Belief Skewness in the Stock Market*

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Abstract

Belief skewness—the asymmetry in investors' cash-flow growth rate expectations—has a negative impact on the stock mean return, controlling for the average bias in beliefs and belief dispersion. When investors are sufficiently optimistic on average, however, the relationship reverses. Belief skewness also has a positive impact on the stock price and a negative impact on the stock volatility. To show this, we first develop a continuous-time general equilibrium model with heterogeneous investors having skewed beliefs. We then use analyst forecast data to construct belief skewness proxies, and verify the model implications for the aggregate market returns empirically.

Keywords: belief heterogeneity, belief skewness, disagreement, sentiment, asset pricing.

JEL classification: D53, G11, G12

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1 Introduction

Investor beliefs are central to asset pricing and characterized by large and persistent heterogeneity (Brunnermeier et al., 2021; Giglio et al., 2021). To summarize belief heterogeneity and analyze its asset pricing implications, the literature has focused on the average belief or bias in beliefs and belief dispersion or the standard deviation of beliefs. This literature has arguably culminated in the elegant and tractable model proposed by Atmaz and Basak (2018) that expresses the main asset pricing quantities as a function of the average belief and belief dispersion. Though important, these first two moments of the belief distribution offer an incomplete picture. Except for the knife-edge case of symmetry, the belief distribution is asymmetric, and belief skewness is a candidate variable to provide additional insights on the nature of heterogeneous beliefs. In this paper, we analyze the theoretical and empirical relationships between belief skewness and the stock market price, returns, and volatility.

To illustrate the importance of considering belief skewness, consider three economies in which a single risky stock is available for trading, each populated by 100 investors. All investors are identical except that they have different beliefs about the stock's expected long-term earnings growth rate. In the first economy, 50 investors believe that the stock's expected long-term earnings growth rate is 3% higher than its actual value, and 50 investors believe that it is 3% lower. In the second economy, 20 investors believe that the stock's expected long-term earnings growth rate is 6% higher than its actual value, and 80 investors believe that it is 1.50% lower. In the third economy, 80 investors believe that the stock's expected long-term earnings growth rate is 1.50% higher than its actual value, and 20 investors believe that it is 6% lower.

Remarkably, in all three economies, the investor-weighted average belief bias is 0% and the associated belief dispersion is 3%. Taking into account the first two belief moments only, one would conclude that belief heterogeneity is the same. However, this is clearly not the case. Beliefs are symmetric in the first economy, positively skewed with a belief skewness of 1.50 in the second economy, and negatively skewed with a belief skewness of

-1.50 in the third economy. Accordingly, the price of the risky asset will differ in each of these three economies.

To capture the main features of the example above and analyze the asset pricing implications of belief skewness, we extend the two-agent model in Atmaz and Basak (2018) to allow for skewed beliefs. Our main theoretical result is that, holding the average bias in beliefs and belief dispersion constant, a higher belief skewness leads to a lower mean stock return. However, this relation switches sign when the average bias is sufficiently large, and a higher belief skewness then leads to a higher mean stock return.

We further show that belief skewness has a positive impact on the stock price and a negative impact on the stock volatility. Our model also allows us to assess the relationships between the first two belief moments and stock characteristics and whether belief skewness affects these relationships. In line with the existing literature, we find that the average belief has a positive impact on the stock price, a negative impact on the stock mean return, and does not affect the stock volatility. Regarding the impact of belief dispersion, we find similar results as Atmaz and Basak (2018), which is not surprising as our model is close to theirs. Specifically, the impact of belief dispersion on the stock mean return is positive, but turns negative for a sufficiently high average belief bias. In both cases, the magnitude of the effect is higher when belief skewness is negative. Belief dispersion also has a negative impact on the stock price and a positive effect on the stock volatility.

We test the predictions of the model using aggregate market returns and analyst forecast data from the Institutional Brokers' Estimate System (I/B/E/S) database. Since Yu (2011) shows that market-level belief dispersion is best calculated with a bottom-up approach; that is, using the value-weighted average of firm-level belief dispersion, we similarly develop bottom-up proxies for belief skewness.[1] These proxies suggest that belief skewness is positive on average and time-varying.

[1] Yu (2011) provides two arguments to support his empirical finding. First, a bottom-up measure offers a better signal-to-noise ratio since it is constructed using thousands of forecasts, whereas there are only a few forecasts for the aggregate market. Second, only a bottom-up measure can properly capture within-market disagreement since investors can simultaneously agree on the market-level prospect but disagree on the firm-level prospects. These two arguments likewise apply to belief skewness.

When controlling for the belief bias and belief dispersion, we find that a one-standard-deviation increase in belief skewness reduces annualized market returns by about 3% to 4%. This relationship is consistent with the prediction of the model that higher belief skewness negatively predicts returns when the belief bias is not too high. Moreover, we find that the relationship between belief skewness and returns is state-dependent, consistent with our model predictions. The negative and significant relationship between belief skewness and returns turns positive and significant when the belief bias is sufficiently high. We further show that the belief-skewness-return relationship is not explained by previously proposed predictors of market returns.

While skewness in our analysis concerns investors' earnings growth rate expectations and not the skewness of earnings or returns, the two could be related.[2] For example, a high expected earnings growth rate of optimistic investors in the introductory example could reflect that these investors attach a positive likelihood to a very high earnings growth rate state. Thus, the skewness of expectations and the skewness of the underlying variable could be correlated. Empirically, we find some evidence that belief skewness positively predicts return skewness on short horizons.

Our study contributes to the literature on belief heterogeneity by studying belief skewness both theoretically and empirically. Our results regarding the average belief bias and belief dispersion confirm existing results. For example, Jouini and Napp (2008) prove that the risk premium is inversely related to the average belief. La Porta (1996) shows that stocks with higher earnings growth expectations underperform in the cross-section, and Campbell and Diebold (2009) provide evidence of market return predictability of the average belief. Regarding belief dispersion, two contrasting hypotheses compete. The first hypothesis posits that belief dispersion proxies for risk, and since investors should be compensated for bearing more risk, greater dispersion should lead to higher returns (e.g., Gebhardt et al., 2001). Conversely, Miller (1977) claims that short-sale constraints exclude pessimistic investors from the market because they want to sell the stock but are

[2]Return skewness has been the focus of many recent asset pricing studies such as Schneider (2019), Dew-Becker (2024), and Gormsen and Jensen (2025), among others.

unable to do so. Such a mechanism leads to overpriced stocks and lower returns. Since this effect increases with belief dispersion, more belief dispersion reduces stock returns. More recently, Atmaz and Basak (2018) reconcile these two competing predictions by showing that even in the absence of short-sales constraints, the relation between belief dispersion and returns can be negative if average beliefs are sufficiently high. Although the empirical evidence is mixed (Buraschi et al., 2014, for example, find that the sign of the relationship is leverage-dependent), most studies are in line with the second hypothesis in the cross-section of stocks (see, e.g., Diether et al., 2002; Golez and Goyenko, 2022; Bali et al., 2023; Goulding et al., 2024) and the time-series of market returns (see, e.g., Yu, 2011; Huang et al., 2021). Theoretical studies of interest include, among others, Scheinkman and Xiong (2003), Kogan et al. (2006), David (2008), Banerjee and Kremer (2010), Bhamra and Uppal (2014), Atmaz and Basak (2018), Beddock and Jouini (2021), and Jouini (2023); Daniel et al. (2024) provide a review on the effect of disagreement on asset prices, and Panageas (2020) provides a general review on heterogeneity in asset pricing.

To the best of our knowledge, only Meng (2015) and Colacito et al. (2016) study belief skewness. Meng (2015) shows that returns decrease with the dispersion in analyst forecasts when belief skewness is negative and increase with analyst forecast dispersion when belief skewness is positive in the cross-section of stocks. Meng rationalizes this empirical finding in a one-period model with risk-neutral investors that are rational on average. Colacito et al. (2016) study the impact of belief skewness at the market level by showing that the skewness of gross domestic product (GDP) short-term growth forecasts negatively predicts market returns. They use a model with a representative investor with Epstein and Zin (1989) preferences who dislikes negative skewness in the future utility profile and a skewed expected growth process for consumption growth to explain their findings. We complement their work by studying the impact of belief skewness in a standard heterogeneous belief setting close to Atmaz and Basak (2018) with expected utility investors and a dividend process that follows a standard geometric Brownian motion. On the empirical side, we use I/B/E/S data, as investors disagree about the long-term expected earnings or cash-flow growth rate in our theoretical model and not the short-term consumption growth rate. Analyst data has the additional advantage of monthly availability rather than semi-annual availability, as for the GDP growth rate forecast data. Finally, Goyal et al. (2024) successfully replicate the negative relation that Colacito et al. (2016) document, but do not find that this relation is significant.[3] Our empirical analysis likewise finds a negative relation that is significant when controlling for the first two beliefs moments. Moreover, the relation switches sign when the average belief bias is high, in line with the model predictions.

The paper proceeds as follows. In Section 2, we present the model and derive the theoretical relationships between belief moments and stock characteristics. Section 3 explains the construction of our proxies, and we discuss our empirical results in Section 4. We conclude in Section 5. Appendix A gathers all proofs, Appendix B reports the construction of the representative agent and the associated alternative belief moments, and Appendix C discusses the parameter values used in the computations for the figures. This paper is accompanied by an Online Appendix that contains robustness checks mentioned in the main text.

2 Model

In this section, we first outline the general equilibrium model in which investors' cash-flow expectations are skewed. We proceed to define the equilibrium belief moments and study how these belief moments affect market outcomes.

2.1 Setup

Our model builds on the two-agent model in Atmaz and Basak (2018). We use similar notations, but adapt the methodology to take belief skewness into account.

[3] Goyal et al. (2024) attribute the source of the diverging results to data differences. They were unable to obtain exactly the same data as Colacito et al. (2016) from the database vendors.

We consider a pure-exchange security market economy with a finite horizon T evolving in continuous time, and posit that we are at time $t < T$. In this economy, a single source of risk is present and represented by the Brownian motion ω defined on the objective probability measure $\mathbb{P}$.

A riskless bond and a risky stock are available for trading. The bond is in zero net supply and pays a riskless interest rate that is set to zero without loss of generality. The stock is in positive net supply of one unit and is a claim on the payoff D_T, which is paid at T. The arrival of news about this final payoff D_T is modelled via the cash-flow news process D_t with dynamics

$$dD_t = \mu D_t dt + \sigma D_t d\omega_t, \tag{1}$$

where $D_0 = 1$ and μ and $\sigma > 0$ are given constants.

The economy is populated by N equally-endowed investors having constant relative risk aversion (CRRA) preferences with a coefficient of relative risk aversion γ. As usually done in this literature (see, for instance, Dumas et al., 2017), we assume that γ is a positive integer. The investors have different beliefs about the dynamics of the cash-flow news process. While they agree on its volatility, they disagree on its drift. Specifically, there are two types of investors, each indexed by their bias in beliefs, θ_1 and θ_2. For $n = 1, 2$, investors with bias θ_n believe that the mean growth rate of the expected payoff is $\mu + \theta_n$ instead of μ. Without loss of generality, we further assume that $\theta_1 < \theta_2$ and thus call investors with bias θ_1 pessimists and those with bias θ_2 optimists. Because the model can accommodate any sign for θ_1 and θ_2, it simply means that pessimists are relatively more pessimistic, or stated differently less optimistic, than optimists.

Each investor chooses an optimal portfolio strategy; that is, the optimal fraction of wealth to be invested in the risky stock. Formally, an investor with bias θ_n maximizes[4]

$$\mathbb{E}_t\left[\eta_{n,T}\frac{W_{n,T}^{1-\gamma}}{1-\gamma}\right], \tag{2}$$

where the final consumption $W_{n,T}$ corresponds to the horizon value of the portfolio, and $\eta_{n,T} = \dfrac{d\mathbb{P}^{\theta_n}}{d\mathbb{P}}$ is the Radon-Nikodym derivative of the subjective measure $\mathbb{P}^{\theta_n}$ with respect to the objective measure $\mathbb{P}$, with the implied likelihood ratio process

$$\eta_{n,t} = \mathbb{E}_t\left[\eta_{n,T}\right] = \exp\left(\frac{\theta_n}{\sigma}\omega_t - \frac{1}{2}\frac{\theta_n^2}{\sigma^2}t\right). \tag{3}$$

Lastly, we do not impose an equal number of pessimists and optimists. Such a situation would lead to a symmetric belief distribution, as in the first economy of the introductory example. Conversely, having more pessimists leads to positive belief skewness, and more optimists leads to negative belief skewness. We thus leave the initial proportion of investors of each type unrestricted.

2.2 Equilibrium belief moments

The model equilibrium is obtained when all investors choose their optimal portfolio strategies and markets clear. The wealth-weighted proportions of each type of investors can then be computed explicitly as the present value of the equilibrium aggregate final consumption of each type divided by the present value of the total payoff (Cox and Huang, 1989).[5] Using these proportions, we can introduce the wealth-weighted belief moments in the next proposition.[6]

[4]For $\gamma = 1$, the objective function becomes $\mathbb{E}_t\left[\eta_{n,T}\ln\left(W_{n,T}\right)\right]$.

[5]As explained in the proof of Proposition 1 in Appendix A, we adapt the concept of exchange equilibrium with transfers introduced in Jouini (2023) to the case of two types of investors to compute the equilibrium. This approach does not require an ex-ante explicit expression for the wealth-weighted belief distribution, but rather starts from Lagrange multipliers that satisfy the first-order conditions. We then obtain the proportion of investors of each type as a function of the Lagrange multipliers.

[6]As shown in Appendix B, we can alternatively define the belief moments based on the representative agent as in Atmaz and Basak (2018). We rather adopt the wealth-weighted approach because it is in the spirit of the introductory example and closer to our empirical proxy. Thus, our notations for the belief moments differ from those in Atmaz and Basak (2018). Specifically, we denote the wealth-weighted average bias, belief dispersion, and belief skewness by m, v, and s, respectively, and we use $\tilde{m}$, $\tilde{v}$, and

Proposition 1 (Belief moments) *The average bias in beliefs, the belief dispersion, and the standardized belief skewness are given by*

$$m_t = \theta_1 + (\theta_2 - \theta_1)(1 - \nu_{1,t}), \tag{4}$$

$$v_t = (\theta_2 - \theta_1)\sqrt{\nu_{1,t}(1 - \nu_{1,t})}, \tag{5}$$

$$s_t = \frac{2\nu_{1,t} - 1}{\sqrt{\nu_{1,t}(1 - \nu_{1,t})}}, \tag{6}$$

respectively, where $\nu_{1,t}$ is the proportion of pessimists given by

$$\nu_{1,t} = \frac{\sum_{k=0}^{\gamma} \frac{k}{\gamma} G_{t,k}}{\sum_{k=0}^{\gamma} G_{t,k}}, \tag{7}$$

where

$$G_{t,k} = \binom{\gamma}{k} \left(\frac{\frac{y_1}{\eta_{1,t}}}{\frac{y_2}{\eta_{2,t}}}\right)^{-\frac{k}{\gamma}} \exp\left(-\frac{1}{2}\left(\frac{k}{\gamma\sigma^2}\left(1 - \frac{k}{\gamma}\right)(\theta_2 - \theta_1)^2 - 2k\left(1 - \frac{1}{\gamma}\right)(\theta_2 - \theta_1)\right)(T - t)\right), \tag{8}$$

and y_1 and y_2 are the Lagrange multipliers of the aggregation of all pessimists and optimists, respectively, given by

$$y_n = f_n(\alpha)\exp((1 - \gamma)\theta_n T), \text{ for } n = 1, 2, \tag{9}$$

where f_n is a function of a free parameter α.

The three belief moments depend on the bias of pessimists, θ_1, the bias difference between optimists and pessimists, $\theta_2 - \theta_1$, and the share of pessimists, $\nu_{1,t}$, which is stochastic. Thus, for a given state of the world, it is possible to construct economies similar to those in the introductory example that match a given average belief bias, belief dispersion, and belief skewness.[7] For instance, we can numerically find sets of values for

$\tilde{s}$ for their representative-agent-based counterparts. Atmaz and Basak (2018) do the opposite.

[7]We have that $\nu_{1,t}$ is stochastic as it depends on $\eta_{1,t}$ and $\eta_{2,t}$ (via Equation (8)) that depend on ω_t. Moreover, we notice that $\nu_{1,t}$ depends on θ_1 and θ_2, but also on the parameter α (via y_1 and y_2 given in Equation (9)). Thus, for a given ω_t, we have three free parameters θ_1, θ_2, and α to match three

θ_1, $\theta_2 - \theta_1$, and $\nu_{1,t}$ so that economies have the same levels of average belief bias and belief dispersion but differ in belief skewness. This methodology allows us to study the impact of a given belief moment, controlling for the two others. We thereby build on Ebert (2015) who shows that the moment-based and tail-based notions of skewness and asymmetry are equivalent for binary risks. This is a crucial feature since a change in the third moment necessarily changes other moments. For binary risks, an increase in skewness that leaves the mean and variance unchanged likewise increases all odd moments. Since all odd moments proxy for skewness, we indeed identify skewness effects in our comparative statics analysis.

Finally, it is important to point out that, although we only consider two types of investors, we choose the functions f_1 and f_2 in Equation (9) so that, for $\omega_t = 0$ and any belief moment values, $\nu_{1,t}$ is never too high nor too low, thus guaranteeing an effective belief heterogeneity in the economy. In other words, we make sure that the economies we study are never economies in which either the pessimists or the optimists control all the wealth, which could lead to irregular behaviours as underlined in Atmaz and Basak (2018). Formally, we impose $\alpha \in]0, 1[$ and choose $f_1(\alpha) = 1/\alpha$ and $f_2(\alpha) = 1/(1-\alpha)$.

2.3 Stock price and its dynamics

In this section, we study the impact of heterogeneous beliefs on the stock price, mean return, and volatility. For each characteristic, we first provide the formulas for the general case in the text and then provide analytical results in the propositions assuming logarithmic utility. We verify numerically that the results with logarithmic utility hold for larger values of relative risk aversion and illustrate such cases with figures.

belief moments. In the remainder of our analysis, we fix $\omega_t = 0$ without loss of generality. We then numerically find values of θ_1, θ_2, and α that match given values of m_t, v_t, and s_t, and obtain the implied value for $\nu_{1,t}$.

2.3.1 Equilibrium stock price

Using the terminology of Cochrane (2001), the stock price is given by the basic pricing equation applied to a continuous-time framework with terminal-date consumption. Simple algebra yields

$$S_t = \exp\left(\left(\mu - \gamma\sigma^2\right)(T-t)\right) D_t \frac{\sum_{k=0}^{\gamma} G_{t,k} \exp\left(\theta_2 (T-t)\right)}{\sum_{k=0}^{\gamma} G_{t,k} \exp\left(\frac{k}{\gamma}\left(\theta_2 - \theta_1\right)(T-t)\right)}. \tag{10}$$

The stock price in the presence of belief heterogeneity is given by the stock price with no belief heterogeneity, $\exp\left(\left(\mu - \gamma\sigma^2\right)(T-t)\right) D_t$, multiplied by a term that depends on the biases of pessimists and optimists, their difference, and the share of pessimists.

In Proposition 2, we show the stock price assuming investors have logarithmic preferences; that is, assuming $\gamma = 1$. This assumption allows us to directly express the stock price as a function of the belief moments and to analytically study the impacts of these moments. Importantly, this case yields similar implications as the general CRRA case, as shown later.

