Bitcoin Mining for Carbon Emission Reduction

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(Comments welcomed)

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Abstract

While Bitcoin mining consumes a huge amount of electricity, does it necessarily translate into increased carbon emission? In an analytical model featuring endogenous renewable energy adoption decisions, we show that with appropriate electricity price policies, the high electricity demand from Bitcoin mining may actually subsidize the capacity building of renewable energy plants and thus *lower* total carbon emission. A key intuition is that unlike other electricity uses, Bitcoin mining intensity can be elastically dialed up or down without disrupting operations, and thus can replace fuel-based electricity generation as an effective shock absorber for the volatile supply of renewable electricity generation. To corroborate this seemingly counterintuitive result at first sight, we are working on quantifying the potential reductions in carbon emission from introducing Bitcoin mining by calibrating the models with empirical data.

Keywords: bitcoin, blockchain, carbon emission, mining, renewable energy, sustainability

1 Introduction

Bitcoin mining (or more generally the operations of proof-of-work blockchains) has long been criticized for its high electricity consumption. Media headlines often depict Bitcoin mining as being estimated to consume as much electricity as a middle-size country (e.g., Norway). Some studies even further extrapolate these estimates and equate electricity consumption to carbon emission and environmental impact. For example, several years ago Mora, Rollins, Taladay, Kantar, Chock, Shimada and Franklin (2018) even made a bold claim that Bitcoin mining can single-handedly push global warming up by 2°C. Given the recent political shift and potential initiatives by the U.S. as well as foreign governments to launch "strategic bitcoin reserves," a more thorough understanding of the environmental impact of Bitcoin mining becomes even more pressing.[1]

Although it is indisputable that Bitcoin mining indeed consumes a huge amount of electricity, we point out that it requires more careful studies to draw a further conclusion of whether the high energy consumption necessarily drives more carbon emission or climate damage. Indeed, if policymakers can design appropriate incentives so that Bitcoin mining is induced to mainly use electricity generated from renewable sources, or if the rise of Bitcoin mining further incentivizes the capacity building of renewable energy plants, the high electricity demand from Bitcoin mining may actually subsidize renewable energy adoption, and thus (maybe counter-intuitively to some people) actually *lower* carbon emission and help fight against global warming. Sporadic media reports on Bitcoin mining consuming excess electricity supply or a high percentage of "clean" energy also suggest an empirical possibility of this desirable outcome,[2] although there are also plenty of counter-arguments doubting the scientific merit of such arguments.[3] Hence, there is an urgent need from the academic

[1] See for example, the Strategic Bitcoin Reserve Bill proposed by U.S. Senator Cynthia Lummis and the latest Washington Post article reporting on Trump's strategic bitcoin reserve plan.

[2] See for example the Forbes article reporting on Texas Bitcoin miners' adoption of renewable energies.

[3] In addition to the usual environment-focused arguments, a Texas Tribune article also reported that

community to develop neutral and rigorous analyses to weigh in the debate and provide guidance for practitioners and regulators, although unfortunately, there is so far little formal academic analysis.

To fill the urgently needed literature gap, this paper develops an analytical model to critically evaluate whether Bitcoin mining can indeed reduce carbon emission, and if so, what policy intervention are needed to make it happen, if any (for example, how should the grid or utility regulators design electricity demand response programs to both stabilize the grid and lower carbon footprint). To reiterate, we formalize the informal reasoning introduced in the previous paragraph in a rigorous analytical model, and demonstrate that the increasing electricity consumption caused by Bitcoin mining indeed may NOT necessarily lead to increasing carbon emission. On the contrary, our analysis shows that with proper demand response programs, Bitcoin mining may actually help *reduce* carbon emissions. However, one can only expect such desirable outcomes if utility regulators adopt well-designed policy interventions, as we derive in the paper.

Several key characteristics of both renewable energy and Bitcoin mining help realize the above-mentioned possibility. First, with respect to renewable energy, electricity generation from renewable sources features (1) high fixed cost (to initially build up capacity), (2) low or even zero variable cost to produce electricity generation (once capacity is already built in place), and (3) imperfect correlation between renewable energy production (supply) and electricity demand. To see these features, note that with respect to (1) and (2), electricity generation from solar/hydro/wind all feature high initial cost to purchase panels/build dams/install turbines, but once in place these renewable sources can also generate electricity almost for free; With respect to (3), electricity supply from solar/hydro/wind goes down during the night/dry season/low winds and may vary with volatile short-term weather changes, yet such supply shocks are unlikely to be perfectly correlated with households or industry

Texas leaders worry that Bitcoin mines may threaten to crash the state power grid.

electricity demands.

Given that electricity has to be consumed as soon as it is produced,[4] characteristic (3) above (imperfect supply-demand correlation) imposes a serious friction against renewable energy adoption: Fuel energy, whose supply is more controllable, has to be in place to synchronize electricity demand and supply, despite significant carbon footprints. On the other hand, the high fixed cost involved in building renewable energy capacity makes it difficult commercially, especially given that much electricity demand has to be met by fuel energies for the reason just mentioned.

All the pain points enumerated above can almost be perfectly resolved by Bitcoin mining, thanks to the latter's two salient characteristics: (4) Bitcoin mining consumes a lot of electricity, and more importantly (5) Bitcoin mining is memoryless (as it only involves brute-force hash operations), meaning that it can be turned off without compromising continued mining operations once turned on again — indeed, Bitcoin mining can endure disruptions at extremely high frequencies (e.g., about 10^{-15} second for the Antminer S19 ASIC mining rig).

Characteristic (4) ensures that Bitcoin mining creates sizable demand for electricity, which helps prospective renewable energy providers overcome the barrier of high fixed cost, especially smooth over time periods during which other household/industry electricity demands are low. On the other hand, during time periods of peak household/industry electricity demands, characteristic (5) ensures that Bitcoin mining can be temporarily shut down to prioritize other difficult-to-disrupt electricity needs. The shutdown can also be implemented voluntarily given well-designed demand response programs, which changes the electricity prices charged to Bitcoin miners depending on the demand/supply imbalance in the grid (indeed, such practices has already been adopted by ERCOT — the electrical grid in the

[4] Although energy storage solutions like batteries/pumps do exist and are developing, their capacities today are still limited to negligible levels compared to total electricity generation.

state of Texas). In short, Bitcoin mining's large electricity demand subsidizes renewable capacity building, its flexible electricity demand substitutes fuel energy for demand-supply synchronization, and their unique combination helps Bitcoin mining relieve reliance on fuel energy and reduce carbon emission.

It is also worth noting that the combination of both large and flexible electricity demand (point (4) and (5) above) is quite unique to Bitcoin mining: Other large electricity consumers (e.g., electrolysis for aluminum production, data centers, or increasingly AI training for large models) cannot be easily turned off and resume later without incurring large disruptions or lengthy "warm-up" periods. Therefore Bitcoin mining enjoys a unique advantage in reducing carbon emission and safeguarding toward a sustainable future.

