}^{\mathrm{T}}\boldsymbol{\Sigma}_f^{-1}\mathbf{1}} \begin{bmatrix} 1 \\ 1 \end{bmatrix}\right) \tag{A22}$$

defining:

$$\Theta_{funding} = \frac{\mathbf{1}^{\mathrm{T}}\boldsymbol{\Sigma}_f^{-1}\begin{bmatrix} \alpha_{EQ} \\ \alpha_{FI} \end{bmatrix}}{\mathbf{1}^{\mathrm{T}}\boldsymbol{\Sigma}_f^{-1}\mathbf{1}} = \frac{\alpha_{EQ}\left(\sigma^2_{f,FI} - cov^{f}_{EQ,FI}\right) + \alpha_{FI}\left(\sigma^2_{f,EQ} - cov^{f}_{EQ,FI}\right)}{\sigma^2_{f,FI} + \sigma^2_{f,EQ} - 2\,cov^{f}_{EQ,FI}} \tag{A23}$$

where the second term can be found using (A18). Given that $\boldsymbol{\Sigma}_f^{-1}$ is positive definite, then the denominator in (A23) is positive:

$$\mathbf{1}^{\mathrm{T}}\boldsymbol{\Sigma}_f^{-1}\mathbf{1} = \frac{\sigma^2_{f,FI} + \sigma^2_{f,EQ} - 2\,cov^{f}_{EQ,FI}}{|\boldsymbol{\Sigma}_f|} > 0 \tag{A24}$$

With definition (A23) we can rewrite (A22):

$$\begin{bmatrix} \partial w^{f}_{EQ}/\partial(RB/\gamma) \\ \partial w^{f}_{FI}/\partial(RB/\gamma) \end{bmatrix} = \frac{1}{0.4\,|\boldsymbol{\Sigma}_f|} \begin{bmatrix} \sigma^2_{f,FI} & -cov^{f}_{EQ,FI} \\ -cov^{f}_{EQ,FI} & \sigma^2_{f,EQ} \end{bmatrix} \begin{bmatrix} \alpha_{EQ} - \Theta_{funding} \\ \alpha_{FI} - \Theta_{funding} \end{bmatrix} \tag{A25}$$

which for equities results in:

$$\frac{\partial w^{f}_{EQ}}{\partial(RB/\gamma)} = \frac{1}{0.4\,|\boldsymbol{\Sigma}_f|} * \left(\sigma^2_{f,FI}\left(\alpha_{EQ} - \Theta_{funding}\right) - cov^{f}_{EQ,FI}\left(\alpha_{FI} - \Theta_{funding}\right)\right) \tag{A26}$$

and noting that from (A23):

$$\alpha_{EQ} - \Theta_{funding} = \frac{\left(\alpha_{EQ} - \alpha_{FI}\right)\left(\sigma^2_{f,EQ} - cov^{f}_{EQ,FI}\right)}{\sigma^2_{f,FI} + \sigma^2_{f,EQ} - 2\,cov^{f}_{EQ,FI}} \tag{A27}$$

$$\alpha_{FI} - \Theta_{funding} = \frac{\left(\alpha_{FI} - \alpha_{EQ}\right)\left(\sigma^2_{f,FI} - cov^{f}_{EQ,FI}\right)}{\sigma^2_{f,FI} + \sigma^2_{f,EQ} - 2\,cov^{f}_{EQ,FI}} \tag{A28}$$

replacing (A27) and (A28) into (A26):

$$\begin{aligned} \frac{\partial w^{f}_{EQ}}{\partial(RB/\gamma)} &= \frac{1}{0.4\,|\boldsymbol{\Sigma}_f|} * \frac{\left(\alpha_{EQ} - \alpha_{FI}\right)\left(\sigma^2_{f,FI}\left(\sigma^2_{f,EQ} - cov^{f}_{EQ,FI}\right) + cov^{f}_{EQ,FI}\left(\sigma^2_{f,FI} - cov^{f}_{EQ,FI}\right)\right)}{\sigma^2_{f,FI} + \sigma^2_{f,EQ} - 2\,cov^{f}_{EQ,FI}} \\ &= \frac{1}{0.4} * \frac{\alpha_{EQ} - \alpha_{FI}}{\sigma^2_{f,FI} + \sigma^2_{f,EQ} - 2\,cov^{f}_{EQ,FI}} \end{aligned} \tag{A29}$$