Proposition 2 (Stock price) *Assume that investors have logarithmic preferences. The stock price is given by*

$$S_t = \exp\left(\left(\mu - \sigma^2\right)(T-t)\right) D_t \frac{2\sqrt{4+s_t^2}\left(\sqrt{4+s_t^2}+s_t\right)\exp\left(\left(m_t + \frac{v_t\left(\sqrt{4+s_t^2}+s_t\right)}{2}\right)(T-t)\right)}{4+\left(\sqrt{4+s_t^2}+s_t\right)^2 \exp\left(v_t\sqrt{4+s_t^2}\,(T-t)\right)}. \tag{11}$$

In the presence of belief heterogeneity,

(i) the stock price increases with the average bias in beliefs,

(ii) the stock price decreases with belief dispersion,

(iii) the stock price increases with belief skewness.

The more optimistic the investors are on average about the future payoff of the stock, the more they are willing to pay for it, which implies a positive relation between the stock price and the average bias in beliefs. The second item follows from the fact that higher belief dispersion leads to extra uncertainty and investors thus require a premium to buy the stock, which leads to lower prices. Finally, the stock price increases with belief skewness because higher belief skewness implies less risk, as investors reduce the subjective probability of very low expected cash-flow growth and increase the subjective probability of very high expected cash-flow growth.

To illustrate our results further and show that they generalize to the CRRA case with $\gamma > 1$, we compare the stock price in economies differing in belief heterogeneity in Figure 1. The figure is obtained for a risk aversion of $\gamma = 2$.[8] Specifically, we consider unbiased and highly-positively-biased economies, average-belief-dispersion and high-belief-dispersion economies, and symmetric, positively-skewed, and negatively-skewed economies. These economies are constructed using empirical moment values given in Table 1. Appendix C explains the construction methodology in detail and how the other parameters are chosen. Average-belief-dispersion and high-belief-dispersion economies are represented by black and grey lines, respectively. Symmetric, positively-skewed, and negatively-skewed economies are represented by solid, plus-signed, and dashed lines, respectively.

Fixing two belief moments, we vary the other moment and observe its impact; that is, we study the impact of a given belief moment controlling for the two others.

Insert Figure 1 here.

Panel A of Figure 1 confirms the positive impact of the average bias in beliefs on the stock price. Moreover, the impact is stronger for economies with lower belief dispersion and more positive belief skewness.[9] As this is true for all levels of average belief bias, we focus on unbiased economies in the other panels for the sake of brevity.

[8] Choosing alternative values of γ does not affect our results qualitatively, although the magnitude of the effects tends to decrease for higher values of risk aversion.

[9] The first effect is observed from the comparison of the black and grey lines, and the second one from the comparison of the solid, plus-signed, and dashed lines. We confirm the second effect in unreported graphs with economies having a higher level of belief dispersion and differing in belief skewness.

In Panel B of Figure 1, we look at the impact of belief dispersion in unbiased symmetric, positively-skewed, and negatively-skewed economies. In Panel C, we focus on the impact of belief skewness in unbiased average-belief-dispersion and high-belief-dispersion economies. In line with our previous observations, we find that the stock price decreases with belief dispersion, and increases with belief skewness. Moreover, it tends to decrease more with belief dispersion in negatively-skewed economies, and to increase more with belief skewness in high-belief-dispersion economies.

2.3.2 Equilibrium mean return and volatility

We now turn to the study of the stock dynamics. Applying Ito's lemma to the stock price, given in Equation (10), yields the stock mean return, μ_{S_t}, and volatility, σ_{S_t}, in the presence of belief heterogeneity

$$\mu_{S_t} = \left(\sigma + \frac{\theta_2 - \theta_1}{\sigma}\left(\frac{\sum_{k=0}^{\gamma}\frac{k}{\gamma}G_{t,k}\exp\left(\frac{k}{\gamma}\left(\theta_2-\theta_1\right)\left(T-t\right)\right)}{\sum_{k=0}^{\gamma}G_{t,k}\exp\left(\frac{k}{\gamma}\left(\theta_2-\theta_1\right)\left(T-t\right)\right)} - \nu_{1,t}\right)\right)$$
$$\times\left(\gamma\sigma + \frac{\theta_2-\theta_1}{\sigma}\frac{\sum_{k=0}^{\gamma}\frac{k}{\gamma}G_{t,k}\exp\left(\frac{k}{\gamma}\left(\theta_2-\theta_1\right)\left(T-t\right)\right)}{\sum_{k=0}^{\gamma}G_{t,k}\exp\left(\frac{k}{\gamma}\left(\theta_2-\theta_1\right)\left(T-t\right)\right)} - \frac{\theta_2}{\sigma}\right), \qquad (12)$$

and

$$\sigma_{S_t} = \sigma + \frac{\theta_2-\theta_1}{\sigma}\left(\frac{\sum_{k=0}^{\gamma}\frac{k}{\gamma}G_{t,k}\exp\left(\frac{k}{\gamma}\left(\theta_2-\theta_1\right)\left(T-t\right)\right)}{\sum_{k=0}^{\gamma}G_{t,k}\exp\left(\frac{k}{\gamma}\left(\theta_2-\theta_1\right)\left(T-t\right)\right)} - \nu_{1,t}\right). \qquad (13)$$

The first term in the expression of the stock mean return in Equation (12) is the stock volatility and the second term is the market price of risk, MPR_t; that is, the expected excess return per unit of volatility. With simple algebra, we can express the market price of risk as

$$\mathrm{MPR}_t = \sigma_{S_t} + (\gamma - 1)\,\sigma - \frac{m_t}{\sigma}. \qquad (14)$$

All else equal, a sufficiently high average bias in beliefs can result in a negative market price of risk. Thus, when investors are overly optimistic, the stock mean return can become negative. As shown below, a sufficiently negative market price of risk explains why the effects of belief dispersion and belief skewness change when investors are very optimistic on average.

As for the stock price, the case of logarithmic preferences allows us to express the stock mean return and volatility as functions of the belief moments and to analytically derive the impacts of belief moments. We derive this specific case in Propositions 3 and 4, and start with stock volatility as the stock mean return depends on it. Figures 2 and 3 further show that the effects of belief moments derived in the propositions generalize to the CRRA case.

Proposition 3 (Stock volatility) *Assume that investors have logarithmic preferences. The stock volatility is given by*

$$\sigma_{S_t} = \sigma + \frac{2v_t\left(\sqrt{4+s_t^2}+s_t\right)\left(\exp\left(v_t\sqrt{4+s_t^2}\,(T-t)\right)-1\right)}{\sigma\left(4+\left(\sqrt{4+s_t^2}+s_t\right)^2\exp\left(v_t\sqrt{4+s_t^2}\,(T-t)\right)\right)}. \tag{15}$$

In the presence of belief heterogeneity,

(i) the stock volatility does not depend on the average bias in beliefs,

(ii) the stock volatility increases with belief dispersion,

(iii) the stock volatility decreases with belief skewness.

We notice that Equation (15) does not depend on m_t. Thus, the first moment of belief heterogeneity does not affect stock volatility. Nevertheless, heterogeneous beliefs still impact stock volatility positively through dispersion and negatively through asymmetry. We further investigate these relationships in Figure 2 that is constructed in the same fashion as Figure 1.

Insert Figure 2 here.

First, Panel A corroborates the independence of stock volatility from the average belief bias. We thus focus on unbiased economies in Panels B and C. In line with the previous literature, Panel B confirms that the stock volatility increases with belief dispersion. Introducing skewed beliefs, we further observe that this positive effect is more pronounced for negatively-skewed economies. Lastly, we see in Panel C that belief skewness has a negative impact on the stock volatility, and that the impact is stronger for high-belief-dispersion economies.

Let us now turn to the stock mean return in the logarithmic case and then show graphically that the results generalize to CRRA utility functions with $\gamma > 1$.

Proposition 4 (Stock mean return) *Assume that investors have logarithmic preferences. The stock mean return is given by*

$$\mu_{S_t} = \sigma_{S_t} \left(\sigma_{S_t} - \frac{m_t}{\sigma} \right). \tag{16}$$

In the presence of belief heterogeneity,

(i) the stock mean return decreases with the average bias in beliefs,

(ii) the stock mean return increases with belief dispersion when $m_t < 2\sigma\sigma_{S_t}$, and decreases with belief dispersion otherwise,

(iii) the stock mean return decreases with belief skewness when $m_t < 2\sigma\sigma_{S_t}$, and increases with belief skewness otherwise.

In line with the literature (see, for instance, Jouini and Napp, 2008), our model yields a stock mean return that decreases with the average bias in beliefs. As explained in Atmaz and Basak (2018), this is directly related to the increasing stock overpricing in the presence of increased average optimism. Regarding belief dispersion and belief skewness, their impact depends on the level of the average belief bias. The optimism level for which the sign of their impact switches, given by $2\sigma\sigma_{S_t}$, is always positive. In other words, the stock mean return increases with belief dispersion and decreases with belief skewness in unbiased (or negatively-biased) economies, but the situations reverses when investors are sufficiently optimistic on average. This high level of optimism ensures that the market price of risk given in Equation (14) is sufficiently negative such that a higher dispersion and a higher belief skewness reduce the stock mean return. The level of optimism for which this sign-switch occurs increases with stock volatility. All else equal and in line with the observations of Proposition 3, this optimism threshold thus increases with belief dispersion and decreases with belief skewness.

We verify these results for the general CRRA case in Figure 3. As suggested by the second and third item of Proposition 4, the impact of belief dispersion and belief skewness reverses for highly-positively-biased economies. Thus, in Panels B and C, we consider unbiased (left sub-figure) and highly-positively-biased (right sub-figure) economies separately.

Insert Figure 3 here.

Panel A supports the negative relation between the stock mean return and the average belief bias for all levels of belief dispersion and belief skewness. Regarding belief dispersion, the two sub-figures in Panel B confirm that the sign of the relation depends on the level of the average belief bias. The fact that the relation is decreasing then increasing in the right sub-figure further suggests that the level of optimism needed to observe a negative relation increases with belief dispersion. The level of average optimism considered in this sub-figure is thereby not high enough for high levels of belief dispersion. In addition, we notice that the stock mean return increases more with belief dispersion in negatively-skewed economies, and that the magnitude of the impact of belief skewness tends to increase with belief dispersion. Lastly, we observe a relation whose sign depends on the level of the average belief bias in Panel C. Comparing the slope of the black and grey lines, the magnitude of the impacts of belief skewness increases with belief dispersion.

3 Belief moment proxies

We use analyst forecasts of the long-term earnings growth rate from the summary statistics file of I/B/E/S to construct our belief moment proxies. Long-term earnings growth rates have two advantages. First, they directly relate to our theoretical framework in which investors disagree on the cash-flow growth rate. Second, they are comparable across firms and time and have been used by prior work, most notably Yu (2011), to construct average belief and belief dispersion proxies.

Our proxies are constructed bottom-up with data from December 1981 to December 2023. We first compute belief moments at the stock level and then aggregate each of them at the market level. This approach follows Yu (2011), who shows that bottom-up disagreement is a better proxy for market-level belief dispersion than top-down disagreement.

Since many different measures of skewness and asymmetry have been proposed in the literature, we use two different skewness measures in our empirical analysis. These measures are robust skewness measures as studied in Kim and White (2004). For each stock i in month t, the mean-median skewness proxy is based on

$$s_{i,t,1} = \frac{m_{i,t} - q_{i,t}}{v_{i,t}}, \tag{17}$$

where $m_{i,t}$, $q_{i,t}$, and $v_{i,t}$ are the long-term earnings growth rate's average, median, and standard deviation, respectively. The second proxy is in the vein of quantile skewness measures. However, since most firms are followed only by few analysts, we do not use the quartile of the distribution but rather the maximum and minimum forecasts.[10] For each stock i in month t, we compute

$$s_{i,t,2} = \frac{\max_{i,t} + \min_{i,t} - 2 \times q_{i,t}}{\max_{i,t} - \min_{i,t}}, \tag{18}$$

[10] When four analysts or less follow a stock, the maximum and minimum forecasts coincide with the 25th and 75th percentiles. When more analysts follow a stock, our skewness measure based on the maximum, minimum, and median has the advantage of using only data available in the I/B/E/S summary file.

where $\max_{i,t}$ and $\min_{i,t}$ are the long-term earnings growth rate's highest and lowest forecast, respectively. When $v_{i,t} = 0$ (and thus $\max_{i,t} = \min_{i,t}$), we set $s_{i,t,1}$ and $s_{i,t,2}$ to zero. Moreover, Hotelling and Solomons (1932) show that $s_{i,t,1}$ is bounded by one in absolute value. It can likewise be shown that max-min skewness $s_{i,t,2}$ is bounded by one in absolute value.

Our belief skewness proxies are the cross-sectional value-weighted averages of the stock-level belief skewness measures for each month.[11] To be precise, for $j = 1, 2$, we have

$$s_{t,j} = \frac{\sum_i \text{MKTCAP}_{i,t} \times s_{i,t,j}}{\sum_i \text{MKTCAP}_{i,t}}, \tag{19}$$

where $\text{MKTCAP}_{i,t}$ denotes the market cap of stock i at the end of month t. We likewise construct proxies for the average belief[12]

$$m_t = \frac{\sum_i \text{MKTCAP}_{i,t} \times m_{i,t}}{\sum_i \text{MKTCAP}_{i,t}}, \tag{20}$$

and belief dispersion

$$v_t = \frac{\sum_i \text{MKTCAP}_{i,t} \times v_{i,t}}{\sum_i \text{MKTCAP}_{i,t}}. \tag{21}$$

Table 1 presents the summary statistics of the proxies for the first three belief moments. The summary statistics suggest that analysts expect earnings per share to grow by 13.08% on average. The forecast standard deviation is 3.45%. These values are comparable to those in Yu (2011), who uses the same sample up to 2005. Our two skewness proxies suggest that forecasts are positively skewed on average, and that skewness varies significantly over time taking negative and positive values. The two skewness proxies show very strong positive correlation with each other at 0.90, and weak negative correlation with the first two belief moments. Thus, the information contained in belief skewness

[11] We winsorize the stock-level variables at the 1% and 99% percentiles at each date before taking the value-weighted averages.

[12] Since the drift μ of the cash-flow news process in (1) is fixed, there is a one-to-one relation between the average belief and the average belief bias in the model. We use average beliefs for simplicity in the empirical analysis. In addition, we show in robustness tests with a sentiment index as an alternative variable and controls for objective cash-flow expectations that the average belief likely also proxies for the average belief bias.

is unlikely to be subsumed by the average belief and belief dispersion.

The high positive correlation between the two belief skewness proxies is also apparent in Figure 4, which plots their six-month moving averages. The figure confirms the pronounced time-variation of belief skewness. Periods of low belief skewness correspond to the second part of the global financial crisis and the outbreak of the coronavirus disease pandemic, whereas belief skewness was high, among others, in 1988 and 2013.

Insert Table 1 and Figure 4 here.

4 Empirical results

In this section, we progress in three stages as we test the predictions of our model and detail our key findings. First, we analyze the relation between belief moments and market returns, and examine whether the relation between belief skewness and market returns depends on the average belief or belief bias. Then, we show that our findings are robust to alternative explanations and control variables. Finally, we examine the relation between belief moments and the volatility and skewness of returns.

4.1 Belief moments and market returns

We use the S&P 500 total return and risk-free rate obtained from Amit Goyal's website to test the theoretical predictions.[13] We first investigate the predictive power of the first three moments of the belief distribution with the following univariate specification

$$R_{t+1\to t+K} = g_0 + g_1 X_t + \epsilon_{t+1\to t+K}, \tag{22}$$

where $R_{t+1\to t+K}$ is the annualized cumulative log excess return of the S&P 500 total return index from month $t+1$ to month $t+K$ for $K = 1, 3, 12$, and X_t is the belief moment proxy.

[13]See `https://sites.google.com/view/agoyal145`.

Table 2 presents the results, which can be summarized as follows. First, in line with the model, the average belief predicts market returns negatively. This relationship is economically significant at all horizons and statistically significant at the three-month and one-year horizons. A one-standard-deviation increase in the average belief is associated with an annualized reduction of realized market returns that ranges from 4% to 6% per year. Second, belief dispersion predicts returns negatively, but the coefficient is significant only at the one-year horizon. The negative relationship is consistent with the empirical results in Yu (2011). Finally, belief skewness predicts returns with a negative sign on average, consistent with the model predictions when the average belief bias is not too high, but the relation is not statistically significant.

Insert Table 2 here.

A key feature of our theoretical analysis is to vary only one moment at a time to identify the relationship between a given moment and expected returns. Since Table 1 shows that belief moments are correlated empirically, we next estimate the multivariate counterpart of Equation (22)

$$R_{t+1\to t+K} = g_0 + g_1 m_t + g_2 v_t + g_3 s_{t,j} + \epsilon_{t+1\to t+K}. \tag{23}$$

The slope on a given belief moment in (23) now corresponds to the moment's marginal effect on market returns when holding the other two moments constant. Table 3 reports the results.

Insert Table 3 here.

The results of the multivariate regression confirm the negative and significant relationship between average beliefs and market returns for the three-month and one-year horizons and belief dispersion and market returns at the one-year horizon documented in the previous table. More importantly, the relation between belief skewness and returns controlling for the first two moments now produces economically larger results for all horizons and significant results at the three-month and one-year horizons. A one-standard-deviation increase in skewness is associated with an annualized reduction in market returns that ranges from 3% to 4% across horizons and belief skewness proxies. These results suggest that controlling for the first and second moments is important to detect skewness effects and suggest a negative relation between belief skewness and market returns—consistent with the model when the average belief is not too high.

We use overlapping data to avoid loss of information. Since the results are more pronounced at the three-month and one-year horizons, we conduct robustness checks summarized in Table OA.1 and Sections A.1 and A.2 of the Online Appendix to ensure that our inference is accurate. First, we show that the negative relation between belief skewness and returns remains statistically significant at the longer horizons with Hansen and Hodrick (1980) and Hodrick (1992) standard errors. Second, we use the specification proposed in Hodrick (1992) to estimate long-horizon effects without overlapping data. In this setting, we also validate our inference with bootstrap simulations and show that the bias inherent in predictive regressions demonstrated by Stambaugh (1999) is small in our setting, especially for the slope coefficients on the belief skewness proxies.

4.2 State-dependent relationship between belief skewness and market returns

In the model, the relationship between belief skewness and the stock mean return is state-dependent and negative only when the average belief is not too high. When the average belief is high enough, the relationship switches sign and higher belief skewness is associated with a higher stock mean return. For $j = 1, 2$, we test this prediction with the following specification

$$R_{t+1\to t+K} = g_0 + g_1 \mathbb{1}_{m_t > \text{perc}} + g_2 s_{t,j} + g_3 \mathbb{1}_{m_t > \text{perc}} \times s_{t,j} + \epsilon_{t+1\to t+K}, \tag{24}$$

where $\mathbb{1}_{m_t > \text{perc}}$ is an indicator variable equal to one if m_t is above a chosen percentile and zero otherwise. Since the model predicts that the belief-skewness-return relationship switches sign only when the average belief is very high, we use the 90th percentile of average beliefs for perc in the baseline analysis. In a robustness check, we also set perc equal to the 75th percentile. Table 4 presents the results.

Insert Table 4 here.