We develop an analytical model to formalize the above reasoning. The model features an arbitrarily given number of Bitcoin miners, a representative non-mining household/industry with volatile electricity demands, and a profit-maximizing grid (which decides *ex ante* the optimal level of renewable capacity to build and *ex post* the optimal level of electricity to generate) taking as given the pricing policies set by the regulator. All model ingredients closely follow the background described earlier in the introduction and include: (1) stochastic energy demand (from non-Bitcoin mining related household/industry needs); (2) stochastic renewable energy supply (or more precisely, the stochastic renewable energy capacity factor, or utilization ratio defined as the ratio of actual electrical energy output over a given period of time to the theoretical maximum electrical energy output over that period; the stochastic renewable energy supply is the product of this stochastic ratio and the renewable capacity in place); (3) costly but controllable electricity supply from fuel energy; (4) high fixed cost for renewable energy capacity building; (5) high (low) carbon emission from fuel (renewable) sources for electricity generation. We will further assume a (6) perfectly controllable electricity demand from Bitcoin mining, which is without loss of generality, as we will prove that even with endogenous Bitcoin mining intensity, the grid can always induce a desirable level of bitcoin mining by setting appropriate pricing policies (as known as demand response programs) assuming miners' profit-maximizing incentives. In other words,

Based on the model setup, we characterize the carbon emission implications from the introduction of Bitcoin mining. First, we demonstrate that introducing Bitcoin mining (probably unsurprisingly) always induces higher investment into the capacity building of renewable energy plants. Second, we also show that under fairly reasonable conditions, there also exists electricity pricing policies (demand response programs) to miners under which the introduction of Bitcoin mining (maybe counterintuitive in first sight) helps reduce carbon emission. For a simple demand response program, we also characterize the conditions under which the carbon emission reduction effect from introducing Bitcoin mining holds. We further demonstrate that the profit-maximizing grid earns a higher profit under the carbon-reducing price policies compared to when Bitcoin mining is banned. All these results suggest that the intuition discussed earlier in the introduction is indeed valid.

The key intuition from our analysis is that in the presence of Bitcoin mining, the optimal level of renewable energy capacity also increases. Therefore, all else equal more electricity demands can be fulfilled from renewable sources. To the extent that mining is rationed during periods with extreme spikes of electricity demand, overall a lower quantity of electricity demands will involve fuel energy, and therefore total emission can go down. On the other hand, we also point out a novel insight that Bitcoin mining should not be rationed too heavily (and therefore, the electricity prices charged to miners should not be too high even during periods with supply shortage from renewable sources). This is because the ex post inefficiency of fuel supported mining is the very reason to induce a higher level of ex ante renewable capacity building.

Finally, in ongoing work, we are working on bringing our model to data to quantify the magnitude of potential carbon reductions from Bitcoin mining by calibrating all model parameters to actual market level data. Preliminary results seem promising, as our model places little restrictions under which our key result holds.

We note that the idea of using mining as a shock absorber is grounded in empirical data and practices. Indeed, there are many examples on this aspect:

- First, Yaish and Zohar (2023) survey relevant anecdotes in its Section 1 and Subsection 2.2 regarding two companies that designed ASICs specifically to be used as absorbers — see layer1.com and blockstream.com/energy.

- Furthermore, back when mining was still legal in China, miners would act as de-facto absorbers, moving between regions when the wet season started and ended, due to the season's impact on renewable energy production and costs — see decrypt.co/46601/bitcoin-hash-rate-drop-attributed-to-chinese-rainy-season, https://cointelegraph.com/news/btc-hash-rate-slumps-amid-seasonal-miner-migration-in-china, and news.bitcoin.com/chinese-bitcoin-miners-migrate-north-after-wet-season/.

- In addition, in the United States, miners have also been reported to adjust their operations to accommodate emergencies and special events, see for example media reports at www.cnbc.com/2022/02/03/winter-storm-descends-on-texas-bitcoin-miners-shut-off-to-protect-ercot.html and www.forbes.com/sites/christopherhelman/2020/05/21/how-this-billionaire-backed-crypto-startup-gets-paid-to-not-mine-bitcoin/#7bdc51947596.

- From a technology perspective, since electrical equipment tends to be sensitive, mining ASICs need to be designed so that they can be rapidly switched on and off (so this is a design consideration rather than an artifact). This is indeed in line with the reality. In fact, miners have been reported to switch "at a minute's notice" or "in real time" — see for example numerous mainstream news articles such as bloomberg.com/news/articles/2021-09-03/bitcoin-hash-crash-rebound-points-to-miners-plugging-back-in, cnbc.com/2022/02/03/winter-storm-descends-on-texas-bitcoin-miners-shut-off-to-protect-ercot.html, and forbes.com/sites/christopherhelman/2020/05/21/how-this-billionaire-backed-crypto-startup-gets-paid-to-not-mine-bitcoin/#7bdc51b97596. Also see technical contents such as support.bitmain.com/hc/en-us/articles/360019738593-Difference-between-Low-Power-Mode-and-Low-Power-Enhanced-Mode and technical details from product manuals such as file12.bitmain.com/shop-product-s3/firmware/4e25b493-58d5-4986-8cff-52006dda2038/2022/01/19/17/S19%20Server%20Manual.pdf from mining equipment manufacturers.

- For completely different purposes, Yaish, Tochner and Zohar (2022) show that miners can repeatedly switch their machines on and off to manipulate mining difficulty in a way which can increase their profits: when block rewards are involved this can increase the average returns from mining, and when block rewards are low this can create interest rate arbitrage between lending pools (as some accrue interest per-block, and some accrue according to the time difference between blocks).

Related literature This paper relates to large literature on bitcoin mining. On the theory side, Biais, Bisiere, Bouvard and Casamatta (2019) model bitcoin mining as a coordination game and presents the possibility of multiple equilibrium. Cong, He and Li (2021) studies the dynamics of mining pool formation and show that the rise of mining pools may lead to significant electricity consumption (though it does not talk about the source of electricity and thus not the implications on renewables). Prat and Walter (2021) propose a model that uses the exchange rate of Bitcoin against the US dollar to predict the computing power of Bitcoin's network. Arnosti and Weinberg (2022) shows that bitcoin mining has the tendency toward significant centralization given even small initial differences in variable costs. Chatzigiannis, Baldimtsi, Griva and Li (2022) develops the optimal rule and a robo-advisor for miners to allocate computing power across mining pools, based on the insight in Cong, He and Li (2021). Bertucci, Bertucci, Lasry and Lions (2024) models bitcoin mining as a mean-field game.