In practice, we tend to find a higher alpha from actively managed equity funds than from fixed income funds, as shown in Table A4. Therefore, recalling in (A24) that the denominator is also positive, $\partial w_{EQ}^{f}/\partial(RB/\gamma)$ is positive and because $w_{EQ}^{f}+w_{FI}^{f}=1$ the change in the allocation to fixed income is $\partial w_{FI}^{f}/(RB/\gamma) = -\partial w_{EQ}^{f}/(RB/\gamma)$ and is negative.

For this reason, increasing RB/γ should lead to an increase in the equity allocation, a consequence of the higher net alpha of active equity funds. For the same reasons, we can explain why the allocation to active funds with positive higher net alpha increases with RB/γ while allocation to passive funds and active funds with lower or negative net alpha decreases.

5.4.4. Allocation to passive versus active funds as a function of tactical views

The objective of this appendix is to explain why, in Section 3.4, when there are tactical views for a given asset class, as the tactical view becomes more positive, the allocation to the passive fund increases, and as the tactical view becomes more negative, the allocation to the passive fund decreases, this while the allocation to active funds of the same asset class remains relatively unchanged for as long as the no short and no leverage constraints are met.

We consider a simplified problem where there is only one asset class and only two funds: one active and one passive. The benchmark is the same for both funds, with volatility σ_b, and is also the only asset in the SAA portfolio. Directional tactical views will be made for this benchmark.

Since the benchmark of the funds is also the only asset in the SAA, equation (3) can be simplified:

$$XR_f^{Active}(t) = \alpha^{Active} + \beta^{Active}\, XR_b(t) + \varepsilon^{Active}(t) \tag{A30}$$

$$XR_f^{Passive}(t) = \alpha^{Passive} + \beta^{Passive}\, XR_b(t) + \varepsilon^{Passive}(t) \tag{A31}$$

with the alphas:

$$\alpha_{fb}^{Active} = 0.5 * \sigma_{fb}^{Active} - OCR^{Active} \tag{A32}$$

$$\alpha_{fb}^{Passive} = -OCR^{Passive} \tag{A33}$$

and with the volatility of the residuals ε given by σ_{fb}^{Active} and $\sigma_{fb}^{Passive}$.

In this case, the matrix $\mathbf{\Sigma_{all}}$ is given by

$$\mathbf{\Sigma}_A = \begin{bmatrix} \left(\beta^{Passive}\sigma_b\right)^2 + \left(\sigma_{fb}^{Passive}\right)^2 & \beta^{Passive}\beta^{Active}\sigma_b^2 & \beta^{Passive}\sigma_b^2 \\ \beta^{Passive}\beta^{Active}\sigma_b^2 & \left(\beta^{Active}\sigma_b\right)^2 + \left(\sigma_{fb}^{Active}\right)^2 & \beta^{Active}\sigma_b^2 \\ \beta^{Passive}\sigma_b^2 & \beta^{Active}\sigma_b^2 & \sigma_b^2 \end{bmatrix} \tag{A34}$$

The variance-covariance matrix for funds $\mathbf{\Sigma}_f$ can be obtained from of $\mathbf{\Sigma}_A$ by removing the last row and column:

$$\mathbf{\Sigma}_f = \begin{bmatrix} \left(\beta^{Passive}\sigma_b\right)^2 + \left(\sigma_{fb}^{Passive}\right)^2 & \beta^{Passive}\beta^{Active}\sigma_b^2 \\ \beta^{Passive}\beta^{Active}\sigma_b^2 & \left(\beta^{Active}\sigma_b\right)^2 + \left(\sigma_{fb}^{Active}\right)^2 \end{bmatrix} \quad \text{(A35)}$$