The results support the state-dependent role of belief skewness for returns when the threshold for the average belief is the 90th percentile. As shown in Panel A of Table 4, when the average belief is below its 90th percentile, the relation between belief skewness and market returns is negative and now significant for all specifications and horizons. Moreover, the interaction of the high average belief indicator and belief skewness is positive and significant in all the specifications and at all horizons. When combined with the slope coefficient on belief skewness, the slope on the interaction implies that the relation between belief skewness and returns is positive when the average belief is above its 90th percentile. The corresponding results are reported in the row $g_2 + g_3$. The relation is positive for all horizons and statistically significant in all but one case.

The results reported in Panel B of Table 4 using the 75th percentile of average beliefs to define the indicator variable yields consistent but weaker and mostly statistically insignificant results. Thus, the data supports the theoretical prediction that the belief-skewness-return relationship switches sign only when average beliefs are very high.

In the Online Appendix, we provide additional robustness checks to corroborate the average-belief-dependent role of belief skewness for returns. Table OA.2 in the Online Appendix summarizes the robustness checks with Hansen and Hodrick (1980) and Hodrick (1992) standard errors and the Hodrick (1992) specification. The results are generally consistent with those presented in the main text and the evidence tends to be strongest with the mean-median belief skewness proxy—just as the magnitude of the t-statistics in Panel A of Table 4 already suggests.

We also show that the state-dependent relation between belief skewness and returns is robust to using alternative definitions for very-high-belief-bias states. As an alternative proxy for excessive optimism, we use Baker and Wurgler's (2006) sentiment index orthogonalized to economic conditions. The results are presented in Table OA.3 of the Online Appendix. Whether we simply use the most recent value of the sentiment index or the 18-month average as in Huang et al. (2021) to define the dummy variable, the estimates in Table OA.3 confirm the state-dependent relation in Panel A of Table 4, but the results are not always statistical significant—especially for the 18-month average sentiment.

In sum, the results show that the belief-skewness-return relationship is state-dependent as predicted by the model. This state-dependent relationship might thereby explain why the unconditional specification studied in the previous section is sometimes unable to detect belief skewness effects.

4.3 Alternative hypotheses and more control variables

In this section, we study whether alternative variables drive the relation between belief moments and the equity premium documented in Sections 4.1 and 4.2. We first augment the specification in (23) to include the following control variables one-by-one: earnings-price ratio, dividend-price ratio, book-to-market ratio, term spread, default yield spread, inflation, net equity expansion, stock variance, Lettau and Ludvigson's (2001) consumption-wealth ratio, Jondeau et al.'s (2019) average stock skewness, Kelly and Jiang's (2014) cross-sectional tail risk, Yu's (2011) analyst disagreement, and a partial least squares aggregation of all control variables following Kelly and Pruitt (2013). The caption of Table 5, Goyal and Welch (2008), and Goyal et al. (2024) contain the details and references for these variables. All variables are from Amit Goyal's website and available at the monthly frequency, except the consumption-wealth ratio for which we use the most recent available value in the monthly regressions.

Insert Table 5 here.

Table 5 presents the results using mean-median belief skewness in Panel A and max-min belief skewness in Panel B. Our findings are robust to the inclusion of the different control variables. Belief skewness predicts returns with a negative sign at all horizons and the coefficients are statistically significant at the 3-month and 12-month horizons. Control variables such as the earnings-price ratio, for which high values proxy for low earnings growth expectations, reduce the magnitude of the average belief coefficient slightly and to insignificance only in Panel B at the one-month horizon. Thus, the predictive power of average beliefs likely stems from its belief bias component, which verifies the suitability of our measures to proxy for the average belief bias.

While belief skewness in our study concerns the expected mean cash-flow growth rate, it could relate to measures of return skewness empirically. In particular, our model with dogmatic beliefs is silent on the origins of investors' beliefs. A very pessimistic forecast, for example, could reflect that the investor attaches a higher likelihood to extremely low growth states. Thus, belief skewness could relate to the average stock skewness and cross-sectional tail risk control variables. In unreported robustness checks, we find that these two measures have a low positive correlation with the mean-median and max-min belief skewness measures. The correlations range from 0.09 to 0.19. Including these control variables also does not alter the inference about the belief-skewness-return relationship.

The penultimate control variable is analyst disagreement of Yu (2011) calculated by Goyal et al. (2024). Since Yu (2011) and Goyal et al. (2024) do not restrict their attention to firms followed by at least three analysts among others, this variable is more volatile than our belief dispersion variable v. Nevertheless, the two proxies have a correlation of 0.92. To avoid multicollinearity issues, we do not include v in the specification with this control variable. The results suggest that the relation between belief dispersion and returns is negative but not statistically significant at the different horizons.

The last control variable is a partial least squares aggregation of all twelve control variables following Kelly and Pruitt (2013). The partial least squares method effectively extracts the information contained in the control variable for subsequent returns. It yields the highest adjusted R^2, but leaves our main results unaffected.

We next include the control variables in the regression specification in (24) to see whether the control variables subsume the state-dependent relation between belief skewness and returns documented in Table 4. We focus on the 90th percentile threshold for the average belief, which is in line with the model predictions. Table 6 presents the results, which confirm the ones in Table 4. In particular, mean-median belief skewness predicts a significantly negative belief-skewness-return relation when the average belief is not too high and a significantly positive belief-skewness-return relation when the average belief is high for all horizons. Max-min belief skewness predicts a qualitatively similar relation that is usually marginally significant at the one-month horizon, insignificant at the 3-month horizon, and significant at the 12-month horizon. These results mirror the ones for that proxy in Table 4.

Insert Table 6 here.

Taken together, we conclude that the multivariate relation between belief skewness and returns documented in Section 4.1 and the state-dependent relation between belief-skewness and returns documented in Section 4.2 are robust to the inclusion of control variables.

4.4 Belief moments and return volatility

In this section, we examine the link between belief moments and stock return volatility. For $j = 1, 2$, we estimate the following regression:

$$\sigma_{t+1\to t+K} = g_0 + g_1 m_t + g_2 v_t + g_3 s_{t,j} + g_4 \sigma_{t-K+1\to t} + \epsilon_{t+1\to t+K}, \tag{25}$$

where $\sigma_{t+1\to t+K}$ is the annualized volatility of daily returns of the S&P 500 total returns in month $t+1$ to month $t+K$ and $\sigma_{t-K+1\to t}$ is its lagged value. In the model, return volatility increases with disagreement, decreases with belief skewness, and does not depend on the belief bias.

Panel A of Table 7 presents the results of the estimation of Equation (25) without the lagged volatility control variable. In line with the model, the relation between disagreement and volatility is positive and significant for all horizons. However, the results do not support the negative relationship between belief skewness and volatility. The estimated coefficients on belief skewness are mostly positive but insignificant. The results for the average belief are mixed since the results suggest a positive but usually insignificant average-belief-volatility relation.

Insert Table 7 here.

The results in Panel B of the table show that controlling for lagged volatility increases the adjusted R^2 from about 2% to 37% at the one-month horizon. Thus, it is perhaps unsurprising that lagged volatility also subsumes the predictive power of disagreement for return volatility. The slope on disagreement is still positive but no longer statistically significant. Including lagged volatility as a control variable also tends to strengthen the evidence for positive average-belief-volatility and belief-skewness-volatility relations.

Taken together, we conclude that the results provide only weak support for the disagreement-volatility prediction of the model and do not support the belief-skewness-volatility and belief-bias-volatility predictions.

4.5 How does belief skewness relate to return skewness?

Belief skewness is not related to return skewness per se since these two concepts concern different quantities—growth forecasts and stock returns. Our model is not able to relate them either since it is set in a geometric Brownian motion framework. Nevertheless, one might ask whether belief skewness and return skewness are related empirically. We investigate this question in this section.

We measure return skewness using the coefficient of skewness as well as the mean-median skewness formula in Equation (17) applied to the daily return of the S&P 500 total returns over a K-month period. The unreported correlation between the two return skewness proxies is 0.55 for the one-month horizon, which highlights the empirical challenge to measure return skewness. We regress each proxy calculated over a K-month horizon on lagged values of the belief moments and the lagged dependent variable in a specification similar to the one in (25), replacing $\sigma.$ with the return skewness proxies. Table 8 reports the results.

Insert Table 8 here.

At the one-month horizon, belief skewness relates positively to both return skewness measures and the relation is statistically significant for mean-median return skewness.[14] At longer horizons, the slope on belief skewness is insignificant but remains positive for mean-median return skewness. For the coefficient of skewness, the relation is also insignificant but switches sign. Moreover, the table does not suggest that belief dispersion relates to return skewness. The average belief is positively related to future return skewness, especially at long horizons. Past skewness controls are included for completeness in all specifications, but they are often insignificant and only marginally improve the adjusted R^2.

Since return skewness is inherently hard to measure, we consider three alternative skewness proxies in Table OA.4 of the Online Appendix. A max-min skewness measure applies Equation (18) to daily returns, and two quantile-based skewness measures use the 10th and 90th percentiles and the 25th and 75th percentile instead of the minimum and maximum in that formula. These additional return skewness proxies confirm a positive relationship between belief skewness and return skewness at the one-month horizon but no reliable relation at longer horizon. Moreover, these measures do not suggest that there is a robust relationship between the average belief and return skewness.

5 Conclusion

This paper examines the theoretical and empirical impact of belief skewness on the stock price, mean return, and volatility. We first develop a general equilibrium model in which investors have skewed cash-flow growth expectations, and derive the implications of these

[14] In results available upon request, we find that the contemporaneous relation between belief skewness and return skewness is positive and significant in three out of four cases.

skewed beliefs on market outcomes. We then test the main model predictions using belief skewness proxies constructed from analysts' long-term earnings growth forecasts.

Our central theoretical result is that belief skewness negatively affects the stock mean return. However, when the average belief bias is sufficiently high, the market price of risk becomes sufficiently negative to turn the impact of belief skewness on the mean return positive. Our empirical analysis confirms the state-dependent nature of this relation. When the investors' average optimism is not too high, a one-standard-deviation increase in belief skewness is associated with a 3% to 4% reduction in annualized market returns. When average optimism is high enough, this relation turns positive and significant. We verify that our results are robust to variety of alternative econometric specifications, definitions of high-average-belief states, and the inclusion of control variables.

Overall, our work emphasizes the importance of considering the third moment of heterogeneous beliefs, and not only their mean and dispersion. Importantly, the interaction between the average belief bias and belief skewness suggests that belief moments jointly shape asset prices in non-trivial ways, and should be incorporated together in both theoretical and empirical asset pricing.

A Proofs

Proof of Proposition 1

Before determining the equilibrium belief moments, we need to compute the model equilibrium. To do so, we adapt the definition of an exchange equilibrium with transfers given in Jouini (2023, section 5) to an economy with two types of investors. The idea is to start from given Lagrange multipliers that solve the first-order conditions and derive the associated individual endowments that solve the individual budget constraints.

Specifically, we assume that the Lagrange multipliers of the aggregation of all pessimists and optimists, denoted by y_1 and y_2, respectively, are given by

$$y_n = f_n(\alpha) \exp\left((1-\gamma)\,\theta_n T\right),$$

for $n = 1, 2$, where α is a free parameter. In the remainder of the analysis, we impose that $\alpha \in]0,1[$, and choose $f_1(\alpha) = 1/\alpha$ and $f_2(\alpha) = 1/(1-\alpha)$.

The equilibrium is characterized by optimal portfolio strategies, which yield portfolios whose horizon value is used for consumption at date T, and a unique state price density. For $n = 1, 2$, let $W_{n,T}$ denote the horizon aggregate consumption of investors with bias θ_n, and let ξ_T denote the state price density. We also introduce the scaling factor ϑ to ensure that $\xi_T = 1$ when $T = 0$ and $\omega_T = 0$. The equilibrium with transfers is obtained when utilities are maximized under the first-order conditions and markets clear; that is, portfolio strategies maximize[15]

$$\mathbb{E}_t\left[\eta_{n,T}\frac{W_{n,T}^{1-\gamma}}{1-\gamma}\right],$$

for $n = 1, 2$, with $\eta_{n,T} W_{n,T}^{-\gamma} - \vartheta^{-\gamma}\xi_T y_n = 0$, and $D_T = W_{1,T} + W_{2,T}$.

[15]For $\gamma = 1$, the objective function becomes $\mathbb{E}_t\left[\eta_{n,T}\ln\left(W_{n,T}\right)\right]$.

The first-order conditions give

$$W_{n,T} = \vartheta y_n^{-\frac{1}{\gamma}} \eta_{n,T}^{\frac{1}{\gamma}} \xi_T^{-\frac{1}{\gamma}},$$

for $n = 1, 2$. Thus, we obtain

$$D_T = \vartheta \left(y_1^{-\frac{1}{\gamma}} \eta_{1,T}^{\frac{1}{\gamma}} + y_2^{-\frac{1}{\gamma}} \eta_{2,T}^{\frac{1}{\gamma}} \right) \xi_T^{-\frac{1}{\gamma}}.$$

This leads to

$$\xi_T = \vartheta^{\gamma} \left(y_1^{-\frac{1}{\gamma}} \eta_{1,T}^{\frac{1}{\gamma}} + y_2^{-\frac{1}{\gamma}} \eta_{2,T}^{\frac{1}{\gamma}} \right)^{\gamma} D_T^{-\gamma}.$$

To ensure that $\xi_T = 1$ when $T = 0$ and $\omega_T = 0$, we set

$$\vartheta = \left(y_1^{-\frac{1}{\gamma}} + y_2^{-\frac{1}{\gamma}} \right)^{-1}.$$

Overall, we have

$$\begin{aligned} W_{n,T} &= \frac{y_n^{-\frac{1}{\gamma}} \eta_{n,T}^{\frac{1}{\gamma}}}{y_1^{-\frac{1}{\gamma}} + y_2^{-\frac{1}{\gamma}}} \xi_T^{-\frac{1}{\gamma}}, \text{ for } n = 1, 2, \\ \xi_T &= \left(\frac{y_1^{-\frac{1}{\gamma}} \eta_{1,T}^{\frac{1}{\gamma}} + y_2^{-\frac{1}{\gamma}} \eta_{2,T}^{\frac{1}{\gamma}}}{y_1^{-\frac{1}{\gamma}} + y_2^{-\frac{1}{\gamma}}} \right)^{\gamma} D_T^{-\gamma}. \end{aligned}$$

The equilibrium wealth-weighted proportion of the investors with bias in beliefs θ_n, denoted by $\nu_{n,t}$, is then determined by solving the budget constraints. Specifically, it is given by the present value of their aggregate consumption divided by the present value of the total payoff (Cox and Huang, 1989)

$$\nu_{n,t} = \frac{\mathbb{E}_t \left[\xi_T W_{n,T} \right]}{\mathbb{E}_t \left[\xi_T D_T \right]}.$$

To derive the explicit expression of $\nu_{n,t}$, we first recall Lemma IA1 from the Online

Appendix of Atmaz and Basak (2018).

Lemma IA1 (Atmaz and Basak, 2018): *Let the process D be as in Equation* (1) *and the random variable M_T be defined as*

$$M_T := \sum_{n=1}^{2} y_n^{-\frac{1}{\gamma}} \eta_{n,T}^{\frac{1}{\gamma}}.$$

Then for any number a and positive integer b, we have

$$\mathbb{E}_t \left[D_T^a M_T^b \right] = D_t^a y_2^{-\frac{b}{\gamma}} \eta_{2,t}^{\frac{b}{\gamma}} \exp\left(a \left(\mu - \frac{1}{2}\sigma^2 \right) (T-t) \right)$$
$$\times \sum_{k=0}^{b} \binom{b}{k} \left(\frac{\frac{y_1}{\eta_{1,t}}}{\frac{y_2}{\eta_{2,t}}} \right)^{-\frac{k}{\gamma}} \exp\left(-\frac{1}{2} \left(\frac{k}{\gamma}\frac{\theta_1^2}{\sigma^2} + \frac{b-k}{\gamma}\frac{\theta_2^2}{\sigma^2} - \left(a\sigma + \frac{k}{\gamma}\frac{\theta_1}{\sigma} + \frac{b-k}{\gamma}\frac{\theta_2}{\sigma} \right)^2 \right) (T-t) \right),$$

where the process η_n is as in Equation (3) *for $n = 1, 2$, and $\binom{b}{k} = \frac{b!}{k!\,(b-k)!}$ is the binomial coefficient.*

We have $\nu_{n,t} = \frac{\mathbb{E}_t\left[\xi_T W_{n,T}\right]}{\mathbb{E}_t\left[\xi_T D_T\right]} = \frac{y_n^{-\frac{1}{\gamma}} \mathbb{E}_t\left[\eta_{n,T}^{\frac{1}{\gamma}} D_T^{1-\gamma} M_T^{\gamma-1} \right]}{\mathbb{E}_t\left[D_T^{1-\gamma} M_T^{\gamma} \right]}$,
and $\eta_{n,T}^{\frac{1}{\gamma}} = \eta_{n,t}^{\frac{1}{\gamma}} D_t^{-\frac{\theta_n}{\gamma\sigma^2}} \exp\left(-\frac{\theta_n}{\gamma\sigma^2}\left(\mu - \frac{1}{2}\sigma^2 \right)(T-t) - \frac{1}{2}\frac{\theta_n^2}{\gamma\sigma^2}(T-t) \right) D_T^{\frac{\theta_n}{\gamma\sigma^2}}$.

Thus, the explicit expression of the numerator is obtained by applying Lemma IA1 with $a = 1 - \gamma + \frac{\theta_n}{\gamma\sigma^2}$ and $b = \gamma - 1$, and the explicit expression of the denominator is obtained by applying Lemma IA1 with $a = 1 - \gamma$ and $b = \gamma$. Overall, simple algebra and rearranging the terms leads to

$$\nu_{1,t} = \frac{\sum_{k=0}^{\gamma} \frac{k}{\gamma} G_{t,k}}{\sum_{k=0}^{\gamma} G_{t,k}},$$

and $\nu_{2,t} = 1 - \nu_{1,t}$, where

$$G_{t,k} = \binom{\gamma}{k} \left(\frac{\frac{y_1}{\eta_{1,t}}}{\frac{y_2}{\eta_{2,t}}} \right)^{-\frac{k}{\gamma}} \exp\left(-\frac{1}{2} \left(\frac{k}{\gamma\sigma^2} \left(1 - \frac{k}{\gamma} \right) (\theta_2 - \theta_1)^2 - 2k \left(1 - \frac{1}{\gamma} \right) (\theta_2 - \theta_1) \right) (T-t) \right).$$

We can then define the wealth-weighted belief moments given by

$$
\begin{aligned}
m_t &= \nu_{1,t}\theta_1 + (1-\nu_{1,t})\,\theta_2, \\
v_t &= \sqrt{\nu_{1,t}\left(\theta_1 - m_t\right)^2 + (1-\nu_{1,t})\left(\theta_2 - m_t\right)^2}, \text{ and} \\
s_t &= \frac{\nu_{1,t}\left(\theta_1 - m_t\right)^3 + (1-\nu_{1,t})\left(\theta_2 - m_t\right)^3}{v_t^3}.
\end{aligned}
$$

Simple algebra then leads to Equations (4), (5), and (6).

To conclude the proof of Proposition 1, let us derive a corollary that shows how θ_1, θ_2, $\theta_2 - \theta_1$, and $\nu_{1,t}$ depend on the belief moments. We will use this corollary in order to express the stock price, mean return, and volatility as functions of the beliefs moments.