On the empirical side, the most related paper to ours is Halaburda and Yermack (2023), who study the operations and financial valuations of 13 cryptocurrency mining companies listed on the NASDAQ stock exchange and have facilities in North America. Their empirical finding largely supports the feasibility of the idea presented here: Specifically, they find that miners using Texas wind power are offline more than other miners, in a more erratic pattern. Their model shows that miners using sustainable energy may be more profitable than those using conventional sources, despite the shutdowns, as they benefit from extremely low prices when there is oversupply of sustainable energy (e.g., strong winds). The model also shows that it may be beneficial for the electric utility to offer miners compensation for curtailment of their activity when there is undersupply of energy (e.g. lack of wind), which we also observe in our sample. This compensation further increases profits of the miners. They find a negative and significant beta between crypto mining stocks and an index of electric utilities, suggesting that ownership of a crypto mining company might provide a useful channel for risk management in the electric power industry. Another related paper is Benetton, Compiani and Morse (2023), who document that cryptocurrency miners' use of local electricity implies higher electricity prices for existing small businesses and households in Upstate New York. However we note that the electricity market in that study does not seem to feature carefully designed demand response programs, so that the empirical findings is not apple-to-apple with our theoretical implications.

Several industry reports also touches on the themes discussed in this paper. For example, Ibañez and Freier (2023) contain numerous verbal arguments on why Bitcoin mining may be environmentally friendly that echos the several key aspects of the current paper, although they do not distill an end-to-end theory. In a Bitcoin Policy Institute report, Margot Paez and Troy Cross document several data patterns observed consistent with bitcoin mining's flexible electricity demand.

2 Model

This section formalizes the key intuitions above in an analytical model. We first lay out a few key modeling ingredients.

2.1 Key model ingredients

The key model ingredients are given as follows:

(1) stochastic energy demand: we assume the quantity of demand for electricity from (non-Bitcoin mining related) household/industry needs within a representative period to follow

$$D = D(\tilde{D}, P), \tag{1}$$

where $\tilde{D}$ is a positive random variable, p is the (unit) electricity price charged to households/industries, and $D(\cdot, \cdot)$ is increasing in $\tilde{D}$ and non-increasing in P.

For simplicity, we will start with $D(\tilde{D}, P) = \tilde{D}$. Other simplifying examples may have $D = \tilde{D} - \lambda P$ where $\lambda > 0$ is the slope of a linear demand function.

(2) stochastic renewable energy supply: for a given level of the renewable capacity k in place (which is determined by costly initial investment ex ante), the quantity of supply of renewable energy within a representative period is given by

$$S = S(\tilde{S}, k), \tag{2}$$

where $\tilde{S}$ is a positive random variable with support in $[0, 1]$ and $S(\cdot, \cdot)$ is increasing in both $\tilde{S}$ and k.

For simplicity, we will assume $S = \tilde{S}k$. In this specification, $\tilde{S}$ as a percentage number has a natural interpretation of being a stochastic utilization rate of renewal energy capacity. In electricity parlance, this ratio is known as the capacity factor, defined as the ratio between actual electrical energy output over a given period of time and the theoretical maximum electrical energy output over that period.

The variable cost of generating electricity from renewable energy is assumed to be zero.

(3) costly but controllable electricity supply from fuel energy: we assume that grid has perfect control over the quantity of supply of fuel-based energy within a representative period. Assuming this quantity is Q, then the (variable) cost incurred would be

$$c \cdot Q, \tag{3}$$

where $c > 0$ captures the (unit) cost of fuels.

(4) high fixed cost for renewable energy capacity building: We assume that it costs the grid

$$C \cdot k \tag{4}$$

to build up the capacity of renewable energy production.

(5) high (low) carbon emission from fuel (renewable) sources for electricity generation: we assume that the emission from any level of renewable energy production is 0, while the emission from Q fuel-based energy production is

$$\theta \cdot Q. \tag{5}$$

Notice that the total emission is proportional to the variable cost incurred by the grid, and hence we will often equate the objective of emission reduction with variable cost reduction.

2.2 Profit-maximizing grid

There are two key decision makers in our model. The first key decision maker is a profit-maximizing grid who decides *ex ante* on how much renewable capacity k to build and *ex post* how much electricity to provide to households/industries or (in their presence) Bitcoin miners (while Bitcoin miners also make endogenous decisions, we will later show that they would not affect our main analysis). The second key decision maker is a regulator (e.g., state-level or regional-level electricity board), who stipulates the price of electricity charged to households/industries or Bitcoin miners.[5]

We assume that the regulator stipulates the grid to charge a fixed price $p > c$ to households/industries. Since p is higher than the variable cost of electricity generation (c in case of fuel-based electricity or 0 in case of renewable-based electricity), the grid will always optimally satisfy *all* households/industry electricity demand.

On the other hand, to bitcoin miners the regulator stipulates a demand response program so that the grid charges a price of $P(\tilde{D}, \tilde{S} \cdot k)$ to Bitcoin miners for every unit of electricity. Under this setup, the grid dynamically adjusts the price charged to bitcoin miners in real time. This assumption makes sense in practice because (1) Bitcoin mining is a memoryless process, so that it can easily be turned on or off and could thus be responsive to price changes; (2) almost all Bitcoin mining today is conducted at large mining farms operating in remote areas with distinct transition lines, and they do actively negotiate electricity price deals with the grid, so that the grid can can effectively price discriminate between households/industries and Bitcoin miners and work out complicated pricing schedules with miners in advance. While it is less feasible to price discriminate against small mining rigs,

[5]Exogenously set price is common in reality. For example, in some states, public service (utility) commissions fully regulate prices, and other states have a combination of unregulated prices (for generators) and regulated prices (for transmission and distribution) — see eia.gov/energyexplained/electricity/prices-and-factors-affecting-prices.php. Our model corresponds to a regulated market where grid vertically integrates power generation and transmission; alternatively, grid can buy electricity from retail market at competitive price and transmit customers — we discuss this case in Section 2.8.

they are negligible in the mining industry anyway.

For a given pricing strategy to miners, and the miners' electricity demand function $d(P, \xi)$, where ξ captures all non-price factors that influences miners' demand function for electricity,[6] the objective function of the grid is now given by

$$\max_k \left\{ \mathbb{E}\left[p\tilde{D} + P(\tilde{D}, \tilde{S}k) \cdot d\left(P(\tilde{D}, \tilde{S}k), \xi\right) - c \cdot \max\{\tilde{D} + d\left(P(\tilde{D}, \tilde{S}k), \xi\right) - \tilde{S}k, 0\} - Ck \right] \right\}, \tag{6}$$

where $p\tilde{D}$ captures the grid's revenue from households/industries, $P(\tilde{D}, \tilde{S}k) \cdot d\left(P(\tilde{D}, \tilde{S}k), \xi\right)$ captures the grid's revenue from Bitcoin miners, $c \cdot \max\{\tilde{D} + d\left(P(\tilde{D}, \tilde{S}k), \xi\right) - \tilde{S}k, 0\}$ captures the grid's cost for serving electricity demand, and Ck captures the grid's cost in building up renewable energy capacity.