And much like in the Appendix 5.4.3. the inverse of $\mathbf{\Sigma}_f$ is given by:

$$\mathbf{\Sigma}_f^{-1} = \frac{1}{|\mathbf{\Sigma}_f|} * \begin{bmatrix} \left(\beta^{Active}\sigma_b\right)^2 + \left(\sigma_{fb}^{Active}\right)^2 & -\beta^{Passive}\beta^{Active}\sigma_b^2 \\ -\beta^{Passive}\beta^{Active}\sigma_b^2 & \left(\beta^{Passive}\sigma_b\right)^2 + \left(\sigma_{fb}^{Passive}\right)^2 \end{bmatrix} \quad \text{(A36)}$$

In this simplified case, the implied returns from the SAA portfolio as in (A4) are just:

$$\bar{\boldsymbol{\mu}}_{\mathrm{SAA}}^{f} = \begin{bmatrix} \beta^{Passive}\sigma_b^2 \\ \beta^{Active}\sigma_b^2 \end{bmatrix} \quad \text{(A37)}$$

and the expected returns for the funds in (A4) derived from tactical views and fund alphas are:

$$\bar{\boldsymbol{\mu}}_f = \begin{bmatrix} \bar{\mu}_{Passive} \\ \bar{\mu}_{Active} \end{bmatrix} \quad \text{(A38)}$$

which, in the absence of tactical views, simplifies into:

$$\bar{\boldsymbol{\mu}}_f = \frac{1}{\gamma}\begin{bmatrix} -OCR^{Passive} \\ 0.5 * \sigma_{fb}^{Active} - OCR^{Active} \end{bmatrix} = \frac{1}{\gamma}\begin{bmatrix} \alpha_{fb}^{Passive} \\ \alpha_{fb}^{Active} \end{bmatrix} \quad \text{(A39)}$$

However, if there is a directional tactical view $S_{directional}$ on the benchmark, with the active portfolio:

$$\boldsymbol{w}_{active} = \begin{bmatrix} 0 \\ 0 \\ S_{directional} * RB\ \sigma_b^{-1} \end{bmatrix} \quad \text{(A40)}$$

the implied returns from the tactical views are $2\lambda\mathbf{\Sigma}_A\boldsymbol{w}_{active}$. Retaining only the rows for funds in the tactical implied returns vector, and adding this tactical term to (A39) results in:

$$\bar{\boldsymbol{\mu}}_f = 2\lambda * S_{directional} * RB\ \sigma_b^{-1}\begin{bmatrix} \beta^{Passive}\sigma_b^2 \\ \beta^{Active}\sigma_b^2 \end{bmatrix} + \frac{1}{\gamma}\begin{bmatrix} -OCR^{Passive} \\ 0.5 * \sigma_{fb}^{Active} - OCR^{Active} \end{bmatrix} \quad \text{(A41)}$$

From now on we assume that the long-only and no leverage constraints are fulfilled. We also remove the funding constraint to facilitate the implementation of the view given that we have a single asset in the SAA portfolio. The optimization problem in (A7) can now be re-written using (A36) for the inverse variance covariance of funds, (A37) for the fund implied returns derived from the SAA, and (A38) for the fund implied returns derived from the tactical view and ignoring the funding Lagrange multiplier:

$$\begin{bmatrix} w^{Passive} \\ w^{Active} \end{bmatrix} = \frac{1}{2\lambda}\frac{1}{|\mathbf{\Sigma}_f|} * \begin{bmatrix} \left(\beta^{Active}\sigma_b\right)^2 + \left(\sigma_{fb}^{Active}\right)^2 & -\beta^{Passive}\beta^{Active}\sigma_b^2 \\ -\beta^{Passive}\beta^{Active}\sigma_b^2 & \left(\beta^{Passive}\sigma_b\right)^2 + \left(\sigma_{fb}^{Passive}\right)^2 \end{bmatrix} * \begin{bmatrix} \bar{\mu}_{Passive} + 2\lambda\beta^{Passive}\sigma_b^2 \\ \bar{\mu}_{Active} + 2\lambda\beta^{Active}\sigma_b^2 \end{bmatrix} \quad \text{(A42)}$$