Formally, we first invert (6) to obtain $\nu_{1,t}$. Second, we plug the expression of $\nu_{1,t}$ in (5) and invert it to obtain $\theta_2 - \theta_1$. Third, we plug $\theta_2 - \theta_1$ and $\nu_{1,t}$ in (4) and invert it to obtain θ_1. Finally, θ_2 is easily obtained from the formulas of $\theta_2 - \theta_1$ and θ_1. Overall, it yields the following corollary.

Corollary 1 *The biases of pessimists and optimists, the bias difference between pessimists and optimists, and the share of pessimists are given by*

$$
\begin{aligned}
\theta_1 &= m_t - \frac{v_t\left(\sqrt{4+s_t^2} - s_t\right)}{2}, && (26) \\
\theta_2 &= m_t + \frac{v_t\left(\sqrt{4+s_t^2} + s_t\right)}{2}, && (27) \\
\theta_2 - \theta_1 &= v_t\sqrt{4+s_t^2}, && (28) \\
\nu_{1,t} &= \frac{1}{2} + \frac{s_t}{2\sqrt{4+s_t^2}}. && (29)
\end{aligned}
$$

For completeness, we report their first partial partial derivatives with respect to the three belief moments. We have

$$
\begin{aligned}
&\frac{\partial\theta_1}{\partial m_t} = 1 > 0, \quad \frac{\partial\theta_1}{\partial v_t} = -\frac{\sqrt{4+s_t^2} - s_t}{2} < 0, \quad \frac{\partial\theta_1}{\partial s_t} = -\frac{v_t}{2}\left(\frac{s_t}{\sqrt{4+s_t^2}} - 1\right) > 0, \\
&\frac{\partial\theta_2}{\partial m_t} = 1 > 0, \quad \frac{\partial\theta_2}{\partial v_t} = \frac{\sqrt{4+s_t^2} + s_t}{2} > 0, \quad \frac{\partial\theta_2}{\partial s_t} = \frac{v_t}{2}\left(\frac{s_t}{\sqrt{4+s_t^2}} + 1\right) > 0,
\end{aligned}
$$

$$
\begin{aligned}
&\frac{\partial\left(\theta_{2}-\theta_{1}\right)}{\partial m_{t}}=0, \quad \frac{\partial\left(\theta_{2}-\theta_{1}\right)}{\partial v_{t}}=\sqrt{4+s_{t}^{2}}>0, \quad \frac{\partial\left(\theta_{2}-\theta_{1}\right)}{\partial s_{t}}=\frac{v_{t} s_{t}}{\sqrt{4+s_{t}^{2}}}<>0, \\
&\frac{\partial \nu_{1,t}}{\partial m_{t}}=0, \quad \frac{\partial \nu_{1,t}}{\partial v_{t}}=0, \quad \frac{\partial \nu_{1,t}}{\partial s_{t}}=\frac{2}{\left(4+s_{t}^{2}\right)^{3/2}}>0.
\end{aligned}
$$

The derivatives and the expressions in the corollary clarify that an increase in belief skewness raises the biases of pessimists and optimists and the share of pessimists. To hold the first two moments fixed and increase skewness, one must therefore not only increase the share of pessimists but also shift biases towards more optimism as in the introductory example.

Proposition 2

By no arbitrage, the stock price is given by

$$
S_{t}=\frac{\mathbb{E}_{t}\left[\xi_{T} D_{T}\right]}{\mathbb{E}_{t}\left[\xi_{T}\right]}=\frac{\mathbb{E}_{t}\left[D_{T}^{1-\gamma} M_{T}^{\gamma}\right]}{\mathbb{E}_{t}\left[D_{T}^{-\gamma} M_{T}^{\gamma}\right]}.
$$

The explicit expression of the numerator is obtained by applying Lemma IA1 of Atmaz and Basak (2018) with $a=1-\gamma$ and $b=\gamma$, and the explicit expression of the denominator is obtained by applying Lemma IA1 with $a=-\gamma$ and $b=\gamma$.

Simple algebra leads to

$$
S_{t}=\exp\left(\left(\mu-\gamma\sigma^{2}\right)(T-t)\right) D_{t} \frac{\sum_{k=0}^{\gamma} G_{t,k} \exp\left(\theta_{2}(T-t)\right)}{\sum_{k=0}^{\gamma} G_{t,k} \exp\left(\frac{k}{\gamma}\left(\theta_{2}-\theta_{1}\right)(T-t)\right)}.
$$

We now study the logarithmic case and assume that $\gamma=1$.

We have $G_{t,0}=1$ and, from (7), $G_{t,1}=\dfrac{\nu_{1,t}}{1-\nu_{1,t}}$. It leads to

$$
S_{t}=\exp\left(\left(\mu-\sigma^{2}\right)(T-t)\right) D_{t} \frac{\exp\left(\theta_{2}(T-t)\right)}{1-\left(1-\exp\left(\left(\theta_{2}-\theta_{1}\right)(T-t)\right)\right) \nu_{1,t}}.
$$

Next, we use Corollary 1 to write this expression as a function of m_t, v_t, and s_t.

We obtain

$$S_t = \exp\left(\left(\mu - \sigma^2\right)(T-t)\right) D_t \frac{2\sqrt{4+s_t^2}\left(\sqrt{4+s_t^2}+s_t\right)\exp\left(\left(m_t + \frac{v_t\left(\sqrt{4+s_t^2}+s_t\right)}{2}\right)(T-t)\right)}{4+\left(\sqrt{4+s_t^2}+s_t\right)^2\exp\left(v_t\sqrt{4+s_t^2}\,(T-t)\right)}.$$

To obtain *(i)*, we compute the first partial derivative of this expression with respect to m_t. We get

$$\frac{\partial S_t}{\partial m_t} = (T-t)\, S_t > 0.$$

Similarly, to obtain *(ii)*, we compute the first partial derivative of S_t with respect to v_t. It leads to

$$\frac{\partial S_t}{\partial v_t} = -\frac{2\left(\sqrt{4+s_t^2}+s_t\right)\left(\exp\left(v_t\sqrt{4+s_t^2}\,(T-t)\right)-1\right)(T-t)}{4+\left(\sqrt{4+s_t^2}+s_t\right)^2\exp\left(v_t\sqrt{4+s_t^2}\,(T-t)\right)} S_t.$$

Since $v_t\sqrt{4+s_t^2}\,(T-t) > 0$, we have $\exp\left(v_t\sqrt{4+s_t^2}\,(T-t)\right) - 1 > 0$. Moreover, $\sqrt{4+s_t^2}+s_t > 0$ for all values of s_t. Consequently, we have $\frac{\partial S_t}{\partial v_t} < 0$.

Finally, to obtain *(iii)*, we compute the first partial derivative of S_t with respect to s_t and obtain

$$\frac{\partial S_t}{\partial s_t} = \frac{2\left(\sqrt{4+s_t^2}+s_t\right)\times S_{t,s_t}}{\left(4+s_t^2\right)\left(4+\left(\sqrt{4+s_t^2}+s_t\right)^2\exp\left(v_t\sqrt{4+s_t^2}\,(T-t)\right)\right)} S_t,$$

where the denominator is always positive, and

$$S_{t,s_t} := 2 + v_t\sqrt{4+s_t^2}\,(T-t) + \left(v_t\sqrt{4+s_t^2}\,(T-t) - 2\right)\exp\left(v_t\sqrt{4+s_t^2}\,(T-t)\right).$$

Defining $X := v_t\sqrt{4+s_t^2}\,(T-t) > 0$, we get $\frac{\partial S_{t,s_t}}{\partial X} = 1 + (X-1)\exp(X)$, and $\frac{\partial^2 S_{t,s_t}}{\partial X^2} = X\exp(X) > 0$.

Consequently, $\frac{\partial S_{t,s_t}}{\partial X}$ increases with X and is always higher than its value obtained for $X=0$. Since $1+(0-1)\exp(0)=0$, we have $\frac{\partial S_{t,s_t}}{\partial X}>0$. Thus, S_{t,s_t} increases with X and is always higher than its value obtained for $X=0$. Since $2+0+(0-2)\exp(0)=0$, we conclude that $S_{t,s_t}>0$.

Overall, it leads to $\frac{\partial S_t}{\partial s_t}>0$, which proves *(iii)*.

Proposition 3

The equilibrium volatility corresponds to the diffusion term of the dynamics of S. It is given in Equation (13) by

$$\sigma_{S_t}=\sigma+\frac{\theta_2-\theta_1}{\sigma}\left(\frac{\sum_{k=0}^{\gamma}\frac{k}{\gamma}G_{t,k}\exp\left(\frac{k}{\gamma}(\theta_2-\theta_1)(T-t)\right)}{\sum_{k=0}^{\gamma}G_{t,k}\exp\left(\frac{k}{\gamma}(\theta_2-\theta_1)(T-t)\right)}-\nu_{1,t}\right).$$

Imposing logarithmic utility ($\gamma=1$) yields

$$\sigma_{S_t}=\sigma+\frac{\theta_2-\theta_1}{\sigma}\left(\frac{\exp((\theta_2-\theta_1)(T-t))\,\nu_{1,t}}{1-(1-\exp((\theta_2-\theta_1)(T-t)))\,\nu_{1,t}}-\nu_{1,t}\right).$$

Using Corollary 1, we obtain

$$\sigma_{S_t}=\sigma+\frac{2v_t\left(\sqrt{4+s_t^2}+s_t\right)\left(\exp\left(v_t\sqrt{4+s_t^2}\,(T-t)\right)-1\right)}{\sigma\left(4+\left(\sqrt{4+s_t^2}+s_t\right)^2\exp\left(v_t\sqrt{4+s_t^2}\,(T-t)\right)\right)}.$$

First, it is straightforward to see that $\frac{\partial\sigma_{S_t}}{\partial m_t}=0$, which yields *(i)*.

Second, we have

$$\frac{\partial\sigma_{S_t}}{\partial v_t}=\frac{2\left(\sqrt{4+s_t^2}+s_t\right)}{\sigma\left(4+\left(\sqrt{4+s_t^2}+s_t\right)^2\exp\left(v_t\sqrt{4+s_t^2}\,(T-t)\right)\right)^2}\times\sigma_{S_t,v_t},$$

where the first term is always positive, and

$$\begin{aligned}\sigma_{S_t,v_t} &\coloneqq v_t\sqrt{4+s_t^2}\,(T-t)\exp\left(v_t\sqrt{4+s_t^2}\,(T-t)\right)\left(4+\left(\sqrt{4+s_t^2}+s_t\right)^2\right)\\ &+\left(\exp\left(v_t\sqrt{4+s_t^2}\,(T-t)\right)-1\right)\left(4+\left(\sqrt{4+s_t^2}+s_t\right)^2\exp\left(v_t\sqrt{4+s_t^2}\,(T-t)\right)\right).\end{aligned}$$

Since $v_t\sqrt{4+s_t^2}\,(T-t)>0$, we have $\exp\left(v_t\sqrt{4+s_t^2}\,(T-t)\right)-1>0$.

Thus, σ_{S_t,v_t} is the sum of two positive terms.

Consequently, $\dfrac{\partial\sigma_{S_t}}{\partial v_t}>0$, which yields *(ii)*.

Third, we have

$$\frac{\partial\sigma_{S_t}}{\partial s_t}=\frac{2v_t\left(\sqrt{4+s_t^2}+s_t\right)}{\sigma\sqrt{4+s_t^2}\left(4+\left(\sqrt{4+s_t^2}+s_t\right)^2\exp\left(v_t\sqrt{4+s_t^2}\,(T-t)\right)\right)^2}\times\sigma_{S_t,s_t},$$

where the first term is always positive, and

$$\begin{aligned}\sigma_{S_t,s_t} &\coloneqq -\left(\sqrt{4+s_t^2}+s_t\right)^2\exp\left(v_t\sqrt{4+s_t^2}\,(T-t)\right)\left(\exp\left(v_t\sqrt{4+s_t^2}\,(T-t)\right)-1\right)\\ &+2\sqrt{4+s_t^2}\left(\sqrt{4+s_t^2}+s_t\right)v_ts_t\,(T-t)\exp\left(v_t\sqrt{4+s_t^2}\,(T-t)\right)\\ &+4\left(\exp\left(v_t\sqrt{4+s_t^2}\,(T-t)\right)-1\right).\end{aligned}$$

Note that $-\left(\sqrt{4+s_t^2}+s_t\right)^2\exp\left(v_t\sqrt{4+s_t^2}\,(T-t)\right)<0$.

Moreover, $\exp(x)-1>x,\ \forall x>0$.

Thus, the first line of σ_{S_t,s_t} is lower than

$$-\left(\sqrt{4+s_t^2}+s_t\right)^2\exp\left(v_t\sqrt{4+s_t^2}\,(T-t)\right)v_t\sqrt{4+s_t^2}\,(T-t).$$

It yields that $\sigma_{S_t,s_t} < \bar{\sigma}_{S_t,s_t}$, where

$$\begin{aligned}\bar{\sigma}_{S_t,s_t} \coloneqq & -\left(\sqrt{4+s_t^2}+s_t\right)^2 \exp\left(v_t\sqrt{4+s_t^2}\,(T-t)\right) v_t\sqrt{4+s_t^2}\,(T-t) \\ & + 2\sqrt{4+s_t^2}\left(\sqrt{4+s_t^2}+s_t\right) v_t s_t\,(T-t)\exp\left(v_t\sqrt{4+s_t^2}\,(T-t)\right) \\ & + 4\left(\exp\left(v_t\sqrt{4+s_t^2}\,(T-t)\right)-1\right) \\ = & -4\left(\left(v_t\sqrt{4+s_t^2}\,(T-t)-1\right)\exp\left(v_t\sqrt{4+s_t^2}\,(T-t)\right)+1\right).\end{aligned}$$

Defining $X \coloneqq v_t\sqrt{4+s_t^2}\,(T-t) > 0$, we easily derive that $\dfrac{\partial\bar{\sigma}_{S_t,s_t}}{\partial X} < 0$.
Thus, $\bar{\sigma}_{S_t,s_t}$ decreases with X and is always lower than its value obtained for $X=0$.
Since $-4\left((0-1)\exp(0)+1\right)=0$, we conclude that $\sigma_{S_t,s_t} < \bar{\sigma}_{S_t,s_t} < 0$.
Overall, it leads to $\dfrac{\partial\sigma_{S_t}}{\partial s_t} < 0$, which yields *(iii)*.

Proposition 4

The equilibrium mean return corresponds to the drift term of the dynamics of S. It is given in Equation (12) by

$$\begin{aligned}\mu_{S_t} = & \left(\sigma+\frac{\theta_2-\theta_1}{\sigma}\left(\frac{\sum_{k=0}^{\gamma}\frac{k}{\gamma}G_{t,k}\exp\left(\frac{k}{\gamma}(\theta_2-\theta_1)(T-t)\right)}{\sum_{k=0}^{\gamma}G_{t,k}\exp\left(\frac{k}{\gamma}(\theta_2-\theta_1)(T-t)\right)}-\nu_{1,t}\right)\right) \\ & \times\left(\gamma\sigma+\frac{\theta_2-\theta_1}{\sigma}\frac{\sum_{k=0}^{\gamma}\frac{k}{\gamma}G_{t,k}\exp\left(\frac{k}{\gamma}(\theta_2-\theta_1)(T-t)\right)}{\sum_{k=0}^{\gamma}G_{t,k}\exp\left(\frac{k}{\gamma}(\theta_2-\theta_1)(T-t)\right)}-\frac{\theta_2}{\sigma}\right).\end{aligned}$$

Identifying the stock volatility and the market price of risk, given in Equations (13) and (14), we have

$$\mu_{S_t} = \sigma_{S_t}\left(\sigma_{S_t}+(\gamma-1)\,\sigma-\frac{m_t}{\sigma}\right).$$

Taking $\gamma=1$ results in

$$\mu_{S_t} = \sigma_{S_t}\left(\sigma_{S_t}-\frac{m_t}{\sigma}\right).$$

The first partial derivatives of μ_{S_t} with respect to the belief moments allows us to derive *(i)* to *(iii)*. We have

$$\begin{aligned}\frac{\partial \mu_{S_t}}{\partial m_t} &= -\frac{\sigma_{S_t}}{\sigma} < 0,\\ \frac{\partial \mu_{S_t}}{\partial v_t} &= \frac{\partial \sigma_{S_t}}{\partial v_t}\left(2\sigma_{S_t} - \frac{m_t}{\sigma}\right),\\ \frac{\partial \mu_{S_t}}{\partial s_t} &= \frac{\partial \sigma_{S_t}}{\partial s_t}\left(2\sigma_{S_t} - \frac{m_t}{\sigma}\right).\end{aligned}$$

As seen in the proof of Proposition 3, $\frac{\partial \sigma_{S_t}}{\partial v_t} > 0$ and $\frac{\partial \sigma_{S_t}}{\partial s_t} < 0$. Thus, the sign of $\frac{\partial \mu_{S_t}}{\partial v_t}$ and $\frac{\partial \mu_{S_t}}{\partial s_t}$ depends on the sign of $2\sigma_{S_t} - \frac{m_t}{\sigma}$. Formally

$$2\sigma_{S_t} - \frac{m_t}{\sigma} > 0 \Leftrightarrow m_t < 2\sigma\sigma_{S_t}.$$

When this condition is satisfied, we thereby have $\frac{\partial \mu_{S_t}}{\partial v_t} > 0$ and $\frac{\partial \mu_{S_t}}{\partial s_t} < 0$.

Conversely, when m_t exceeds this threshold, $\frac{\partial \mu_{S_t}}{\partial v_t} < 0$ and $\frac{\partial \mu_{S_t}}{\partial s_t} > 0$; that is, the impacts reverse when investors are sufficiently optimistic on average.

B Representative agent

The representative agent is the investor whose wealth corresponds to the total wealth of the economy and whose belief is given by a weighted average of individual investors' beliefs.

From the second welfare theorem, it is easy to show that the utility function of the representative agent is given by

$$U\left(D_T\right) = \left(y_1^{-\frac{1}{\gamma}}\eta_{1,T}^{\frac{1}{\gamma}} + y_2^{-\frac{1}{\gamma}}\eta_{2,T}^{\frac{1}{\gamma}}\right)^{\gamma}\frac{D_T^{1-\gamma}}{1-\gamma}.$$

We thus identify $Z_T := \left(y_1^{-\frac{1}{\gamma}}\eta_{1,T}^{\frac{1}{\gamma}} + y_2^{-\frac{1}{\gamma}}\eta_{2,T}^{\frac{1}{\gamma}}\right)^{\gamma} = M_T^{\gamma}$ as the Radon-Nikodym derivative of the representative investor's subjective belief $\mathbb{P}^R$ with respect to the true belief $\mathbb{P}$, where M_T is defined in Lemma IA1 of Atmaz and Basak (2018).

Because Z is a martingale, we have

$$Z_t = \mathbb{E}_t\left[Z_T\right]$$
$$= y_2^{-1}\eta_{2,t}\sum_{k=0}^{\gamma}\binom{\gamma}{k}\left(\frac{\frac{y_1}{\eta_{1,t}}}{\frac{y_2}{\eta_{2,t}}}\right)^{-\frac{k}{\gamma}}\exp\left(-\frac{1}{2}\left(\frac{k}{\gamma}\frac{\theta_1^2}{\sigma^2}+\frac{\gamma-k}{\gamma}\frac{\theta_2^2}{\sigma^2}-\left(\frac{k}{\gamma}\frac{\theta_1}{\sigma}+\frac{\gamma-k}{\gamma}\frac{\theta_2}{\sigma}\right)^2\right)(T-t)\right),$$

where the second equality is obtained by applying Lemma IA1 with $a=0$ and $b=\gamma$.