2.3 Environmentally-aware regulators

We assume that regulator is environmentally-aware — in that it aims to reduce total emission. Then with the understanding of the grid's profit-maximization problem (6), the regulator's problem is given by

$$\begin{aligned} &\min_{P(\cdot,\cdot,\cdot),p} \mathbb{E}_{\tilde{D},\tilde{S}} \left[\theta \cdot \max\left\{ \tilde{D} + d\left(P(\tilde{D}, \tilde{S}k), \xi\right) - \tilde{S} \cdot k, 0 \right\} \right], \text{ s.t.} \\ k \in \arg\max_k &\left\{ \mathbb{E}\left[p\tilde{D} + P(\tilde{D}, \tilde{S}k) \cdot d\left(P(\tilde{D}, \tilde{S}k), \xi\right) - c \cdot \max\{\tilde{D} + d\left(P(\tilde{D}, \tilde{S}k), \xi\right) - \tilde{S}k, 0\} - Ck \right] \right\}. \end{aligned} \tag{7}$$

In words, the regulator chooses the optimal electricity price to households/industries p, and the optimal demand response program (i.e., pricing policy) to Bitcoin miners $P(\cdot, \cdot, \cdot)$, in order to minize emission (which is proportional to the total excess electricity demand over

[6] ξ may include for example (1) Bitcoin price, (2) the level of mining fixed reward, which halves roughly every four years, and (3) the level of transaction fees paid to miners, which is affected by transaction frequencies or new feature introductions such Bitcoin NFT protocols like Ordinal/Rune.

supply from renewal sources, if any), subject to the grid's optimal decision over renewable capacity k (recall that since we assume the regulator specify $p > c$, the grid always serves all electricity demand — the only variable for the grid to optimize over is k).

2.4 Simplifying the grid's problem (6)

A quick reflection will reveal that the revenue terms in the grid's problem (6) do not actually affect the grid's decisions. This is because on the one hand, the revenue from households/industries $p\tilde{D}$ is completely independent of the grid's decisions, while the the revenue from bitcoin miners $P(\tilde{D}, \tilde{S}k) \cdot d\left(P(\tilde{D}, \tilde{S}k), \xi\right)$ can be interpreted as miners' operating costs, which will also be independent of the electricity price miners are charged. This is because although a higher electricity price will have the direct effect of discouraging mining, this direct effect will be counteracted by an indirect effect from Bitcoin mining difficulty adjustment.

We will formally establish the above argument in Section 2.4.1. That said, even without going into details of the industrial organization of Bitcoin mining, it is immediate that under a first-order approximation of assuming perfect competition among Bitcoin miners, miners' operating costs equal to mining revenues, and the latter may be argued to be independent of the grid's decisions. Indeed, total mining revenue within a given period equals the product of (1) number of blocks produced within the period, which is hard-coded in Bitcoin's protocol given its on average 10 min block time; (2) block reward which consists of fixed reward and exogenous transaction fee revenues; (3) Bitcoin price which is arguably exogenous to the electricity price set by the regulator (this assumption is reasonable in current environments, but may need relaxation if going forward; that said, to the extent that Bitcoin price is influenced by many factors beyond electricity price, the assumption is likely to hold approximately, even if not perfectly; we plan to discuss more about this later or in future research). Therefore, as a first approximation one may interpret our findings in Section 2.5 as a reasonable approximation of the reality already.

The next section will show that the insight we discuss in the above paragraph also applies to settings in which bitcoin mining is not perfectly competitive.

2.4.1 Characterizing the miner demand function

It is our intention to assume a general miner demand function $d(\cdot,\cdot)$. Here we conduct a simply analysis to illustrate how $d(\cdot,\cdot)$ may look like.

As we have already seen in the discussion above, if the mining industry is perfectly competitive, then we can reasonably assume the grid's revenue from Bitcoin miners is independent of the prices charged to miners. Now we instead follow the literature (e.g., Cong, He and Li (2021); Arnosti and Weinberg (2022)) to consider an oligopolistic mining industry. Specifically, assume n symmetric miners. Then each miner i's objective is

$$\max_{d_i} R\frac{d_i}{d_i+\sum_{j\neq i} d_j} - d_i P \tag{8}$$

where d_i is miner i's computing power allocation, R is the total dollar-based block reward within a representative period, and P is the electricity price.

By first order condition, in equilibrium we have

$$R\frac{\sum_{j\neq i} d_j}{(d_i+\sum_{j\neq i} d_j)^2} = P.$$

Summing over all i-s we have that

$$R\frac{(n-1)\sum_j d_j}{(\sum_j d_j)^2} = nP,$$

and thus $\sum_j d_j = R\frac{(n-1)}{nP}$. In other words, we have that

$$d = \frac{(n-1)}{n}\frac{R}{P} \tag{9}$$

Then again we show that with oligopolistic miners, we still have $Pd = \frac{(n-1)}{n} R$ to be exogenous and independent of the grid's decisions.

2.4.2 Demand response program

Equation (9) gives a formula regarding how total bitcoin mining demand responds to the charged electricity price P as well as the bitcoin market (captured by R) and the mining industry conditions (captured by n). Equation (9) is a convenient outcome for demand response programs, in that one can reverse-engineer the real-time price of electricity charged to bitcoin miners to produce any given pattern of the bitcoin mining demand. More precisely, one can just set $P = \frac{(n-1)}{n} \frac{R}{d}$.

Even our goal is to have regulators to initiate demand response programs, that is, to dynamically adjusts the price charged to bitcoin miners and thus indirectly influences the intensity of Bitcoin mining, given Equation (9) and the interpretation above, we see that we can equivalently assume that the regulator can explicitly set the electricity demand from Bitcoin miners $d(\tilde{D}, \tilde{S}k)$. Adopting this treatment will significantly simplify our later analysis, even though it is without loss of generality. We will assume so in all subsequent analysis.

2.5 Duality: The grid's cost minimization problem

It is easily verified that by crossing out the revenue terms that are independent from the grid's decisions and assuming (without loss of generality) perfectly controllable $d(\cdot, \cdot)$, the profit-maximization objective of the grid will be a duality of a cost-minimization problem. In this section, we formally present the grid's cost-minimization problem (with the simplification in 2.4.2 of assuming the regulator being able to directly control the mining intensity through the demand response program), and will then for simplicity always use on the grid's cost-minimization problem instead of its profit-maximization problem in all remaining discussions unless otherwise specified.

To state the grid's cost minimization problem, we have the regulator to stipulate the quantity of electricity demand d from bitcoin mining as a function of household/industry electricity demand D — the realization of $\tilde{D}$, the renewable supply factor S — realization of $\tilde{S}$, the cost of fuel c, and existing renewable capacity k.

With the regulator's ability to control the quantity of electricity demand from bitcoin mining and the optimal decision to fulfill all electricity demands from household/industry electricity needs and Bitcoin mining (because recall the implication of $p > c$), the grid aims to minimize its expected total (initial and variable) cost. Formally, the grid's problem is given by:

$$\min_{k} \left\{ C \cdot k + c \cdot \mathbb{E}[\max\{\tilde{D} + d(\tilde{D}, \tilde{S}) - \tilde{S} \cdot k, 0\}] \right\}. \tag{10}$$

Notice that Equation 10 is the flip coin of the cost terms in the grids profit-maximization problem (6).