Let us define the ratio of volatilities of excess returns relative to the beta exposure of each fund to the benchmark $r_\sigma = \sigma_{fb}^{Active}/\sigma_{fb}^{Passive}$, which is typically larger than 1, and the ratio of fund betas relative to the only benchmark in this example $r_\beta = \beta^{Active}/\beta^{Passive}$. Using these definitions in (A42):

$$\begin{bmatrix} w^{Passive} \\ w^{Active} \end{bmatrix} = \frac{1}{2\lambda}\frac{1}{|\mathbf{\Sigma}_f|} * \begin{bmatrix} \left(r_\beta\beta^{Passive}\sigma_b\right)^2 + \left(\sigma_{fb}^{Active}\right)^2 & -r_\beta\left(\beta^{Passive}\sigma_b\right)^2 \\ -r_\beta\left(\beta^{Passive}\sigma_b\right)^2 & \left(\beta^{Passive}\sigma_b\right)^2 + \left(\sigma_{fb}^{Passive}\right)^2 \end{bmatrix} * \begin{bmatrix} \bar{\mu}_{Passive} + 2\lambda\beta^{Passive}\sigma_b^2 \\ \bar{\mu}_{Active} + 2\lambda r_\beta\beta^{Passive}\sigma_b^2 \end{bmatrix} \quad \text{(A43)}$$

We shall now use (A41) in (A43), first while ignoring tactical views and then including a tactical view to see the difference. For the case $S_{directional} = 0$, and recalling (A32) and (A33):

$$\begin{bmatrix} w_{S_d=0}^{Passive} \\ w_{S_d=0}^{Active} \end{bmatrix} = \frac{\beta^{Passive}\sigma_b^2}{|\mathbf{\Sigma}_f|} * \begin{bmatrix} \left(\sigma_{fb}^{Active}\right)^2 \\ r_\beta\left(\sigma_{fb}^{Passive}\right)^2 \end{bmatrix} + \frac{RB}{0.4\gamma} * \frac{1}{|\mathbf{\Sigma}_f|}\begin{bmatrix} \alpha_{fb}^{Passive}\left(\left(r_\beta\beta^{Passive}\sigma_b\right)^2 + \left(\sigma_{fb}^{Active}\right)^2\right) - \alpha_{fb}^{Active} r_\beta\left(\beta^{Passive}\sigma_b\right)^2 \\ \alpha_{fb}^{Active}\left(\left(\beta^{Passive}\sigma_b\right)^2 + \left(\sigma_{fb}^{Passive}\right)^2\right) - \alpha_{fb}^{Passive} r_\beta\left(\beta^{Passive}\sigma_b\right)^2 \end{bmatrix} \quad \text{(A44)}$$

which means that the final portfolio of funds can be decomposed into:

$$\begin{bmatrix} w_{S_d=0}^{Passive} \\ w_{S_d=0}^{Active} \end{bmatrix} = \begin{bmatrix} w_{\text{SAA}}^{Passive} \\ w_{\text{SAA}}^{Active} \end{bmatrix} + \begin{bmatrix} w_\alpha^{Passive} \\ w_\alpha^{Active} \end{bmatrix} \quad \text{(A45)}$$

with the second term that scales with RB/γ and a first term that is independent of it. Moreover, with the ratio of the weight in the first term

$$\frac{w_{\text{SAA}}^{Passive}}{w_{\text{SAA}}^{Active}} = \frac{r_\sigma^2}{r_\beta} \quad \text{(A46)}$$

and with $w_\alpha^{Passive} < 0$ and $w_\alpha^{Active} > 0$. Therefore, RB/γ can be used to calibrate the proportion of passive versus active and reduce the impact of r_σ^2/r_β in the final allocation.