Applying Ito's lemma to Z_t yields

$$\frac{dZ_t}{Z_t}=\sum_{k=0}^{\gamma}\frac{G_{t,k}\exp\left(k\left(\frac{1}{\gamma}-1\right)(\theta_2-\theta_1)(T-t)\right)}{\sum_{j=0}^{\gamma}G_{t,j}\exp\left(j\left(\frac{1}{\gamma}-1\right)(\theta_2-\theta_1)(T-t)\right)}\left(\frac{k}{\gamma}\frac{\theta_1}{\sigma}+\left(1-\frac{k}{\gamma}\right)\frac{\theta_2}{\sigma}\right)d\omega_t.$$

We define the time-t representative-agent-based average bias in beliefs, denoted by $\tilde{m}_t$, as the time-t (stochastic) bias in beliefs of the representative investor. From the dynamics of Z we get that it is given by

$$\begin{aligned}\tilde{m}_t &= \frac{\sum_{k=0}^{\gamma}\frac{k}{\gamma}G_{t,k}\exp\left(k\left(\frac{1}{\gamma}-1\right)(\theta_2-\theta_1)(T-t)\right)}{\sum_{k=0}^{\gamma}G_{t,k}\exp\left(k\left(\frac{1}{\gamma}-1\right)(\theta_2-\theta_1)(T-t)\right)}\theta_1\\ &\quad+\left(1-\frac{\sum_{k=0}^{\gamma}\frac{k}{\gamma}G_{t,k}\exp\left(k\left(\frac{1}{\gamma}-1\right)(\theta_2-\theta_1)(T-t)\right)}{\sum_{k=0}^{\gamma}G_{t,k}\exp\left(k\left(\frac{1}{\gamma}-1\right)(\theta_2-\theta_1)(T-t)\right)}\right)\theta_2\\ &= h_{1,t}\theta_1+h_{2,t}\theta_2,\end{aligned}$$

where $h_{1,t}=\frac{\sum_{k=0}^{\gamma}\frac{k}{\gamma}G_{t,k}\exp\left(k\left(\frac{1}{\gamma}-1\right)(\theta_2-\theta_1)(T-t)\right)}{\sum_{k=0}^{\gamma}G_{t,k}\exp\left(k\left(\frac{1}{\gamma}-1\right)(\theta_2-\theta_1)(T-t)\right)}$ and $h_{2,t}=1-h_{1,t}$.

We use these weights to construct the time-t representative-agent-based belief dispersion and belief skewness

$$\tilde{v}_t=\sqrt{h_{1,t}\left(\theta_1-\tilde{m}_t\right)^2+h_{2,t}\left(\theta_2-\tilde{m}_t\right)^2},$$
$$\tilde{s}_t=\frac{h_{1,t}\left(\theta_1-\tilde{m}_t\right)^3+h_{2,t}\left(\theta_2-\tilde{m}_t\right)^3}{\tilde{v}_t^3}.$$

Note that, in the case of logarithmic preferences ($\gamma = 1$), we have

$$\exp\left(k\left(\frac{1}{\gamma} - 1\right)(\theta_2 - \theta_1)(T - t)\right) = 1.$$

Thus, $h_{1,t} = \nu_{1,t}$ and we conclude that, in such a case, the representative-agent-based belief moments coincide with the wealth-weighted belief moments.

C Parameter values

In this appendix, we explain how we choose the parameter values of all the figures in Section 2.3. Most of them derive from the empirical belief moments given in Table 1.

First, we notice that different values of t and ω_t do not qualitatively affect our results. Thus, we choose $t = 0$ and $\omega_t = 0$ for convenience and without loss of generality. Second, we set $T = 10$ and $\gamma = 2$ as in Atmaz and Basak (2018). Again, different values for these parameters yield qualitatively similar results.

Regarding the cash-flow news process, we choose the drift μ to match the mean value of the empirical average belief over our sample, which is reported in Table 1. We further choose the cash-flow news volatility σ so that the stock volatility σ_S equals 18% in the symmetric economy with no average bias in beliefs and average belief dispersion (defined below). Formally, we numerically find the values of θ_1, $\theta_2 - \theta_1$, $\nu_{1,t}$, and σ so that (4), (5), (6), and (13) match given values. Overall, it yields $\mu = 13.08\%$ and $\sigma = 12.33\%$.

In Section 2.3, we also compare economies that differ in belief moments. Specifically, we consider unbiased and highly-positively-biased economies, average-belief-dispersion and high-belief-dispersion economies, and symmetric, positively-skewed, and negatively-skewed economies.

Unbiased economies are economies in which the average bias in beliefs is zero. The average belief bias in the highly-positively-biased economies is computed as the difference between the maximum value of the empirical average belief over our sample and its mean value, both reported in Table 1: $20.89\% - 13.08\% = 7.81\%$.

Belief dispersion equals the mean value of the empirical belief dispersion over our sample in the average-belief-dispersion economies; that is, it equals 3.45% as shown in Table 1. In high-belief-dispersion economies, the belief dispersion is one empirical standard deviation higher than its empirical mean value: $3.45\% + 0.62\% = 4.07\%$.

Belief skewness is zero in the symmetric economies. Skewness in the positively-skewed economies equals the maximum value of the empirical max-min belief skewness proxy over our sample. The value is 0.21 as shown in Table 1. While the skewness measures in our theoretical and empirical analysis differ, Ghysels et al. (2016) show that the coefficient of skewness equals the quantile skewness times a positive constant that exceeds one. Thus, our approach to calibrate the coefficient of skewness to a quantile type measure can be seen as conservative. Notice also that max-min skewness is bounded by one in absolute value, while the coefficient of skewness can take any value on the real line. For the negatively-skewed economies, we set the skewness equal to -0.21 to compare economies with similar levels of absolute belief skewness.

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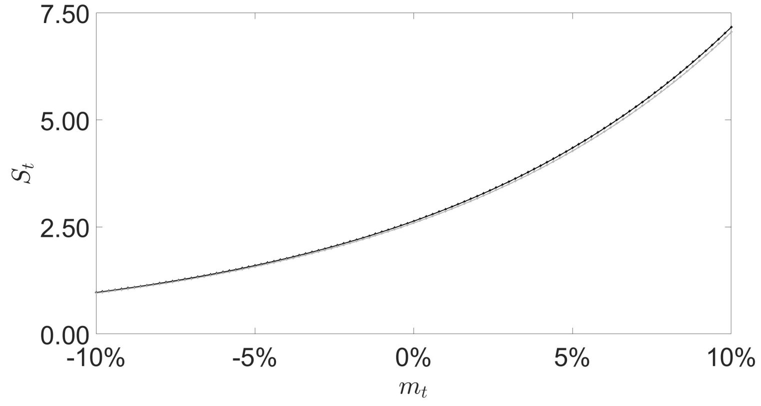
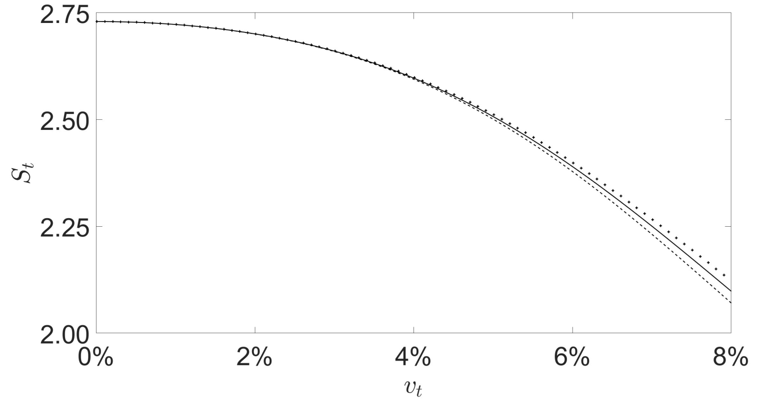
Figure 1: Evolution of the stock price depending on belief moments

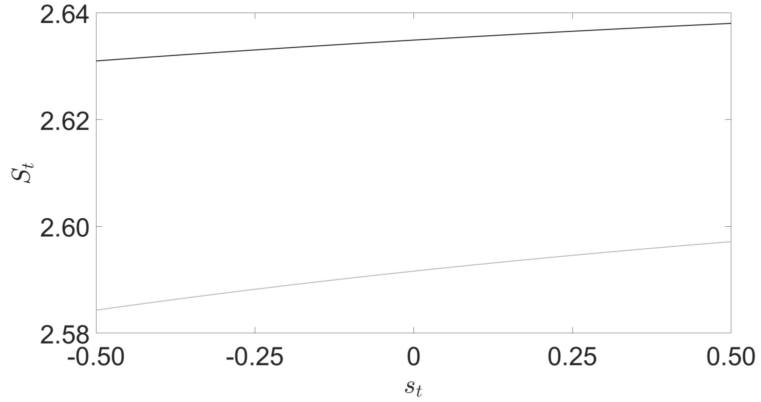
Panel A: Average bias in beliefs



Panel B: Belief dispersion

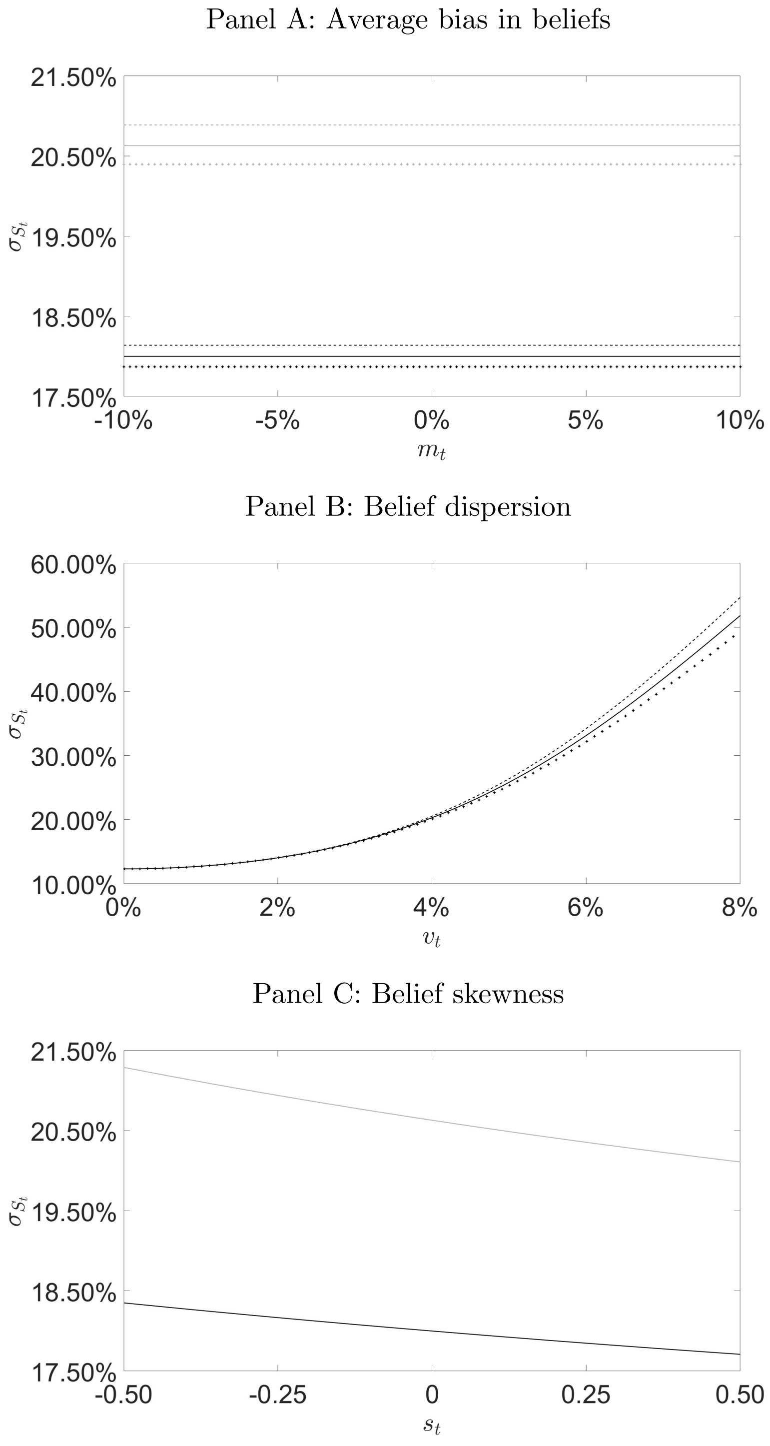


Panel C: Belief skewness



The figure presents the evolution of the stock price, S_t, depending on the belief moments. Panel A (B, C) shows the evolution depending on the average bias in beliefs (belief dispersion, belief skewness), m_t (v_t, s_t). Black (grey) lines represent average- (high-) belief-dispersion economies; that is, economies in which belief dispersion equals 3.45% (4.07%). Solid (plus-signed, dashed) lines represent symmetric (positively-skewed, negatively-skewed) economies; that is, economies in which belief skewness equals 0 (0.21, -0.21). The other parameter values are: $\mu = 13.08\%$, $\sigma = 12.33\%$, $t = 0$, $T = 10$, $\omega_t = 0$, and $\gamma = 2$; and we set $f_1(\alpha) = 1/\alpha$ and $f_2(\alpha) = 1/(1-\alpha)$. Appendix C explains how these values are chosen.

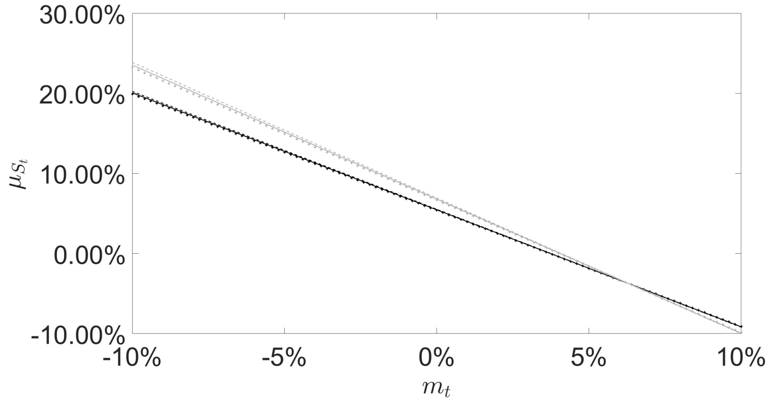
Figure 2: Evolution of the stock volatility depending on belief moments



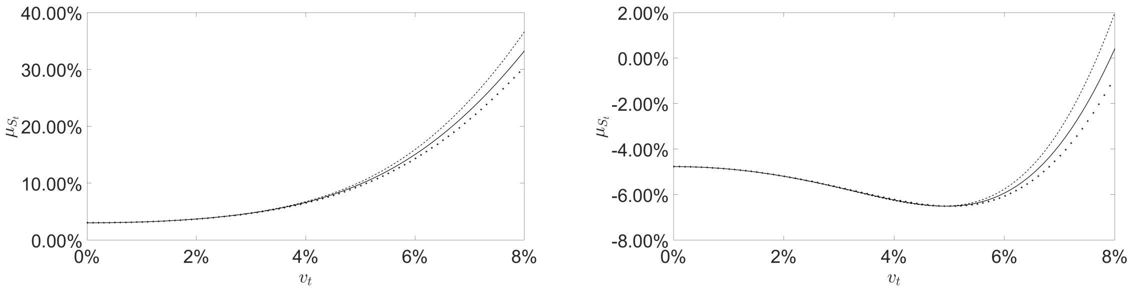
The figure presents the evolution of the stock volatility, σ_{S_t}, depending on the belief moments. Panel A (B, C) shows the evolution depending on the average bias in beliefs (belief dispersion, belief skewness), m_t (v_t, s_t). Black (grey) lines represent average-(high-) belief-dispersion economies; that is, economies in which belief dispersion equals 3.45% (4.07%). Solid (plus-signed, dashed) lines represent symmetric (positively-skewed, negatively-skewed) economies; that is, economies in which belief skewness equals 0 (0.21, -0.21). The other parameter values are: $\mu = 13.08\%$, $\sigma = 12.33\%$, $t = 0$, $T = 10$, $\omega_t = 0$, and $\gamma = 2$; and we set $f_1(\alpha) = 1/\alpha$ and $f_2(\alpha) = 1/(1-\alpha)$. Appendix C explains how these values are chosen.

Figure 3: Evolution of the stock mean return depending on belief moments

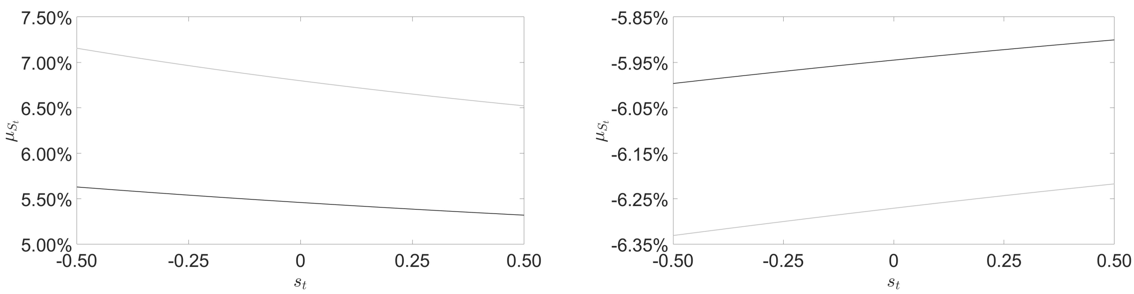
Panel A: Average bias in beliefs



Panel B: Belief dispersion

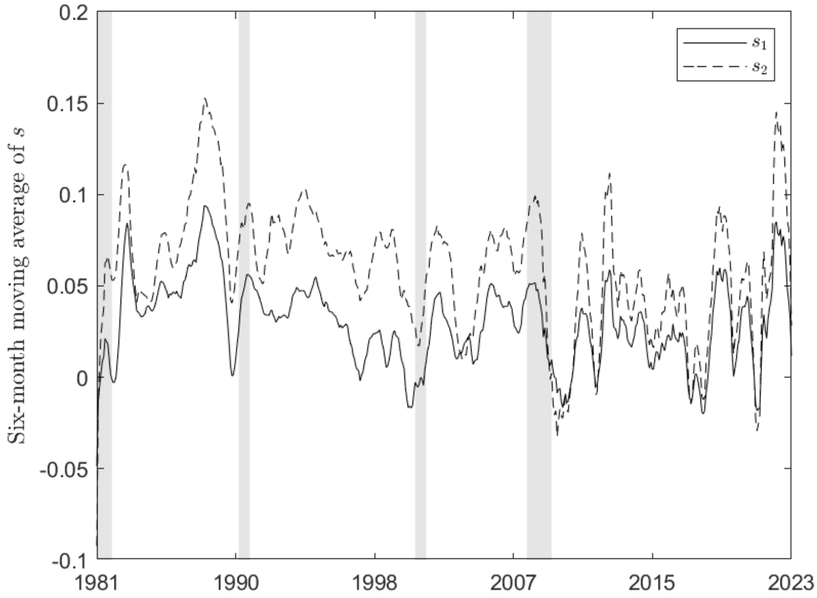


Panel C: Belief skewness



The figure presents the evolution of the stock mean return, μ_{S_t}, depending on the belief moments. Panel A (B, C) shows the evolution depending on the average bias in beliefs (belief dispersion, belief skewness), m_t (v_t, s_t). In Panels B and C, the left (right) sub-figure represents unbiased (highly-positively-biased) economies; that is, economies in which the average bias in beliefs equals 0% (7.81%). Black (grey) lines represent average- (high-) belief-dispersion economies; that is, economies in which belief dispersion equals 3.45% (4.07%). Solid (plus-signed, dashed) lines represent symmetric (positively-skewed, negatively-skewed) economies; that is, economies in which belief skewness equals 0 (0.21, -0.21). The other parameter values are: $\mu = 13.08\%$, $\sigma = 12.33\%$, $t = 0$, $T = 10$, $\omega_t = 0$, and $\gamma = 2$; and we set $f_1(\alpha) = 1/\alpha$ and $f_2(\alpha) = 1/(1-\alpha)$. Appendix C explains how these values are chosen.