2.6 The implication of introducing Bitcoin mining

We are interested in comparing the scenario with optimally regulated Bitcoin mining intensity (induced via an optimal demand response program) and the one in which bitcoin mining is banned (that is, $d = 0$). We are specifically interested in how the introduction of Bitcoin mining affects the grid's incentive in building renewable capacity and the overall effect on total carbon emission. Our first result is summarized in the following proposition.

Proposition 1 (The impact of introducing bitcoin mining)**.** *Compared to a world in which Bitcoin mining is not allowed, introducing Bitcoin mining leads to*

(1) more renewable capacity building (i.e., a higher k);

(3) potentially lower total emission (i.e., a lower $\theta \cdot \mathbb{E}[\max\{\tilde{D} + d(\tilde{D}, \tilde{S}) - \tilde{S} \cdot k, 0\}]$).

The intuition for the proof of Proposition 1 can be visualized in Figure 1.

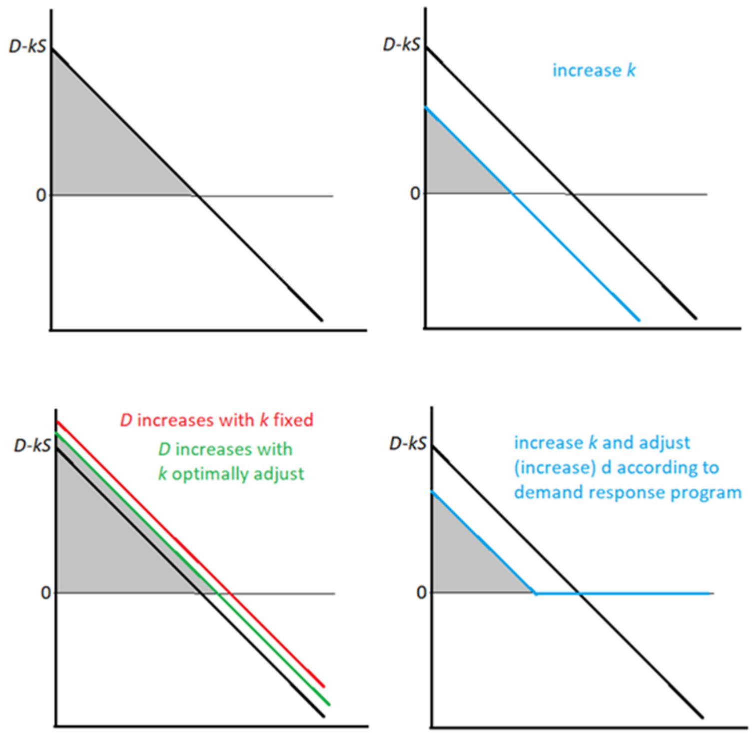


Figure 1: Visualization of the centrally-planned problem

Note that in Figure 1, the upper-left panel illustrates, for a given level of renewable capacity k and a given realization of household/industry demand $\tilde{D}$ and renewable factor $\tilde{S}$, the grey area illustrates (in proportion) the grid's total variable cost and total emission in the baseline case in which no bitcoin mining is introduced. On the upper-right corner, we see that a higher level of renewable capacity k can reduce the grid's total variable cost and total emission (the grey area), but such savings will be counteracted by a higher initial cost of renewable capacity building (not illustrated in the graphs). In order for the grid to increase its optimal level, one may seek an increase in extra electricity demand.

In the lower-left panel, we illustrate a naive approach. In this graph, the quantity of demand increases (from black to red), and the optimal capacity adjusts accordingly (increases, as the green line illustrates). However, (by an intuition similar to that of the Le Chatelier's principle) the change increase in k is not large enough to offset the increase in D nor the grey area, and total emission will increases if we just naively add more electricity demand.

The lower-right panel demonstrates how we can add electricity demand smartly. We only add extra electricity demand (from Bitcoin mining) when household/industry demand exceeds renewable supply, and otherwise cap extra electricity demand from Bitcoin mining to only absorb unconsumed renewable supply (in electricity grid parlance, we "curtail" the electricity demand from bitcoin miners when the load from household/industry exceeds the supply from renewable sources). By adding extra electricity demand (from Bitcoin mining) when household/industry demand exceeds renewable supply, we incentivize the grid to invest in renewable capacity to reduce the probability of such situation happens.

By capping extra electricity demand from Bitcoin mining to only absorb unconsumed renewable supply when household/industry demand does not meet renewable supply, we may be able to not increase emission in this situation. It turns out the lower chance of having emission at all may sometimes trump the extra emission when this does happen, and thus introducing bitcoin mining (with proper control of its intensity) would lower total emission.

2.6.1 A concrete example

In subsequent analysis, we illustrate the intuition of the previous section using a concrete example. Specifically, we assume that $\tilde{D} \sim \text{Uniform}[0, \bar{D}]$ and $\tilde{S} \sim \text{Uniform}[0, \bar{S}]$. We also assume that $\tilde{D}$ and $\tilde{S}$ are independent of each other. Then (10) becomes

$$\min_k \left\{ C \cdot k + c \cdot \frac{1}{\bar{D}\bar{S}} \int_0^{\bar{D}} \int_0^{\bar{S}} \max\{D + d(D, S) - S \cdot k, 0\} dS dD \right\}. \tag{11}$$

We now consider two cases: without bitcoin mining and with bitcoin mining (and a specifically chosen demand response program).

Without bitcoin mining Without bitcoin mining, we get the grid's cost minization problem to be

$$\min_k \left\{ C \cdot k + c \cdot \frac{1}{\bar{D}\bar{S}} \int_0^{\bar{D}} \int_0^{\bar{S}} \max\{D - S \cdot k, 0\} dS dD \right\}.$$

We separate two cases:

If $k\bar{S} \geq \bar{D}$, then the objective becomes

$$\begin{aligned} & \min_k \left\{ C \cdot k + c \cdot \frac{1}{\bar{D}\bar{S}} \int_0^{\bar{D}} \int_0^{\frac{D}{k}} (D - S \cdot k) dS dD \right\} && (12) \\ = \; & \min_k \left\{ C \cdot k + c \cdot \frac{1}{\bar{D}\bar{S}} \int_0^{\bar{D}} \frac{D^2}{2k} dD \right\} && (13) \\ = \; & \min_k \left\{ C \cdot k + c \cdot \frac{1}{\bar{S}} \frac{\bar{D}^2}{6k} \right\} \geq 2\sqrt{\frac{\bar{D}^2 C c}{6\bar{S}}}. && (14) \end{aligned}$$

with equality when $k = \sqrt{\frac{\bar{D}^2 c}{6\bar{S}C}}$.