If we repeat the exercise while considering $S_{directional} \neq 0$ in (A41) to rederive (A45), we can show that the ratio of the differences in the weights allocated to passive and to active arising from introducing directional views are:

$$\frac{w_{S_d \neq 0}^{Passive} - w_{S_d=0}^{Passive}}{w_{S_d \neq 0}^{Active} - w_{S_d=0}^{Active}} = \frac{2\lambda * S_{directional} * RB\ \sigma_b^{-1} * \left(\sigma_{fb}^{Active}\right)^2 * \beta^{Passive} \sigma_b^2}{2\lambda * S_{directional} * RB\ \sigma_b^{-1} * \left(\sigma_{fb}^{Passive}\right)^2 * r_\beta\ \beta^{Passive} \sigma_b^2} = \frac{r_\sigma^2}{r_\beta} \tag{A47}$$

In practice, the exposure of the two funds to the benchmark is likely similar, so that $r_\beta \sim 1$, whereas the square of the ratio of alphas should be $r_\sigma^2 \gg 1$. Therefore, the difference in weights generated by the introduction of a tactical view is mainly driven by the change in the allocation to the passive fund, both for positive directional views, increasing it, and for negative views, decreasing it, and the change in weights goes in the same direction for both passive and active.

5.4.5. Tracking error of TAA portfolio with tactical views

In this appendix we show, using a simple example with a single tactical view, that the tracking error of the TAA portfolio relative to the SAA portfolio increases as views become more extreme, whether positive or negative. Indeed, in this example, the square of the tracking error of a TAA portfolio incorporating views is driven by two distinct components: The square of the tracking error of the allocation without tactical views, and an additional term equal to the square of the tracking error of the asset class for which the tactical view is expressed, scaled by the square of the score used for the directional view.

We consider a universe with several funds with benchmarks that are used in the SAA portfolio. As in previous examples, we assume that the long-only and no-short constraints are satisfied. This allows us to use (A6), ignoring the respective Lagrangian multipliers in the optimization equation (A5), which simplifies the analysis.

In this case, the implied return in (A6) is:

$$\boldsymbol{\mu}_f = \overline{\boldsymbol{\mu}}_f + 2\lambda\, \overline{\boldsymbol{\mu}}_{\text{SAA}}^f \tag{A48}$$

Here, as before, $\overline{\boldsymbol{\mu}}_{\text{SAA}}^f$ is the vector of implied returns $\boldsymbol{\Sigma}_A \boldsymbol{w}_{\text{SAA}}^A$ derived from the SAA portfolio and where only rows for funds are kept. In turn, $\overline{\boldsymbol{\mu}}_f$ is the vector with the implied returns from tactical views and net fund alpha as in (26) where, for simplicity, we ignore the term in κ. This last vector can be decomposed into its two components:

$$\overline{\boldsymbol{\mu}}_f = \overline{\boldsymbol{\mu}}_f^{tactical} + \overline{\boldsymbol{\mu}}_f^{alpha} \tag{A49}$$

The second term $\overline{\boldsymbol{\mu}}_f^{alpha}$ is:

$$\overline{\boldsymbol{\mu}}_f^{alpha} = \frac{1}{\gamma} \begin{bmatrix} IR_{\alpha_{fb}^{fund_1}} \sigma_{fb}^{fund_1} - OCR_f^{fund_1} \\ \cdots \\ IR_{\alpha_{fb}^{fund_n}} \sigma_{fb}^{fund_n} - OCR_f^{fund_n} \end{bmatrix} \tag{A50}$$

and $\overline{\boldsymbol{\mu}}_f^{tactical}$ is constructed from the implied returns of the tactical view $2\lambda \boldsymbol{\Sigma}_A \boldsymbol{w}_{active}$ by retaining only the rows for funds.

Hence, (A6) with no tactical views is:

$$\boldsymbol{w}_{\text{TAA}}^{f,no\ tactical} = \frac{1}{2\lambda}\boldsymbol{\Sigma}_f^{-1}\left(\bar{\boldsymbol{\mu}}_f^{alpha} + 2\lambda\ \bar{\boldsymbol{\mu}}_{\text{SAA}}^{f} - \delta_{funding}^{no\ views}\mathbf{1}\right) \tag{A51}$$

and with tactical views is:

$$\boldsymbol{w}_{\text{TAA}}^{f,with\ tactical} = \frac{1}{2\lambda}\boldsymbol{\Sigma}_f^{-1}\left(\bar{\boldsymbol{\mu}}_f^{tactical} + \bar{\boldsymbol{\mu}}_f^{alpha} + 2\lambda\ \bar{\boldsymbol{\mu}}_{\text{SAA}}^{f} - \delta_{funding}^{with\ views}\mathbf{1}\right) \tag{A52}$$