Figure 4: Time-series of belief skewness



The figure plots the six-month moving average of the two belief skewness measures over time. The solid line represents the mean-median belief skewness proxy, s_1, and the dashed line represents the max-min belief skewness proxy, s_2. The sample period is from December 1981 to December 2023.

Table 1: Descriptive statistics
The table shows the mean, standard deviation, skewness, kurtosis, minimum, and maximum value of the proxies and their correlations and autocorrelations. The proxy for the belief bias or average belief is m, the belief dispersion proxy is v, the belief skewness proxy based on the mean-median difference is s_1, and the belief skewness proxy based on the minimum and maximum is s_2. The sample period is from December 1981 to December 2023 yielding 505 monthly observations.

	Mean	Std	Min	Max	Skew	Kurt	Correlation with			AR(1)
							v	s_1	s_2	
m	13.08	1.97	10.06	20.89	1.62	5.95	0.28	-0.31	-0.13	0.98
v	3.45	0.62	2.17	5.77	0.67	2.85		-0.36	-0.37	0.93
s_1	0.03	0.03	-0.06	0.13	0.05	3.52			0.90	0.75
s_2	0.06	0.04	-0.10	0.21	-0.31	4.22				0.72

Table 2: Belief moments and market returns
The table reports the regression results of $R_{t+1\to t+K} = g_0 + g_1 X_t + \epsilon_{t+1\to t+K}$, where $R_{t+1\to t+K}$ is the annualized cumulative log excess return of the S&P500 total return index from month $t+1$ to month $t+K$ and X_t is the belief moment proxy indicated in the row header. The dependent variable is annualized and in percent and its horizon K varies through the columns from one month (1m) to one year (12m). The independent variables are standardized to have unit variance and zero mean to facilitate the interpretation. The t-statistics of the slope coefficients are reported in parentheses and computed with Newey and West (1987) standard errors with K lags. Intercepts are not tabulated and the adjusted R^2 is reported in percent. The sample period is from December 1981 to December 2023. *, **, and *** denote significance at the 10%, 5%, and 1% levels, respectively.

Horizon	1m			3m			12m		
	b		Adj R^2	b		Adj R^2	b		Adj R^2
m	-4.4*	(-1.81)	0.49	-4.7**	(-2.53)	2.18	-6.2***	(-4.34)	15.59
v	-1.9	(-0.85)	-0.07	-1.8	(-0.97)	0.16	-3.8**	(-2.24)	5.95
s_1	-2.0	(-0.80)	-0.05	-2.0	(-1.06)	0.22	-0.1	(-0.07)	-0.20
s_2	-2.4	(-0.99)	0.01	-2.5	(-1.45)	0.47	-1.2	(-0.79)	0.35

Table 3: Belief moments and market returns

The table extends the analysis of Table 2 by considering the first three belief moment variables jointly in the regression. Each column reports the estimates from a separate regression. The calculations otherwise follow those in Table 2. *, **, and *** denote significance at the 10%, 5%, and 1% levels, respectively.

Horizon	1m		3m		12m	
m	-5.2*	-4.3*	-5.5***	-4.6**	-6.1***	-5.5***
	(-1.90)	(-1.65)	(-2.71)	(-2.35)	(-4.21)	(-3.61)
v	-2.1	-2.2	-1.9	-2.0	-3.3**	-3.5**
	(-0.83)	(-0.83)	(-0.94)	(-0.95)	(-2.00)	(-2.05)
s_1	-4.4		-4.4**		-3.2**	
	(-1.55)		(-2.35)		(-2.05)	
s_2		-3.8		-3.8**		-3.0**
		(-1.38)		(-2.07)		(-1.99)
Adj R^2	0.67	0.55	3.49	3.14	20.66	20.52

Table 4: State-dependent relation between belief skewness and market returns
For $j = 1, 2$, the table reports the estimates of the regression $R_{t+1\to t+K} = g_0 + g_1 \mathbb{1}_{m_t>\text{perc}} + g_2 s_{t,j} + g_3 \mathbb{1}_{m_t>\text{perc}} \times s_{t,j} + \epsilon_{t+1\to t+K}$, where $\mathbb{1}_{m_t>\text{perc}}$ is an indicator variable equal to one if m_t is above a chosen percentile and zero otherwise. The variable $s_{t,j}$ is standardized to have unit variance and zero mean to facilitate the interpretation. The t-statistics of the slope coefficients are reported in parentheses and computed with Newey and West (1987) standard errors with K lags. We also report the sum of g_2 and g_3 in the indicated row. Panel A reports the results for the 90th percentile and Panel B for the 75th percentile. *, **, and *** denote significance at the 10%, 5%, and 1% levels, respectively.

Horizon	1m		3m		12m	
Panel A. Indicator variable using the 90th percentile						
$\mathbb{1}_{m>\text{perc}}$	11.5	-2.2	0.5	-9.3*	-11.1**	-16.3***
	(1.09)	(-0.26)	(0.10)	(-1.81)	(-2.45)	(-3.27)
s_1	-4.8*		-4.7**		-2.8**	
	(-1.80)		(-2.55)		(-2.12)	
$\mathbb{1}_{m>\text{perc}} \times s_1$	28.5***		20.2***		12.0***	
	(3.30)		(3.26)		(4.21)	
s_2		-4.4*		-4.2**		-2.8**
		(-1.67)		(-2.31)		(-2.15)
$\mathbb{1}_{m>\text{perc}} \times s_2$		19.5**		13.0*		9.3***
		(2.04)		(1.75)		(2.62)
Adj R^2	1.52	0.86	4.55	3.41	17.88	17.42
$g_2 + g_3$	23.7***	15.1*	15.5***	8.8	9.2***	6.5**
	(2.89)	(1.65)	(2.61)	(1.23)	(3.77)	(1.97)
Panel B. Indicator variable using the 75th percentile						
$\mathbb{1}_{m>\text{perc}}$	-7.6	-7.0	-8.7*	-7.8	-5.9	-6.2
	(-1.23)	(-1.20)	(-1.69)	(-1.58)	(-1.17)	(-1.21)
s_1	-4.5		-3.7*		-2.0	
	(-1.40)		(-1.83)		(-1.38)	
$\mathbb{1}_{m>\text{perc}} \times s_1$	5.3		2.7		4.0	
	(1.01)		(0.61)		(1.18)	
s_2		-4.4		-3.7*		-2.5*
		(-1.51)		(-1.91)		(-1.73)
$\mathbb{1}_{m>\text{perc}} \times s_2$		7.3		4.1		5.1*
		(1.34)		(0.90)		(1.83)
Adj R^2	0.24	0.35	1.65	1.76	4.25	5.36
$g_2 + g_3$	0.8	2.8	-1.0	0.4	2.1	2.6
	(0.20)	(0.62)	(-0.27)	(0.10)	(0.66)	(1.10)

Table 5: Belief moments and market returns controlling for alternative predictors

The table reports the estimates of the specification in (23) augmented one-by-one with the control variable listed in the row header. The estimates for the slope on the control variable are in the column Ctr. The control variables are the dividend to price ratio (d_p; difference between log of dividends and log of price), the earnings-price ratio (e_p; difference between log of earnings and log of price), the book-to-market ratio (b_m; ratio of book value to market value for the Dow Jones Industrial Average), the term spread (tms; difference between the long term yield on government bonds and the Treasury-bill), the default yield spread (dfy; difference between BAA and AAA-rated corporate bond yields), inflation (infl; from the Consumer Price Index), net equity expansion (ntis; ratio of 12-month moving sums of net issues by NYSE listed stocks divided by the total end-of-year market capitalization of NYSE stocks), stock variance (svar; sum of squared daily returns on the S&P 500), Lettau and Ludvigson's (2001) consumption-wealth ratio of (cay), Jondeau et al.'s (2019) average stock skewness (skvw), Kelly and Jiang's (2014) cross-sectional tail risk (tail), Yu's (2011) analyst disagreement (disag), and a partial least squares aggregation of all control variables (all_pls). For more details on these variables, see Goyal and Welch (2008) and Goyal et al. (2024). All variables are from Amit Goyal's website and available at the monthly frequency, except cay for which we use the most recent available value in the monthly regressions. Panel A uses the mean-median belief skewness proxy s_1 and Panel B uses the max-min belief skewness proxy s_2. T-statistics use Newey and West (1987) standard errors with K lags. *, **, and *** denote significance at the 10%, 5%, and 1% levels, respectively.

Horizon: 1m

Horizon	m		v		s		Ctr		Adj R^2
Panel A. Specification with s_1									
d_p	-4.6*	(-1.7)	-1.6	(-0.6)	-4.6	(-1.6)	2.5	(0.9)	0.7
e_p	-4.7*	(-1.7)	-1.7	(-0.7)	-4.8*	(-1.7)	3.2	(1.1)	0.8
b_m	-4.6*	(-1.7)	-2.0	(-0.8)	-4.5	(-1.6)	2.1	(0.8)	0.6
tms	-5.7**	(-2.1)	-1.8	(-0.7)	-4.6	(-1.6)	-1.9	(-0.8)	0.6
dfy	-5.5**	(-2.0)	-2.0	(-0.8)	-4.4	(-1.6)	-1.0	(-0.3)	0.5
infl	-5.0*	(-1.8)	-2.1	(-0.8)	-4.2	(-1.5)	-1.3	(-0.5)	0.5
ntis	-5.7**	(-2.0)	-1.7	(-0.6)	-4.4	(-1.5)	1.5	(0.5)	0.5
svar	-5.3*	(-2.0)	-2.0	(-0.8)	-4.6	(-1.6)	-3.0	(-0.6)	0.8
cay	-5.1*	(-1.9)	-3.7	(-1.3)	-4.5	(-1.6)	-4.3*	(-1.7)	1.0
skvw	-4.8*	(-1.8)	-2.4	(-0.9)	-4.1	(-1.4)	-2.2	(-0.7)	0.6
tail	-4.5	(-1.5)	-1.7	(-0.7)	-4.4	(-1.5)	1.8	(0.7)	0.6
disag	-5.5**	(-2.0)			-3.8	(-1.4)	-0.3	(-0.1)	0.5
all_pls	-3.3	(-1.1)	-1.5	(-0.6)	-4.5	(-1.6)	5.9*	(1.8)	1.6
Panel B. Specification with s_2									
d_p	-3.7	(-1.4)	-1.7	(-0.6)	-3.9	(-1.4)	2.4	(0.9)	0.5
e_p	-3.8	(-1.4)	-1.8	(-0.7)	-4.2	(-1.5)	3.1	(1.1)	0.7
b_m	-3.7	(-1.4)	-2.0	(-0.8)	-3.9	(-1.4)	2.1	(0.8)	0.5
tms	-4.8*	(-1.9)	-1.9	(-0.7)	-4.0	(-1.4)	-1.9	(-0.8)	0.5
dfy	-4.6*	(-1.8)	-2.0	(-0.8)	-3.8	(-1.4)	-0.9	(-0.3)	0.4
infl	-4.1	(-1.6)	-2.1	(-0.8)	-3.6	(-1.3)	-1.4	(-0.5)	0.4
ntis	-4.9*	(-1.7)	-1.7	(-0.6)	-3.8	(-1.4)	1.6	(0.5)	0.4
svar	-4.3*	(-1.7)	-2.0	(-0.8)	-3.9	(-1.4)	-2.9	(-0.6)	0.6
cay	-4.1	(-1.6)	-3.5	(-1.2)	-3.6	(-1.3)	-4.0	(-1.6)	0.9
skvw	-4.0	(-1.5)	-2.4	(-0.9)	-3.4	(-1.2)	-2.3	(-0.7)	0.5
tail	-3.6	(-1.2)	-1.8	(-0.7)	-3.7	(-1.4)	1.8	(0.7)	0.4
disag	-4.7*	(-1.9)			-3.1	(-1.2)	-0.3	(-0.1)	0.4
all_pls	-2.4	(-0.8)	-1.5	(-0.6)	-3.7	(-1.3)	5.8*	(1.7)	1.4

Horizon: 3m

Horizon	m		v		s		Ctr		Adj R^2
Panel A. Specification with s_1									
d_p	-4.8**	(-2.4)	-1.2	(-0.6)	-4.6**	(-2.4)	3.0	(1.3)	4.1
e_p	-5.1**	(-2.4)	-1.5	(-0.7)	-4.8**	(-2.4)	3.1	(1.2)	4.3
b_m	-4.8**	(-2.3)	-1.7	(-0.9)	-4.5**	(-2.4)	2.8	(1.2)	4.1
tms	-6.0***	(-2.9)	-1.6	(-0.8)	-4.6**	(-2.4)	-1.9	(-1.0)	3.7
dfy	-5.6***	(-2.7)	-1.9	(-0.9)	-4.4**	(-2.4)	-0.2	(-0.1)	3.3
infl	-5.5***	(-2.7)	-1.9	(-0.9)	-4.4**	(-2.4)	0.3	(0.1)	3.3
ntis	-6.5***	(-3.0)	-1.1	(-0.5)	-4.3**	(-2.3)	2.7	(0.9)	4.0
svar	-5.5***	(-2.8)	-1.9	(-0.9)	-4.4**	(-2.4)	-0.7	(-0.2)	3.4
cay	-5.4***	(-2.7)	-3.1	(-1.5)	-4.5**	(-2.5)	-3.2**	(-2.0)	4.3
skvw	-5.0**	(-2.4)	-2.3	(-1.1)	-3.9**	(-2.1)	-2.8*	(-1.8)	4.1
tail	-5.2**	(-2.3)	-1.8	(-0.9)	-4.4**	(-2.3)	0.6	(0.3)	3.3
disag	-5.7***	(-2.9)			-4.0**	(-2.2)	-0.8	(-0.4)	3.2
all_pls	-3.9*	(-1.7)	-1.4	(-0.7)	-4.5**	(-2.4)	4.8*	(1.8)	5.4
Panel B. Specification with s_2									
d_p	-3.9*	(-1.9)	-1.3	(-0.6)	-4.0**	(-2.1)	2.9	(1.2)	3.7
e_p	-4.1**	(-2.0)	-1.6	(-0.8)	-4.2**	(-2.1)	3.0	(1.2)	3.9
b_m	-3.9*	(-1.9)	-1.8	(-0.9)	-3.9**	(-2.1)	2.7	(1.1)	3.7
tms	-5.1**	(-2.6)	-1.7	(-0.9)	-4.0**	(-2.1)	-2.0	(-1.0)	3.3
dfy	-4.6**	(-2.3)	-2.0	(-0.9)	-3.8**	(-2.1)	-0.2	(-0.1)	3.0
infl	-4.6**	(-2.4)	-2.0	(-0.9)	-3.8**	(-2.0)	0.1	(0.1)	3.0
ntis	-5.7***	(-2.7)	-1.2	(-0.6)	-3.8**	(-2.0)	2.8	(1.0)	3.7
svar	-4.6**	(-2.4)	-2.0	(-1.0)	-3.8**	(-2.1)	-0.6	(-0.2)	3.0
cay	-4.4**	(-2.3)	-3.0	(-1.4)	-3.7**	(-2.0)	-3.0*	(-1.9)	3.8
skvw	-4.2**	(-2.1)	-2.3	(-1.1)	-3.4*	(-1.8)	-2.9*	(-1.9)	3.8
tail	-4.3*	(-1.9)	-1.9	(-0.9)	-3.8**	(-2.1)	0.7	(0.3)	3.0
disag	-4.9**	(-2.6)			-3.4*	(-1.9)	-0.9	(-0.5)	2.9
all_pls	-3.0	(-1.3)	-1.5	(-0.7)	-3.8**	(-2.0)	4.6*	(1.8)	5.0

Horizon: 12m

Horizon	m		v		s		Ctr		Adj R^2
Panel A. Specification with s_1									
d_p	-5.6***	(-3.9)	-2.8	(-1.6)	-3.4**	(-2.1)	2.3*	(1.7)	22.3
e_p	-5.8***	(-3.9)	-3.0*	(-1.9)	-3.5**	(-2.2)	2.4*	(1.7)	22.8
b_m	-5.5***	(-3.8)	-3.1*	(-1.8)	-3.3**	(-2.1)	2.6**	(2.0)	23.0
tms	-6.3***	(-3.8)	-3.3*	(-1.9)	-3.2**	(-2.1)	-0.4	(-0.3)	20.6
dfy	-5.6***	(-3.6)	-3.6**	(-2.2)	-3.1**	(-2.0)	1.8	(1.5)	21.7
infl	-5.9***	(-4.0)	-3.3**	(-2.0)	-3.0**	(-2.0)	-1.4*	(-1.7)	21.3
ntis	-7.1***	(-4.5)	-2.5*	(-1.8)	-3.1**	(-2.1)	2.7	(1.1)	23.0
svar	-6.1***	(-4.2)	-3.3**	(-2.0)	-3.1**	(-2.0)	1.2*	(1.7)	21.1
cay	-6.1***	(-4.1)	-4.7**	(-2.6)	-3.2**	(-2.2)	-3.5***	(-3.0)	24.9
skvw	-6.1***	(-4.1)	-3.3**	(-2.0)	-3.1**	(-2.0)	-0.2	(-0.3)	20.5
tail	-5.4***	(-3.6)	-3.0*	(-1.9)	-3.1**	(-2.1)	1.8	(1.3)	21.5
disag	-6.5***	(-4.4)			-2.6*	(-1.8)	-2.0	(-1.1)	18.4
all_pls	-4.9***	(-3.4)	-3.0*	(-1.8)	-3.3**	(-2.2)	3.6***	(2.7)	25.1
Panel B. Specification with s_2									
d_p	-4.9***	(-3.2)	-2.9*	(-1.7)	-3.2**	(-2.1)	2.3	(1.5)	22.1
e_p	-5.1***	(-3.3)	-3.1*	(-1.9)	-3.3**	(-2.1)	2.4	(1.6)	22.6
b_m	-4.8***	(-3.2)	-3.3*	(-1.9)	-3.2**	(-2.0)	2.5*	(1.8)	22.8
tms	-5.6***	(-3.4)	-3.4**	(-2.0)	-3.1**	(-2.0)	-0.4	(-0.3)	20.4
dfy	-4.9***	(-3.1)	-3.8**	(-2.2)	-3.0**	(-2.0)	1.8	(1.4)	21.6
infl	-5.3***	(-3.4)	-3.4**	(-2.0)	-2.9**	(-2.0)	-1.5*	(-1.8)	21.3
ntis	-6.5***	(-4.2)	-2.7*	(-1.9)	-3.0**	(-2.1)	2.7	(1.2)	22.9
svar	-5.4***	(-3.6)	-3.5**	(-2.0)	-3.0*	(-2.0)	1.3*	(1.9)	21.1
cay	-5.4***	(-3.5)	-4.7**	(-2.5)	-2.9*	(-2.0)	-3.3***	(-2.9)	24.2
skvw	-5.4***	(-3.6)	-3.5**	(-2.1)	-3.0*	(-1.9)	-0.2	(-0.2)	20.4
tail	-4.8***	(-3.0)	-3.2**	(-2.0)	-3.0**	(-2.0)	1.8	(1.3)	21.4
disag	-5.9***	(-3.9)			-2.4*	(-1.8)	-2.2	(-1.1)	18.2
all_pls	-4.3***	(-2.8)	-3.1*	(-1.9)	-3.0**	(-2.0)	3.5**	(2.5)	24.7

Table 6: State-dependent relation between belief skewness and market returns with control variables

The table reports the estimates of the specification in (24) augmented with the control variable listed in the row header. The control variables and the construction of the table otherwise follows the one of Table 5. *, **, and *** denote significance at the 10%, 5%, and 1% levels, respectively.