If $k\bar{S} < \bar{D}$, then the objective becomes

$$\begin{aligned} & \min_k \left\{ C \cdot k + c \cdot \frac{1}{\bar{D}\bar{S}} \int_0^{\bar{S}} \int_{kS}^{\bar{D}} (D - S \cdot k) \, dD dS \right\} && (15) \\ = \; & \min_k \left\{ C \cdot k + c \cdot \frac{1}{\bar{D}\bar{S}} \int_0^{\bar{S}} \left(\frac{1}{2}\bar{D}^2 - \frac{1}{2}(kS)^2 - S \cdot k\bar{D} + (kS)^2 \right) dS \right\} && (16) \\ = \; & \min_k \left\{ C \cdot k + c \cdot \frac{1}{\bar{D}\bar{S}} \left(\frac{1}{2}\bar{D}^2\bar{S} - \frac{1}{2}\bar{S}^2 \cdot k\bar{D} + \frac{1}{6}k^2\bar{S}^3 \right) \right\} && (17) \\ = \; & \min_k \left\{ C \cdot k + c \cdot \frac{1}{\bar{D}} \left(\frac{1}{2}\bar{D}^2 - \frac{1}{2}\bar{S} \cdot k\bar{D} + \frac{1}{6}k^2\bar{S}^2 \right) \right\} && (18) \\ \geq \; & -\frac{3\bar{D}}{2c\bar{S}^2} \left(\frac{1}{2}\bar{S}c - C \right)^2 + \frac{1}{2}c\bar{D}. && (19) \end{aligned}$$

with equality when $k = \frac{3\bar{D}}{c\bar{S}^2}(\frac{1}{2}\bar{S}c - C)$.

Therefore, summarizing the above two cases, we have

$$\begin{cases} \text{If } C > \frac{1}{2}\bar{S}c, & k^* = 0, & \text{total cost} = \frac{1}{2}c\bar{D} \\ & & \text{variable cost} = \frac{1}{2}c\bar{D} \\ \text{If } \frac{1}{6}\bar{S}c < C \leq \frac{1}{2}\bar{S}c, & k^* = \frac{3\bar{D}}{c\bar{S}^2}(\frac{1}{2}\bar{S}c - C), & \text{total cost} = -\frac{3\bar{D}}{2c\bar{S}^2}\left(\frac{1}{2}\bar{S}c - C\right)^2 + \frac{1}{2}c\bar{D} \\ & & \text{variable cost} = \frac{1}{8}c\bar{D} + \frac{3\bar{D}}{2c\bar{S}^2}C^2 \\ \text{If } C \leq \frac{c\bar{S}}{6}, & k^* = \sqrt{\frac{\bar{D}^2 c}{6\bar{S}C}}, & \text{total cost} = 2\sqrt{\frac{\bar{D}^2 Cc}{6\bar{S}}} \\ & & \text{variable cost} = \sqrt{\frac{\bar{D}^2 Cc}{6\bar{S}}}. \end{cases} \tag{20}$$

With bitcoin mining With bitcoin mining, we show that there exists some stipulated quantity of electricity demand from bitcoin mining $d(D, S \cdot k)$ (which means that there exists some demand response programs based on what we learned in Section 2.4.2) that leads to lower emission when the grid chooses its optimal renewable capacity building decision.

Suppose we specify

$$d(D,S) = \begin{cases} S \cdot k - D, & \text{if } S \cdot k - D \geq 0 \\ d, & \text{if } S \cdot k - D < 0 \end{cases}$$

where $d > 0$, that is, we let bitcoin mining consume a positive amount of electricity when household/industry needs exceed renewable supply, and absorb extra supply from renewable when otherwise. A positive d effectively serves as penalty to the under-building of renewable capacity. Then in this case, the grid's objective becomes

$$\min_k \left\{ C \cdot k + c \cdot \frac{1}{\bar{D}\bar{S}} \int_0^{\bar{D}} \int_{0 \leq S \leq \bar{S}, S \cdot k < D} (D + d - S \cdot k) dSdD \right\} \tag{21}$$

We again separate two cases:

If $k\bar{S} \geq \bar{D}$, then the objective becomes

$$\begin{aligned}
& \min_k \left\{ C \cdot k + c \cdot \frac{1}{\bar{D}\bar{S}} \int_0^{\bar{D}} \int_0^{\frac{D}{k}} (D + d - S \cdot k) dS dD \right\} && (22) \\
= \; & \min_k \left\{ C \cdot k + c \cdot \frac{1}{\bar{D}\bar{S}} \int_0^{\bar{D}} \left(\frac{1}{2} \frac{D^2}{k} + d \frac{D}{k} \right) dD \right\} && (23) \\
= \; & \min_k \left\{ C \cdot k + c \cdot \frac{1}{\bar{D}\bar{S}} \left(\frac{1}{6} \frac{\bar{D}^3}{k} + d \frac{\bar{D}^2}{2k} \right) \right\} && (24) \\
\geq \; & 2\sqrt{\frac{Cc}{\bar{S}} \left(\frac{\bar{D}^2}{6} + d\frac{\bar{D}}{2} \right)}. && (25)
\end{aligned}$$

with equality when $k = \sqrt{\frac{c}{\bar{S}} \left(\frac{\bar{D}^2}{6C} + d\frac{\bar{D}}{2C} \right)}$.

If $k\bar{S} < \bar{D}$, then the objective becomes

$$\begin{aligned}
& \min_k \left\{ C \cdot k + c \cdot \frac{1}{\bar{D}\bar{S}} \int_0^{\bar{S}} \int_{kS}^{\bar{D}} (D + d - S \cdot k) dD dS \right\} && (26) \\
= \; & \min_k \left\{ C \cdot k + c \cdot \frac{1}{\bar{D}\bar{S}} \int_0^{\bar{S}} \left(\frac{1}{2}(\bar{D}^2 - (kS)^2) + (d - kS)(\bar{D} - kS) \right) dS \right\} && (27) \\
= \; & \min_k \left\{ C \cdot k + c \cdot \frac{1}{\bar{D}\bar{S}} \left((\frac{1}{2}\bar{D}^2 + d\bar{D})\bar{S} - \frac{1}{2}(d + \bar{D})k\bar{S}^2 + \frac{1}{6}k^2\bar{S}^3 \right) \right\} && (28) \\
= \; & \min_k \left\{ C \cdot k + c \cdot \frac{1}{\bar{D}} \left((\frac{1}{2}\bar{D}^2 + d\bar{D}) - \frac{1}{2}(d + \bar{D})k\bar{S} + \frac{1}{6}k^2\bar{S}^2 \right) \right\} && (29) \\
\geq \; & c(\frac{1}{2}\bar{D} + d) - \frac{3}{2}\frac{\bar{D}}{c\bar{S}^2} \left(C - \frac{c}{\bar{D}}\frac{1}{2}(d + \bar{D})\bar{S} \right)^2. && (30)
\end{aligned}$$

with equality when $k = \frac{3}{2}(d + \bar{D})\frac{1}{\bar{S}} - 3C\frac{\bar{D}}{c\bar{S}^2}$.