The difference between the two:

$$\begin{aligned}\Delta\boldsymbol{w}_{\text{TAA}}^{f,tactical} &= \boldsymbol{w}_{\text{TAA}}^{f,with\ tactical} - \boldsymbol{w}_{\text{TAA}}^{f,no\ tactical} \\ &= \frac{1}{2\lambda}\boldsymbol{\Sigma}_f^{-1}\left(\bar{\boldsymbol{\mu}}_f^{tactical} - \left(\delta_{funding}^{with\ tactical} - \delta_{funding}^{no\ tactical}\right)\mathbf{1}\right)\end{aligned} \tag{A53}$$

where:

$$\delta_{funding}^{with\ tactical} - \delta_{funding}^{no\ tactical} = \frac{\mathbf{1}^{\mathrm{T}}\boldsymbol{\Sigma}_f^{-1}\ \bar{\boldsymbol{\mu}}_f^{tactical}}{\mathbf{1}^{\mathrm{T}}\boldsymbol{\Sigma}_f^{-1}\mathbf{1}} \tag{A54}$$

In this example we consider one directional tactical view only, expressed on the benchmark index representing a given asset class *j,* so that:

$$\boldsymbol{w}_{active} = S_{directional} * RB\ \sigma_{index_j}^{-1}\begin{bmatrix} 0 \\ \cdots \\ 0 \\ 1 \\ 0 \\ \cdots \\ 0 \end{bmatrix} \quad \begin{matrix} \text{index}_1 \\ \cdots \\ \text{index}_{j-1} \\ \text{index}_j \\ \text{index}_{j+1} \\ \cdots \\ \text{fund}_{n_f} \end{matrix} \tag{A55}$$

spanning all benchmark indices and all funds. In this case, the vector $\bar{\boldsymbol{\mu}}_f$ of the implied returns from tactical views can be written as the product:

$$\bar{\boldsymbol{\mu}}_f^{tactical} = S_{directional} * \bar{\boldsymbol{\mu}}_{f,S_d=1}^{tactical} \tag{A56}$$

between the directional score $S_{directional}$ for the tactical view and $\bar{\boldsymbol{\mu}}_{f,S_d=1}^{tactical}$, the implied returns of a similar tactical view but with directional score $S_{directional} = 1$ in (A55), obtained from $2\lambda\boldsymbol{\Sigma}_A\boldsymbol{w}_{active}$ by retaining only the rows for funds. This means that the difference in (A54) is proportional to the tactical score:

$$(\delta_{funding}^{with\ tactical} - \delta_{funding}^{no\ tactical}) \propto S_{directional} \tag{A57}$$

And then, we can also write (A53) as the product between the directional score $S_{directional}$ and $\Delta\boldsymbol{w}_{\text{TAA},\ S_d=1}^{f,tactical}$ for $S_{directional} = 1$ as:

$$\Delta\boldsymbol{w}_{\text{TAA}}^{f,tactical} = S_{directional} * \Delta\boldsymbol{w}_{\text{TAA},\ S_d=1}^{f,tactical} \tag{A58}$$

The tracking error of the portfolio with tactical views relative to the SAA portfolio is, by definition:

$$\left(TE_{\boldsymbol{w}_{\text{TAA}}^{with\ tactical}}\right)^2 = \left(\boldsymbol{w}_{\text{TAA}}^{A,with\ tactical} - \boldsymbol{w}_{\text{SAA}}^{A}\right)^{\text{T}} \boldsymbol{\Sigma}_A \left(\boldsymbol{w}_{\text{TAA}}^{A,with\ tactical} - \boldsymbol{w}_{\text{SAA}}^{A}\right) \quad \text{(A59)}$$