Horizon	1m											
	$\mathbb{1}_{m>\text{perc}}$		s		$\mathbb{1}_{m>\text{perc}} \times s$		Ctr		Adj R^2	$g_2 + g_3$		
Panel A. Specification with s_1												
d_p	15.5	(1.4)	-5.3*	(-1.9)	**28.89**	(3.35)	3.4	(1.3)	1.7	23.5***	(2.9)	
e_p	15.7	(1.4)	-5.5**	(-2.0)	**29.53**	(3.43)	4.0	(1.3)	1.8	24.1***	(3.0)	
b_m	14.4	(1.3)	-5.0*	(-1.8)	**28.63**	(3.32)	2.7	(1.0)	1.6	23.6***	(2.9)	
tms	10.1	(1.0)	-5.1*	(-1.8)	**29.07**	(3.35)	-2.1	(-0.8)	1.5	23.9***	(2.9)	
dfy	11.0	(1.0)	-4.9*	(-1.8)	**28.61**	(3.31)	-0.8	(-0.2)	1.3	23.7***	(2.9)	
infl	11.7	(1.1)	-4.7*	(-1.7)	**28.23**	(3.28)	-1.5	(-0.5)	1.4	23.6***	(2.9)	
ntis	11.3	(1.1)	-4.8*	(-1.8)	**28.44**	(3.27)	0.3	(0.1)	1.3	23.6***	(2.9)	
svar	11.6	(1.1)	-5.1*	(-1.9)	**28.81**	(3.33)	-3.1	(-0.6)	1.7	23.7***	(2.9)	
cay	9.0	(0.8)	-4.6*	(-1.7)	**27.51**	(3.09)	-3.0	(-1.3)	1.6	22.9***	(2.7)	
skvw	11.8	(1.1)	-4.4	(-1.6)	**28.25**	(3.31)	-2.2	(-0.7)	1.5	23.8***	(3.0)	
tail	15.8	(1.4)	-5.1*	(-1.9)	**28.85**	(3.47)	3.3	(1.3)	1.7	23.8***	(3.0)	
disag	11.4	(1.1)	-4.8*	(-1.7)	**28.52**	(3.31)	0.1	(0.0)	1.3	23.7***	(2.9)	
all_pls	19.2	(1.7)	-5.3*	(-1.9)	**28.65**	(3.43)	6.8**	(2.0)	2.8	23.4***	(3.0)	
Panel B. Specification with s_2												
d_p	2.0	(0.2)	-4.8*	(-1.8)	**19.97**	(2.09)	3.5	(1.3)	1.0	15.1*	(1.7)	
e_p	2.1	(0.2)	-5.0*	(-1.8)	**20.70**	(2.16)	4.0	(1.3)	1.2	15.7*	(1.7)	
b_m	0.8	(0.1)	-4.5*	(-1.7)	**19.64**	(2.06)	2.7	(1.0)	0.9	15.1*	(1.7)	
tms	-3.6	(-0.4)	-4.7*	(-1.7)	**19.85**	(2.08)	-2.0	(-0.8)	0.8	15.2*	(1.7)	
dfy	-2.7	(-0.3)	-4.5*	(-1.7)	**19.66**	(2.06)	-0.9	(-0.3)	0.7	15.2*	(1.7)	
infl	-2.1	(-0.2)	-4.2	(-1.6)	**18.94**	(1.99)	-1.2	(-0.5)	0.7	14.7	(1.6)	
ntis	-2.6	(-0.3)	-4.4*	(-1.7)	**19.37**	(2.04)	0.7	(0.2)	0.7	15.0	(1.6)	
svar	-2.0	(-0.2)	-4.5*	(-1.7)	**19.83**	(2.09)	-3.0	(-0.6)	1.0	15.3*	(1.7)	
cay	-3.7	(-0.4)	-3.9	(-1.5)	**20.85**	(2.26)	-3.9*	(-1.7)	1.2	16.9*	(1.9)	
skvw	-1.6	(-0.2)	-3.9	(-1.5)	**19.66**	(2.11)	-2.5	(-0.8)	0.9	15.8*	(1.8)	
tail	3.4	(0.4)	-4.7*	(-1.8)	**21.17**	(2.28)	3.8	(1.5)	1.1	16.5*	(1.9)	
disag	-2.2	(-0.2)	-4.4	(-1.6)	**19.45**	(2.04)	0.0	(-0.0)	0.7	15.1	(1.6)	
all_pls	7.4	(0.8)	-4.7*	(-1.8)	**22.05**	(2.44)	7.3**	(2.1)	2.3	17.3**	(2.0)	

Horizon	3m										
	$\mathbb{1}_{m>\text{perc}}$		s		$\mathbb{1}_{m>\text{perc}} \times s$		Ctr		Adj R^2	$g_2 + g_3$	
Panel A. Specification with s_1											
d_p	4.6	(0.8)	-5.2***	(-2.6)	20.6***	(3.3)	3.4	(1.5)	5.4	15.4***	(2.6)
e_p	4.3	(0.7)	-5.2***	(-2.6)	21.2***	(3.4)	3.4	(1.2)	5.4	15.9***	(2.7)
b_m	3.7	(0.7)	-4.9**	(-2.6)	20.3***	(3.3)	2.9	(1.2)	5.1	15.5***	(2.6)
tms	-0.7	(-0.1)	-5.1***	(-2.7)	21.1***	(3.3)	-2.3	(-1.2)	4.9	16.0***	(2.6)
dfy	0.4	(0.1)	-4.7***	(-2.6)	20.2***	(3.3)	-0.2	(-0.1)	4.4	15.5***	(2.6)
infl	0.5	(0.1)	-4.7**	(-2.6)	20.2***	(3.3)	0.0	(0.0)	4.4	15.5***	(2.6)
ntis	-0.7	(-0.1)	-4.7**	(-2.5)	19.7***	(3.2)	1.4	(0.5)	4.6	15.0**	(2.5)
svar	0.6	(0.1)	-4.8***	(-2.6)	20.3***	(3.3)	-0.7	(-0.2)	4.4	15.5***	(2.6)
cay	-1.6	(-0.3)	-4.5**	(-2.5)	19.3***	(3.2)	-2.4*	(-1.9)	5.0	14.8***	(2.6)
skvw	1.3	(0.3)	-4.2**	(-2.2)	20.0***	(3.4)	-2.8*	(-1.9)	5.2	15.9***	(2.8)
tail	3.3	(0.5)	-4.9***	(-2.7)	20.7***	(3.5)	1.9	(0.9)	4.7	15.8***	(2.8)
disag	0.9	(0.2)	-4.8**	(-2.6)	20.2***	(3.2)	-0.4	(-0.2)	4.4	15.4***	(2.6)
all_pls	7.3	(1.2)	-5.0***	(-2.7)	20.8***	(3.7)	5.3**	(2.0)	7.0	15.8***	(2.9)
Panel B. Specification with s_2											
d_p	-5.1	(-0.9)	-4.6**	(-2.4)	13.5*	(1.8)	3.4	(1.4)	4.3	8.9	(1.2)
e_p	-5.6	(-0.9)	-4.7**	(-2.3)	14.1*	(1.9)	3.4	(1.2)	4.3	9.4	(1.3)
b_m	-6.1	(-1.1)	-4.3**	(-2.3)	13.2*	(1.8)	2.8	(1.1)	4.0	8.9	(1.2)
tms	-10.8**	(-2.1)	-4.6**	(-2.5)	13.7*	(1.8)	-2.3	(-1.2)	3.7	9.1	(1.2)
dfy	-9.5*	(-1.9)	-4.2**	(-2.3)	13.1*	(1.8)	-0.3	(-0.1)	3.2	8.9	(1.2)
infl	-9.3*	(-1.8)	-4.2**	(-2.3)	13.1*	(1.7)	0.2	(0.1)	3.2	8.9	(1.2)
ntis	-10.4**	(-2.0)	-4.3**	(-2.3)	12.6*	(1.8)	1.8	(0.7)	3.6	8.4	(1.2)
svar	-9.2*	(-1.8)	-4.2**	(-2.3)	13.1*	(1.8)	-0.7	(-0.2)	3.3	8.9	(1.2)
cay	-10.3**	(-2.1)	-3.8**	(-2.1)	14.2**	(2.1)	-3.0**	(-2.2)	4.1	10.4	(1.6)
skvw	-8.3	(-1.6)	-3.6*	(-1.9)	13.5*	(1.9)	-3.0**	(-2.0)	4.2	9.9	(1.4)
tail	-5.9	(-0.9)	-4.3**	(-2.4)	14.2**	(2.0)	2.1	(1.0)	3.6	9.9	(1.4)
disag	-8.9	(-1.6)	-4.3**	(-2.3)	12.9*	(1.7)	-0.5	(-0.2)	3.2	8.6	(1.2)
all_pls	-1.3	(-0.2)	-4.4**	(-2.4)	15.6**	(2.2)	5.6**	(2.0)	6.1	11.2*	(1.7)

Horizon	12m										
	$\mathbb{1}_{m>\text{perc}}$		s		$\mathbb{1}_{m>\text{perc}} \times s$		Ctr		Adj R^2	$g_2 + g_3$	
Panel A. Specification with s_1											
d_p	-7.5	(-1.5)	-3.3**	(-2.5)	12.5***	(4.5)	3.0**	(2.0)	20.9	9.2***	(3.8)
e_p	-8.0*	(-1.7)	-3.2**	(-2.5)	12.9***	(4.6)	2.7*	(1.8)	20.4	9.6***	(4.1)
b_m	-8.0*	(-1.6)	-3.0**	(-2.3)	12.2***	(4.4)	2.7*	(1.8)	20.5	9.2***	(3.9)
tms	-11.4**	(-2.4)	-2.8**	(-2.1)	12.2***	(4.2)	-0.6	(-0.4)	17.8	9.4***	(3.8)
dfy	-10.1**	(-2.3)	-2.7**	(-2.1)	11.8***	(4.2)	1.5	(1.3)	18.6	9.2***	(3.8)
infl	-10.7**	(-2.3)	-2.6**	(-2.1)	11.8***	(4.2)	-1.9**	(-2.2)	19.2	9.2***	(3.7)
ntis	-12.5***	(-3.0)	-2.8**	(-2.1)	11.4***	(4.2)	1.8	(0.8)	19.1	8.6***	(3.5)
svar	-11.1**	(-2.4)	-2.7**	(-2.0)	11.9***	(4.1)	1.2	(1.6)	18.3	9.2***	(3.7)
cay	-13.1**	(-2.3)	-2.5**	(-2.0)	11.1***	(3.4)	-2.4**	(-2.3)	20.0	8.6***	(2.9)
skvw	-11.0**	(-2.4)	-2.7**	(-2.1)	12.0***	(4.2)	-0.2	(-0.3)	17.7	9.3***	(3.8)
tail	-6.7	(-1.3)	-3.0**	(-2.2)	12.8***	(5.2)	3.0*	(2.0)	20.8	9.8***	(5.1)
disag	-9.9**	(-2.0)	-3.1**	(-2.1)	12.0***	(4.2)	-1.5	(-0.8)	18.5	8.9***	(3.6)
all_pls	-5.8	(-1.1)	-3.1**	(-2.4)	12.6***	(4.6)	4.2***	(2.8)	24.0	9.5***	(4.1)
Panel B. Specification with s_2											
d_p	-12.5**	(-2.4)	-3.3**	(-2.4)	9.8***	(2.8)	3.0*	(1.9)	20.5	6.5**	(2.0)
e_p	-13.3**	(-2.6)	-3.3**	(-2.4)	10.2***	(2.9)	2.8*	(1.8)	20.0	6.9**	(2.1)
b_m	-13.1**	(-2.5)	-3.0**	(-2.3)	9.5***	(2.7)	2.7*	(1.7)	20.0	6.6**	(2.0)
tms	-16.7***	(-3.2)	-2.9**	(-2.2)	9.4***	(2.6)	-0.6	(-0.4)	17.4	6.5**	(2.0)
dfy	-15.4***	(-3.1)	-2.7**	(-2.0)	9.0**	(2.5)	1.4	(1.1)	18.0	6.3*	(1.9)
infl	-16.1***	(-3.2)	-2.6**	(-2.1)	8.6**	(2.5)	-1.7**	(-2.0)	18.5	6.0*	(1.8)
ntis	-17.6***	(-3.8)	-3.0**	(-2.1)	8.9***	(2.7)	2.1	(0.9)	19.0	5.9*	(1.9)
svar	-16.3***	(-3.2)	-2.7**	(-2.1)	9.1**	(2.5)	1.2*	(1.7)	17.9	6.4*	(1.9)
cay	-17.3***	(-3.5)	-2.4*	(-1.9)	10.3***	(2.7)	-2.7***	(-3.0)	20.2	7.8**	(2.3)
skvw	-16.2***	(-3.3)	-2.8**	(-2.1)	9.3***	(2.6)	-0.2	(-0.3)	17.3	6.5**	(2.0)
tail	-11.3**	(-2.2)	-3.0**	(-2.3)	11.1***	(3.3)	3.2**	(2.1)	20.8	8.1***	(2.8)
disag	-14.9***	(-2.8)	-3.3**	(-2.2)	9.0***	(2.6)	-1.6	(-0.9)	18.2	5.8*	(1.8)
all_pls	-10.2**	(-2.0)	-3.0**	(-2.3)	11.3***	(3.3)	4.4***	(2.9)	24.1	8.2***	(2.7)

Table 7: Belief moments and market return volatility

The table presents the results of the following regression specification: $\sigma_{t+1\to t+K} = g_0 + g_1 m_t + g_2 v_t + g_3 s_{t,j} + g_4 \sigma_{t-K+1\to t} + \epsilon_{t+1\to t+K}$, where $\sigma_{t+1\to t+K}$ is the annualized volatility of daily returns in percent of the S&P 500 total returns in month $t+1$ to month $t+K$ and $\sigma_{t-K+1\to t}$ is its lagged value. Panel A reports the estimation of the regression results without the lagged volatility control variable and Panel B with control variable. T-statistics use Newey and West (1987) standard errors with K lags. *, **, and *** denote significance at the 10%, 5%, and 1% levels, respectively.

Horizon	1m		3m		12m	
	Panel A. Without volatility control					
m	0.6	0.7	0.8	0.7	1.2**	1.0
	(1.28)	(1.54)	(1.34)	(1.28)	(1.98)	(1.55)
v	1.1**	1.2**	1.2*	1.4**	1.5*	1.7*
	(2.05)	(2.21)	(1.91)	(2.03)	(1.70)	(1.75)
s_1	-0.3		0.4		0.9	
	(-0.40)		(0.64)		(1.16)	
s_2		0.1		0.7		1.1
		(0.17)		(1.11)		(1.37)
Adj R^2	1.94	1.89	2.45	2.83	6.85	7.62
	Panel B. With volatility control					
m	0.5	0.4	0.9*	0.7	1.3**	1.2**
	(1.51)	(1.53)	(1.88)	(1.64)	(2.55)	(1.99)
v	0.5	0.5	0.6	0.7	0.9	1.0
	(1.38)	(1.47)	(1.09)	(1.24)	(1.08)	(1.18)
s_1	0.1		0.7		0.8	
	(0.27)		(1.39)		(1.01)	
s_2		0.2		0.8*		1.0
		(0.55)		(1.67)		(1.25)
$\sigma_{t-K+1\to t}$	5.7***	5.7***	4.1***	4.1***	1.7***	1.7**
	(7.49)	(7.51)	(6.44)	(6.45)	(2.63)	(2.57)
Adj R^2	36.97	37.00	24.38	24.62	11.24	12.01

Table 8: Belief moments and market return skewness

The table studies the relation between belief moments and return skewness using the following specification: $\text{sk}_{t+1\to t+K} = g_0 + g_1 m_t + g_2 v_t + g_3 s_{t,j} + g_4 \text{sk}_{t-K+1\to t} + \epsilon_{t+1\to t+K}$. In Panel A, $\text{sk}_{t+1\to t+K}$ equals the coefficient of skewness in percent of the daily S&P 500 total returns in month $t+1$ to month $t+K$, whereas Panel B uses mean-median skewness in Equation (17) of daily S&P 500 total returns in the K-month period. T-statistics use Newey and West (1987) standard errors with K lags. *, **, and *** denote significance at the 10%, 5%, and 1% levels, respectively.

Horizon	1m		3m		12m	
	Panel A. Coefficient of return skewness					
m	4.6*	3.7	9.5***	9.2***	16.4***	17.1***
	(1.71)	(1.41)	(3.02)	(2.97)	(3.51)	(3.20)
v	-0.5	-0.5	-1.1	-1.6	1.3	0.6
	(-0.15)	(-0.15)	(-0.32)	(-0.46)	(0.28)	(0.13)
s_1	4.1		1.4		-2.9	
	(1.41)		(0.40)		(-0.40)	
s_2		3.5		-0.2		-4.0
		(1.12)		(-0.05)		(-0.65)
$\text{sk}_{t-K+1\to t}$	0.9	1.0	11.8***	11.9***	9.4	9.1
	(0.33)	(0.35)	(3.42)	(3.48)	(1.23)	(1.18)
Adj R^2	(-0.06)	(-0.15)	5.84	5.80	8.06	8.19
	Panel B. Mean-median return skewness					
m	1.0	0.6	0.9	0.7	1.3***	1.2***
	(1.53)	(1.07)	(1.53)	(1.35)	(2.93)	(2.82)
v	0.1	0.1	-0.2	-0.2	0.1	0.0
	(0.10)	(0.14)	(-0.31)	(-0.29)	(0.18)	(0.01)
s_1	1.6**		0.6		0.4	
	(2.25)		(1.15)		(1.06)	
s_2		1.4**		0.6		0.1
		(2.02)		(0.97)		(0.30)
$\text{sk}_{t-K+1\to t}$	0.3	0.3	1.0**	1.0**	0.5	0.5
	(0.47)	(0.45)	(2.09)	(2.05)	(1.00)	(0.95)
Adj R^2	0.49	0.31	1.79	1.70	7.42	6.87

Online Appendix to Accompany the Paper

"Belief Skewness in the Stock Market"

(For Online Publication)

This Online Appendix summarizes robustness checks mentioned in the main text.

A Robustness checks

A.1 Alternative standard errors

To ensure accurate inference, we have used Newey and West (1987) standard errors throughout our analysis. In Panel A of Table OA.4, we also show that the results in Table 3 in the main text are robust to using Hansen and Hodrick (1980) and Hodrick (1992) standard errors. Even though these alternative standard errors generally reduce the magnitude of the t-statistics, the relation between belief skewness and market returns controlling for the first two belief moments documented in Table 3 is still significantly negative at the three-month and one-year horizons.