Therefore, summarizing the above two cases, we have

$$\begin{cases} \text{If } C > \frac{d+\bar{D}}{2\bar{D}} c\bar{S}, & k^* = 0, & \text{total cost} = c\left(\frac{1}{2}\bar{D} + d\right) \\ & & \text{variable cost} = c\left(\frac{1}{2}\bar{D} + d\right) \\ \text{If } \frac{3d+\bar{D}}{6\bar{D}} c\bar{S} < C \leq \frac{d+\bar{D}}{2\bar{D}} c\bar{S}, & k^* = \frac{3}{2}\frac{d+\bar{D}}{\bar{S}} - 3C\frac{\bar{D}}{c\bar{S}^2}, & \text{total cost} = c(\frac{1}{2}\bar{D} + d) - \frac{3}{2}\frac{\bar{D}}{c\bar{S}^2}\left(C - \frac{c}{\bar{D}}\frac{1}{2}(d+\bar{D})\bar{S}\right)^2 \\ & & \text{variable cost} = \frac{c}{8}\bar{D} + \frac{1}{4}cd + \frac{3}{2}\frac{\bar{D}}{c\bar{S}^2}C^2 - \frac{3}{8}\frac{c}{\bar{D}}d^2 \\ \text{If } C \leq \frac{3d+\bar{D}}{6\bar{D}} c\bar{S}, & k^* = \sqrt{\frac{c}{\bar{S}}\left(\frac{\bar{D}^2}{6C} + d\frac{\bar{D}}{2C}\right)}, & \text{total cost} = 2\sqrt{\frac{Cc}{\bar{S}}\left(\frac{\bar{D}^2}{6} + d\frac{\bar{D}}{2}\right)} \\ & & \text{variable cost} = \sqrt{\frac{Cc}{\bar{S}}\left(\frac{\bar{D}^2}{6} + d\frac{\bar{D}}{2}\right)}. \end{cases} \tag{31}$$

Comparing the case with and without Bitcoin mining A comparison between (20) and (31) immediately reveals that

- In case $\frac{c\bar{S}}{6} < C \leq \frac{3d+\bar{D}}{6\bar{D}} c\bar{S}$, mining has lower variable cost if $\sqrt{\frac{Cc}{\bar{S}}\left(\frac{\bar{D}^2}{6} + d\frac{\bar{D}}{2}\right)} < \frac{1}{8}c\bar{D} + \frac{3\bar{D}}{2c\bar{S}^2}C^2$ which holds when

$$d < \frac{\bar{D}\left(c\bar{S} - 6C\right)^2\left(3c^2\bar{S}^2 + 4cC\bar{S} + 12C^2\right)}{96c^3C\bar{S}^3}.$$

- In case $\frac{1}{2}c\bar{S} < C \leq \frac{d+\bar{D}}{2\bar{D}} c\bar{S}$, mining has lower variable cost if $\frac{c}{8}\bar{D} + \frac{1}{4}cd + \frac{3}{2}\frac{\bar{D}}{c\bar{S}^2}C^2 - \frac{3}{8}\frac{c}{\bar{D}}d^2 < \frac{1}{2}c\bar{D}$ which holds when

$$d > \frac{1}{3}\bar{D}\left(1 + \frac{2\sqrt{9C^2 - 2c^2\bar{S}^2}}{c\bar{S}}\right)$$

or

$$d < \frac{1}{3}\bar{D}\left(1 - \frac{2\sqrt{9C^2 - 2c^2\bar{S}^2}}{c\bar{S}}\right).$$

In sum, we prove that there exists a well-planned quantity of electricity demand from bitcoin mining that reduces total emission under the grid's optimal renewable capacity building decision. This result demonstrate how Bitcoin mining can actually contribute to sustainability objectives with the right policies.

2.6.2 Optimal d

To help gain further intuition of the results in Section 2.6.1, we analyze the properties of the variable cost function $\mathrm{Var}(d)$, defined piecewise as follows (following Equation (31)):

$$\mathrm{Var}(d) = \begin{cases} c\left(\frac{\bar{D}}{2} + d\right), & \text{if } \bar{C} > c\bar{S}\frac{d+\bar{D}}{2\bar{D}}, \\ \frac{c\bar{D}}{8} + \frac{cd}{4} + \frac{3}{2}\frac{\bar{D}}{c\bar{S}^2}\bar{C}^2 - \frac{3}{8}\frac{c}{\bar{D}}d^2, & \text{if } \frac{(3d+\bar{D})}{6\bar{D}}c\bar{S} < \bar{C} \leq c\bar{S}\frac{d+\bar{D}}{2\bar{D}}, \\ \sqrt{\frac{c\bar{C}}{\bar{S}}\left(\frac{\bar{D}^2}{6} + \frac{d\bar{D}}{2}\right)}, & \text{if } \bar{C} \leq \frac{(3d+\bar{D})}{6\bar{D}}c\bar{S}, \end{cases}$$

where $c > 0$, $\bar{D} > 0$, $\bar{S} > 0$, $\bar{C} > 0$, and $d \geq 0$.

Note that $\mathrm{Var}(d)$ is proportional to the total carbon emission.

We proceed in the following steps:

1. **Behavior of Var(d) in Each Case:**

   (a) **Case 1:** When $\bar{C} > c\bar{S}\frac{d+\bar{D}}{2\bar{D}}$, the function is:

   $$\mathrm{Var}(d) = c\left(\frac{\bar{D}}{2} + d\right).$$

   This is linear in d, with:

   $$\frac{d\mathrm{Var}}{dd} = c > 0.$$

   Hence, $\mathrm{Var}(d)$ is strictly increasing in d in this region.

   (b) **Case 2:** When $\frac{(3d+\bar{D})}{6\bar{D}}c\bar{S} < \bar{C} \leq c\bar{S}\frac{d+\bar{D}}{2\bar{D}}$, the function is:

   $$\mathrm{Var}(d) = \frac{c\bar{D}}{8} + \frac{cd}{4} + \frac{3}{2}\frac{\bar{D}}{c\bar{S}^2}\bar{C}^2 - \frac{3}{8}\frac{c}{\bar{D}}d^2.$$

Taking the derivative with respect to d,

$$\frac{d\text{Var}}{dd} = \frac{c}{4} - \frac{3}{4}\frac{c}{\bar{D}}d.$$

Setting $\frac{d\text{Var}}{dd} = 0$, we find the critical point:

$$d = \frac{\bar{D}}{3}.$$

i. For $d < \frac{\bar{D}}{3}$, $\frac{d\text{Var}}{dd} > 0$, so $\text{Var}(d)$ is increasing.

ii. For $d > \frac{\bar{D}}{3}$, $\frac{d\text{Var}}{dd} < 0$, so $\text{Var}(d)$ is decreasing.