using $\boldsymbol{w}_{\text{TAA}}^{A,with\ tactical} = \boldsymbol{w}_{\text{TAA}}^{A,no\ tactical} + \Delta\boldsymbol{w}_{\text{TAA}}^{A,tactical}$ and (A58):

$$\left(TE_{\boldsymbol{w}_{\text{TAA}}^{with\ tactical}}\right)^2 = \left(TE_{\boldsymbol{w}_{\text{TAA}}^{no\ tactical}}\right)^2 + S_{directional}^2 \left(\Delta\boldsymbol{w}_{\text{TAA},\,S_d=1}^{f,tactical}\right)^{\text{T}} \boldsymbol{\Sigma}_f\, \Delta\boldsymbol{w}_{\text{TAA},\,S_d=1}^{f,tactical}$$

$$+2\left(\Delta\boldsymbol{w}_{\text{TAA}}^{A,tactical}\right)^{\text{T}} \boldsymbol{\Sigma}_A \left(\boldsymbol{w}_{\text{TAA}}^{A,no\ tactical} - \boldsymbol{w}_{\text{SAA}}^{A}\right) \quad \text{(A60)}$$

because the rows for SAA assets are the same as zero for both $\Delta\boldsymbol{w}_{\text{TAA}}^{A,tactical}$ and $\boldsymbol{w}_{\text{TAA}}^{A,no\ tactical}$, the last term in (A60) can be simplified:

$$\left(\Delta\boldsymbol{w}_{\text{TAA}}^{A,tactical}\right)^{\text{T}} \boldsymbol{\Sigma}_A \left(\boldsymbol{w}_{\text{TAA}}^{A,no\ tactical} - \boldsymbol{w}_{\text{SAA}}^{A}\right) = \left(\Delta\boldsymbol{w}_{\text{TAA}}^{f,tactical}\right)^{\text{T}} \left(\boldsymbol{\Sigma}_f\, \boldsymbol{w}_{\text{TAA}}^{f,no\ tactical} - \overline{\boldsymbol{\mu}}_{\text{SAA}}^{f}\right)$$

$$= \frac{1}{2\lambda} \left(\Delta\boldsymbol{w}_{\text{TAA}}^{f,tactical}\right)^{\text{T}} \left(\overline{\boldsymbol{\mu}}_f^{alpha} - \delta_{funding}^{no\ tactical}\mathbf{1}\right) \quad \text{(A61)}$$

which, recalling that the sum of all elements of the vector $\Delta\boldsymbol{w}_{\text{TAA}}^{f,tactical}$ is zero, then simplifies further into:

$$\left(\Delta\boldsymbol{w}_{\text{TAA}}^{A,tactical}\right)^{\text{T}} \boldsymbol{\Sigma}_A \left(\boldsymbol{w}_{\text{TAA}}^{A,no\ tactical} - \boldsymbol{w}_{\text{SAA}}^{A}\right) = \frac{1}{2\lambda} \left(\Delta\boldsymbol{w}_{\text{TAA}}^{f,tactical}\right)^{\text{T}} \left(\overline{\boldsymbol{\mu}}_f^{alpha}\right) \quad \text{(A62)}$$

If $\overline{\boldsymbol{\mu}}_f^{alpha} \sim \mathbf{0}$, e.g. if γ is large enough, then (A60) is:

$$\left(TE_{\boldsymbol{w}_{\text{TAA}}^{with\ tactical}}\right)^2 \sim \left(TE_{\boldsymbol{w}_{\text{TAA}}^{no\ tactical}}\right)^2 + S_{directional}^2 \left(\Delta\boldsymbol{w}_{\text{TAA},\,S_d=1}^{f,tactical}\right)^{\text{T}} \boldsymbol{\Sigma}_f\, \Delta\boldsymbol{w}_{\text{TAA},\,S_d=1}^{f,tactical} \quad \text{(A63)}$$

Demonstrating that, under such conditions, the square of the tracking error of the allocation with tactical views is equal to the sum of the tracking error of the allocation without tactical views and a term which scales with the square of the directional view.

The result above also holds when more than one tactical view is considered by keeping all tactical views frozen but the one. It can also be extended to tactical views.

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