Panel A of Table OA.2 similarly shows that the variation of the relation between belief skewness and market returns as a function of high average beliefs is significant for Hansen and Hodrick (1980) and Hodrick (1992) standard errors for all horizons with the mean-median belief skewness proxy. For the max-min belief skewness proxy, the results are still consistent with the Newey and West (1987) results at the one-month and one-year horizon in the main text for Hansen and Hodrick (1980) standard errors. Moreover, the positive relation in high-average-belief states is no longer statistically significant with Hodrick (1992) standard errors. These results suggest that the state-dependent results are stronger with the mean-median belief skewness proxy.

A.2 Moving average regressors and bootstrap

Following Hodrick (1992), we study the long-horizon relation with the regression:

$$R_{t+1} = g_0 + g_1 m^{\mathrm{MA}}_{t-K+1\to t} + g_2 v^{\mathrm{MA}}_{t-K+1\to t} + g_3 s^{\mathrm{MA}}_{t-K+1\to t,j} + \epsilon_{t+1}, \tag{A.1}$$

where $j = 1, 2$, R_{t+1} is the annualized monthly log excess return of the S&P 500 total return index in percent, and $\cdot^{\mathrm{MA}}_{t-K+1\to t}$ denotes the K-month average of the indicated variable. The intuition for the regression in (A.1) is that the relation between a predictor variable X_t and the K-month log excess return $R_{t+1\to t+K}$ depends on $\mathrm{Cov}\left(R_{t+1} + \cdots + R_{t+K},\ X_t\right)$, which is equivalent to $\mathrm{Cov}\left(R_{t+1}, X_{t-K+1} + \cdots + X_t\right)$ for stationary series. Thus, the regression in (A.1) effectively analyzes horizon effects without overlapping returns. Panel B of Table OA.4 contains the regression results. These results confirm the previous results both in terms of magnitude and significance. Notice hereby that the adjusted regression R^2 is lower than previously for the three-month and one-year horizons because the dependent variable at these horizons is the monthly return.

As Stambaugh (1999) shows, coefficients in predictive regressions can be biased, especially if the predictor variable follows a persistent autoregressive process and the innovations in the predictor variable and returns are correlated. Below, we study the magnitude of a possible bias in the predictive regression with bootstrap simulations.

The bootstrap follows Yu (2011), adapted to our setting with multiple predictor variables. We set the true coefficients equal to the estimates of (A.1) in Panel B of Table OA.3. The belief moments follow a first-order autoregressive process with the coefficients given in Table 1 in the paper. The residuals are drawn with replacement from their joint empirical distribution and used to construct 10,000 artificial samples. The bias equals the difference between the average estimate computed with the artificial samples and the estimate computed with the actual sample.

The bias is reported in Panel B of Table OA.4 below the t-statistics. The bias is small relative to the coefficient estimates in all specifications. The bias for the belief dispersion is thereby of similar magnitude than the one reported by Yu (2011) on a shorter period. The bias for the belief skewness proxies is of an order of magnitude lower than the one for the average belief and belief dispersion. For example, the bias for the slope on s_1 is -0.053 compared to a coefficient estimate of -5.98, whereas it is -0.165 for belief dispersion, which has a slope coefficient of -5.83. The lower bias for the belief skewness proxies is perhaps not surprising since the belief skewness proxies have relatively low auto-correlation and a lower auto-correlation reduces the absolute value of the bias.

Following Yu (2011), we also conduct a second bootstrap simulation that is identical to the first simulation except that the slope coefficients in A.1 are set to zero. We then compute the p-value for the null hypothesis that a given coefficient on the belief moments is zero by comparing its actual t-statistics with the t-statistic obtained in a second bootstrap simulation. The resulting p-values are reported in brackets in Panel B of Table OA.1 and lead to the same inference than the t-statistics.

In Panel B of Table OA.2, we report the estimates of the state-dependent regression specification using moving average variables:

$$R_{t+1} = g_0 + g_1 \mathbb{1}_{m^{\text{MA}}_{t-K+1\to t} > \text{perc}} + g_2 s^{\text{MA}}_{t-K+1\to t,j} + g_3 \mathbb{1}_{m^{\text{MA}}_{t-K+1\to t} > \text{perc}} \times s^{\text{MA}}_{t-K+1\to t,j} + \epsilon_{t+1\to t+K}. \tag{A.2}$$

In that panel, we also report p-values computed for the null hypothesis of no predictability using the artificial data from the second bootstrap simulation used in Panel B of Table OA.4.

The estimated coefficients for (A.2) in Panel B of Table OA.2 are similar in magnitude across the different horizons and have the same sign than in the baseline analysis. When the (moving average of the) average belief is above its 90th percentile, the relation between the (moving average of) belief skewness and returns is significantly positive in all but one case. When the (moving average of the) average belief is below its 90th percentile, the relation between the (moving average of) belief skewness and returns is still significantly negative at the one-month and one-quarter horizons. Taken together, these results corroborate the baseline results.

A.3 Alternative proxy for average belief or belief bias

In this section of the online appendix, we use Baker and Wurgler's (2006) sentiment index orthogonalized to economic conditions as an alternative proxy for excessive optimism. Table OA.3 reports the results for a dummy variable based on the most recent value of the sentiment index in Panel A and the 18-month average as in Huang et al. (2021) in Panel B. The estimates confirm the state-dependent relation in Panel A of Table 4 for both dummy versions. In Panel A that uses the most recent value of the sentiment index, the coefficients are thereby statistically significant in eight out of twelve cases. With the 18-month average in Panel B, there is statistical significance only at the three-month horizon.

A.4 Alternative return skewness proxies

In this section of the online appendix, we consider three alternative return skewness proxies. A max-min skewness measure applies Equation (18) to daily returns, and two quantile based skewness measures use the 10th and 90th percentiles and the 25th and 75th percentile instead of the minimum and maximum in that formula. Table OA.4 replicates Table 8 in the paper for these alternative proxies. The return skewness proxies confirm a positive relationship between belief skewness and return skewness at the one-month horizon that is statistically significant for the two quantile based skewness proxies. At longer horizons, the relation is not reliable.

Table OA.1: Belief moments and market returns: alternative standard errors and moving average regressors

Panel A replicates Table 3 with t-statistics in parenthesis using Hansen and Hodrick (1980) and Hodrick (1992) standard errors in the first and second row below the estimate, respectively. Panel B reports, for $j = 1, 2$, the estimates of the regression: $R_{t+1} = g_0 + g_1 m^{\text{MA}}_{t-K+1\to t} + g_2 v^{\text{MA}}_{t-K+1\to t} + g_3 s^{\text{MA}}_{t-K+1\to t,j} + \epsilon_{t+1}$, where R_{t+1} is the annualized monthly log excess return of the S&P 500 total return index, $\cdot^{\text{MA}}_{t-K+1\to t}$ denotes the K-month average of the indicated variable, and all independent variables are standardized to have unit variance and zero mean. The t-statistics in parenthesis below the estimates are computed with Newey and West (1987) standard errors. The number with three decimal places below the t-statistics is the bootstrapped bias of the coefficient estimates. The number in brackets is a two-tailed bootstrapped p-value for the null hypothesis of no predictability. We explain the two bootstrap simulations in Section A.2. *, **, and *** denote significance at the 10%, 5%, and 1% levels, respectively.

Horizon	1m		3m		12m	
Panel A. Hansen and Hodrick (1980) and Hodrick (1992)						
m	-5.2**	-4.3*	-5.5**	-4.6**	-6.1***	-5.5***
	(-1.98)	(-1.74)	(-2.46)	(-2.13)	(-3.48)	(-2.94)
	(-1.82)*	(-1.56)	(-1.98)**	(-1.68)*	(-2.24)**	(-2.00)**
v	-2.1	-2.2	-1.9	-2.0	-3.3*	-3.5*
	(-0.85)	(-0.85)	(-0.85)	(-0.87)	(-1.72)	(-1.76)
	(-0.81)	(-0.81)	(-0.81)	(-0.84)	(-1.53)	(-1.61)
s_1	-4.4*		-4.4**		-3.2*	
	(-1.69)		(-2.25)		(-1.82)	
	(-1.43)		(-2.01)**		(-1.88)*	
s_2		-3.8		-3.8**		-3.0*
		(-1.53)		(-2.07)		(-1.74)
		(-1.26)		(-1.80)*		(-1.91)*
Adj R^2	0.67	0.55	3.49	3.14	20.66	20.52
Panel B. Moving average regressors						
m	-5.2*	-4.3*	-5.6**	-4.3*	-7.5***	-5.7***
	(-1.90)	(-1.65)	(-2.31)	(-1.84)	(-3.59)	(-2.77)
	-0.165	-0.167	-0.173	-0.180	-0.178	-0.183
	[0.06]*	[0.11]	[0.03]**	[0.08]*	[0.00]***	[0.02]**
v	-2.1	-2.2	-3.1	-3.3	-5.8**	-6.5**
	(-0.83)	(-0.83)	(-1.22)	(-1.26)	(-2.03)	(-2.07)
	-0.138	-0.141	-0.154	-0.152	-0.165	-0.159
	[0.41]	[0.41]	[0.24]	[0.23]	[0.07]*	[0.06]*
s_1	-4.4		-5.8**		-6.0*	
	(-1.55)		(-2.31)		(-1.84)	
	0.005		0.047		-0.053	
	[0.14]		[0.02]**		[0.09]*	
s_2		-3.8		-5.3**		-6.0*
		(-1.38)		(-2.17)		(-1.78)
		-0.009		0.052		-0.040
		[0.18]		[0.03]**		[0.10]
Adj R^2	0.67	0.55	1.04	0.91	2.04	2.09

Table OA.2: State-dependent relation between belief skewness and market returns: alternative standard errors and moving average specification
Panel A replicates Table 4 with t-statistics in parenthesis using Hansen and Hodrick (1980) and Hodrick (1992) standard errors in the first and second row below the estimate, respectively. Panel B reports the estimates of the regression: $R_{t+1} = g_0 + g_1 \mathbb{1}_{m^{\text{MA}}_{t-K+1\to t}>\text{perc}} + g_2 s^{\text{MA}}_{t-K+1\to t,j} + g_3 \mathbb{1}_{m^{\text{MA}}_{t-K+1\to t}>\text{perc}} \times s^{\text{MA}}_{t-K+1\to t,j} + \epsilon_{t+1\to t+K}$. The t-statistics in parenthesis are computed with Newey and West (1987) standard errors. The two-tailed bootstrapped p-value for the null hypothesis of no predictability is reported in brackets. *, **, and *** denote significance at the 10%, 5%, and 1% levels, respectively.

Horizon	1m		3m		12m	
	Panel A. Hansen and Hodrick (1980) and Hodrick (1992)					
$\mathbb{1}_{m>\text{perc}}$	11.5	-2.2	0.5	-9.3*	-11.1**	-16.3***
	(1.21)	(-0.27)	(0.11)	(-1.92)	(-2.50)	(-2.95)
	(0.90)	(-0.23)	(0.05)	(-1.09)	(-1.33)	(-1.90)*
s_1	-4.8*		-4.7**		-2.8*	
	(-1.94)		(-2.43)		(-1.90)	
	(-1.67)*		(-2.17)**		(-1.56)	
$\mathbb{1}_{m>\text{perc}} \times s_1$	28.5***		20.2***		12.0***	
	(3.87)		(3.05)		(4.53)	
	(2.55)**		(2.32)**		(2.06)**	
s_2		-4.4*		-4.2**		-2.8*
		(-1.82)		(-2.28)		(-1.90)
		(-1.55)		(-1.98)**		(-1.69)*
$\mathbb{1}_{m>\text{perc}} \times s_2$		19.5**		13.0		9.3**
		(2.11)		(1.60)		(2.32)
		(2.00)**		(1.62)		(1.31)
Adj R^2	1.52	0.86	4.55	3.41	17.88	17.42
$g_2 + g_3$	23.7***	15.1*	15.5**	8.8	9.2***	6.5*
	(3.45)	(1.70)	(2.45)	(1.11)	(4.55)	(1.80)
	(2.19)**	(1.62)	(1.83)*	(1.14)	(1.66)*	(0.94)
	Panel B. Moving average regressors					
$\mathbb{1}_{m>\text{perc}}$	11.5	-2.2	6.1	-7.9	8.2	-12.3***
	(1.09)	(-0.26)	(0.78)	(-1.25)	(0.77)	(-2.60)
	[0.30]	[0.81]	[0.47]	[0.25]	[0.54]	[0.06]*
s_1	-4.8*		-5.6**		-2.9	
	(-1.80)		(-2.35)		(-1.28)	
	[0.08]*		[0.02]**		[0.23]	
$\mathbb{1}_{m>\text{perc}} \times s_1$	28.5***		24.7***		27.7***	
	(3.30)		(2.87)		(2.92)	
	[0.00]***		[0.02]**		[0.06]*	
s_2		-4.4*		-5.0**		-3.3
		(-1.67)		(-2.25)		(-1.45)
		[0.11]		[0.03]**		[0.18]
$\mathbb{1}_{m>\text{perc}} \times s_2$		19.5**		14.7*		17.9***
		(2.04)		(1.65)		(3.03)
		[0.07]*		[0.16]		[0.05]**
Adj R^2	1.52	0.86	1.48	0.97	1.31	1.31
$g_2 + g_3$	23.7***	15.1*	19.2**	9.6	24.8***	14.5***
	(2.89)	(1.65)	(2.32)	(1.12)	(2.73)	(2.69)
	[0.01]**	[0.14]	[0.06]*	[0.34]	[0.09]*	[0.09]*

Table OA.3: Sentiment-dependent relation between belief skewness and market returns

The table contains a variation of the analysis in Panel A of Table 4 that uses Baker and Wurgler's (2006) sentiment index orthogonalized to macroeconomic conditions, SENT_t, instead of the bias for the dummy variable. Panel A uses the level of the sentiment index and Panel B uses its 18-month rolling average. The table is otherwise constructed as Table 4 in the main text. *, **, and *** denote significance at the 10%, 5%, and 1% levels, respectively.

Horizon	1m		3m		12m	
Panel A. Indicator variable using sentiment						
$\mathbb{1}_{\text{SENT}>\text{perc}}$	-12.5	-13.2	-13.0**	-14.0*	-13.0***	-14.3**
	(-1.22)	(-1.26)	(-1.96)	(-1.95)	(-2.85)	(-2.52)
s_1	-4.2		-4.8**		-2.2	
	(-1.63)		(-2.47)		(-1.44)	
$\mathbb{1}_{\text{SENT}>\text{perc}} \times s_1$	7.9		10.4**		7.1***	
	(1.03)		(2.44)		(2.61)	
s_2		-4.3*		-4.8**		-2.6*
		(-1.66)		(-2.55)		(-1.75)
$\mathbb{1}_{\text{SENT}>\text{perc}} \times s_2$		6.2		7.9*		3.9**
		(0.82)		(1.87)		(2.18)
Adj R^2	0.60	0.55	4.23	3.78	11.25	9.97
$g_2 + g_3$	3.7	1.9	5.7	3.1	4.9**	1.3
	(0.51)	(0.27)	(1.48)	(0.83)	(2.04)	(1.20)
Panel B. Indicator variable using average sentiment						
$\mathbb{1}_{\text{SENT}>\text{perc}}$	-11.2	-11.4	-9.4	-8.4	-6.7	-5.6
	(-1.28)	(-1.34)	(-1.35)	(-1.17)	(-0.97)	(-0.76)
s_1	-2.0		-3.3		-0.8	
	(-0.75)		(-1.55)		(-0.41)	
$\mathbb{1}_{\text{SENT}>\text{perc}} \times s_1$	2.0		9.7***		5.6	
	(0.26)		(2.94)		(1.47)	
s_2		-2.8		-3.8**		-1.5
		(-1.09)		(-1.99)		(-0.89)
$\mathbb{1}_{\text{SENT}>\text{perc}} \times s_2$		3.9		9.7***		2.6
		(0.51)		(3.38)		(1.07)
Adj R^2	(-0.08)	0.04	1.57	1.82	2.09	1.37
$g_2 + g_3$	0.1	1.1	6.5**	5.9***	4.9	1.2
	(0.01)	(0.15)	(2.53)	(2.73)	(1.43)	(0.65)

Table OA.4: Belief moments and market return skewness: alternative return skewness measures

The calculations and construction of the table follow the one for Table 8 in the main text, but use different return skewness measures. Panel A applies the skewness formula in Equation (18) to the daily returns in the K-month period. The skewness measures in Panels B and C use the same formula, but use the 10th and 90th percentiles and 25th and 75th percentiles of daily returns in the K-month period instead of their minimum and maximum. *, **, and *** denote significance at the 10%, 5%, and 1% levels, respectively.

Horizon	1m		3m		12m	
	Panel A. Max-min return skewness					
m	2.1**	1.7*	2.9***	2.6***	2.9**	3.0**
	(2.06)	(1.79)	(2.80)	(2.64)	(2.04)	(2.06)
v	-0.3	-0.3	-0.5	-0.6	2.8*	2.7*
	(-0.27)	(-0.26)	(-0.41)	(-0.47)	(1.84)	(1.70)
s_1	1.6		1.3		-0.1	
	(1.53)		(1.19)		(-0.08)	
s_2		1.3		0.8		-0.5
		(1.21)		(0.76)		(-0.33)
$\text{sk}_{t-K+1\to t}$	0.3	0.4	4.0***	4.0***	1.9	1.9
	(0.32)	(0.33)	(3.34)	(3.35)	(0.96)	(0.95)
Adj R^2	0.20	0.09	6.71	6.49	8.11	8.17
	Panel B. 90th-10th-percentile return skewness					
m	0.8	0.4	0.0	-0.1	1.2	1.1
	(0.75)	(0.37)	(-0.06)	(-0.16)	(1.31)	(1.24)
v	-0.3	-0.3	-0.1	-0.2	-0.4	-0.6
	(-0.26)	(-0.24)	(-0.11)	(-0.17)	(-0.35)	(-0.51)
s_1	1.9*		0.3		0.4	
	(1.88)		(0.41)		(0.63)	
s_2		1.7*		0.1		-0.2
		(1.66)		(0.17)		(-0.26)
$\text{sk}_{t-K+1\to t}$	0.6	0.6	2.1***	2.1***	0.7	0.7
	(0.57)	(0.56)	(2.82)	(2.81)	(0.97)	(0.90)
Adj R^2	0.16	0.00	1.92	1.88	2.11	1.97
	Panel C. 75th-25th-percentile return skewness					
m	0.2	-0.6	0.5	0.2	0.5	0.6
	(0.15)	(-0.49)	(0.46)	(0.21)	(0.52)	(0.66)
v	0.9	1.1	0.2	0.3	0.6	0.5
	(0.75)	(0.86)	(0.14)	(0.23)	(0.57)	(0.40)
s_1	3.6**		1.3		-0.5	
	(2.51)		(1.15)		(-0.52)	
s_2		3.4**		1.4		-0.8
		(2.55)		(1.19)		(-1.02)
$\text{sk}_{t-K+1\to t}$	1.2	1.1	0.2	0.1	2.0*	2.0*
	(1.03)	(0.99)	(0.18)	(0.13)	(1.67)	(1.69)
Adj R^2	0.94	0.86	(-0.29)	(-0.23)	4.79	5.21

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