Therefore, $\text{Var}(d)$ is increasing for small d, reaches a maximum at $d = \frac{\bar{D}}{3}$, and then decreases.

(c) **Case 3:** When $\bar{C} \leq \frac{(3d+\bar{D})}{6\bar{D}}c\bar{S}$, the function is:

$$\text{Var}(d) = \sqrt{\frac{c\bar{C}}{\bar{S}}\left(\frac{\bar{D}^2}{6} + \frac{d\bar{D}}{2}\right)}.$$

Taking the derivative with respect to d,

$$\frac{d\text{Var}}{dd} = \frac{1}{2} \cdot \frac{\frac{c\bar{C}\bar{D}}{2\bar{S}}}{\sqrt{\frac{c\bar{C}}{\bar{S}}\left(\frac{\bar{D}^2}{6} + \frac{d\bar{D}}{2}\right)}} > 0.$$

Since the derivative is positive, $\text{Var}(d)$ is strictly increasing in d in this region.

2. **Continuity of Var(d) at Transition Points:** The function $\text{Var}(d)$ is continuous at the boundaries between the cases:

(a) At the boundary between Case 2 and Case 3, the transition occurs when:

$$\bar{C} = \frac{(3d + \bar{D})}{6\bar{D}}c\bar{S}.$$

Substituting this into both definitions of $\text{Var}(d)$, it is straightforward to verify that the two expressions match.

3. **Comparing Var(d) at $d = 0$ and $d = d_{\textbf{transition}}$:** Let $d_{\text{transition}} = \bar{D}\left(2\frac{\bar{C}}{c\bar{S}} - \frac{1}{3}\right)$ denote the transition point between Case 2 and Case 3.

   (a) **At** $d = 0$ (in Case 2):
$$\text{Var}(0) = \frac{c\bar{D}}{8} + \frac{3}{2}\frac{\bar{D}}{c\bar{S}^2}\bar{C}^2.$$

   (b) **At** $d = d_{\textbf{transition}}$ (in Case 3):
$$\text{Var}(d_{\text{transition}}) = \sqrt{\frac{c\bar{C}}{\bar{S}}\left(\frac{\bar{D}^2}{6} + \frac{d_{\text{transition}}\bar{D}}{2}\right)}.$$

   By direct comparison, the behavior of $\text{Var}(d)$ depends on the parameter values. For certain $\bar{C}$, it is possible that:
$$\text{Var}(d_{\text{transition}}) < \text{Var}(0).$$

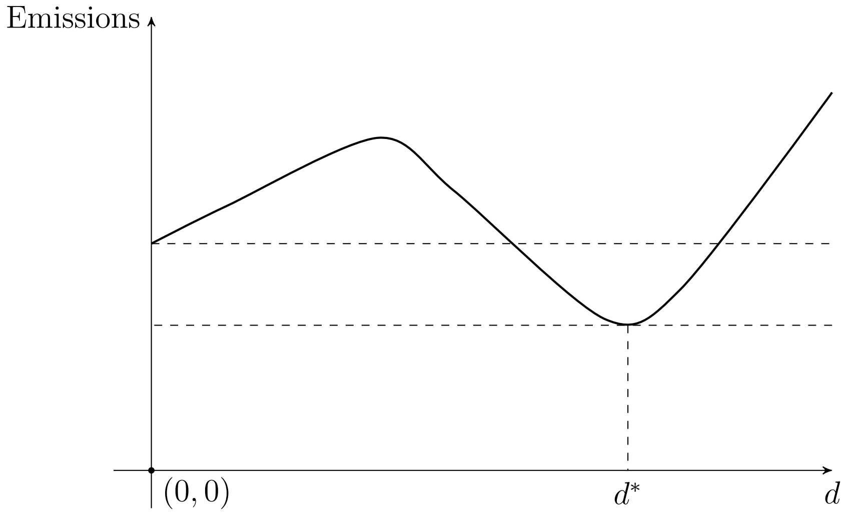
4. **Summary:**

   (a) The variable cost $\text{Var}(d)$ is not strictly increasing in $d \geq 0$; it increases in Cases 1 and 3, but increases then decreases in Case 2.

   (b) There exists a non-empty range of $\bar{C}$ such that $\text{Var}(d_{\text{transition}}) < \text{Var}(0)$, demonstrating that $\text{Var}(d)$ at some positive $d > 0$ can be strictly less than at $d = 0$.

Figure 2 summarizes how total emission changes with the stipulated d. The total emission may initially increases as d increasing, reflecting the direct outcome of more allowed bitcoin mining enabled by fuel energy. However, as d further increases, a second force begins to dominate: the grid's desire to lower the probability of household/industry demand exceeding

Figure 2: Total emission as a function of d



renewable supply (in which case the grid serves Bitcoin miners with fuel energy and induce unit cost of c) incentives the grid to increase renewable capacity building. In fact, the carbon emission-minimizing d^* will be achieved when the renewable capacity k is set when $\bar{D} = \bar{S}k$, and in this case the total emission is lower than when there is no mining at all (i.e., $d = 0$). As d further increases above d^*, total emission goes up again.

2.7 Bitcoin mining subsidizing renewable capacity building

In Section 2.5 and we demonstrate how a cost-minimizing grid may optimally increase renewable capacity building in face of extra electricity demand from bitcoin miners and a well-designed bitcoin mining electricity demand pattern (either through direct control or indirect influence through demand response programs) stipulated by an environmentally-aware regulator may reduce carbon emission. Careful readers may notice that we specify the emission-reducing bitcoin mining quantity to be chosen by the regulator, and may ask whether we can have it chosen by the grid itself? The answer is no, indeed, if if the grid were to be able to decide the price charged to miners (or equivalently the mining intensity per Section 2.4.2), the cost-minimizing total quantity would be to set bitcoin mining to zero.

This observation may give the impression that the grid has to suffer in order to facilitate carbon emission reduction. This section will show that this is not the case.

The crux of the above observation is that in the simplified cost-minization model the grid is not directly compensated by miners. Instead miners either absorb extra renewable supply (which does not affect the grid's cost) or add additional demand for fuel energy (which increases the grid's cost). The situation changes, however, if we go back to the original profit-maximization problem (6) in which the grid can be directly compensated by miners.

In this section, we explicitly calculate the total profit to the grid under the emission-reducing mining demands and the grid's profit when there is no mining, and characterize conditions under which the former is higher than the latter. This result can be interpreted as evaluating to what extent the introduction of Bitcoin mining can help subsidize renewable energy adoption and compensate for lower total emission.

In the uniform distribution example we had in Section 2.5, in the case of $\frac{3d+\bar{D}}{6\bar{D}}c\bar{S} < C \leq \frac{1}{2}c\bar{S}$, we have the grid's profit in the case of no bitcoin mining to be

$$\frac{1}{2}p\bar{D} - \frac{c}{8}\bar{D} - \frac{3}{2}\frac{\bar{D}}{c\bar{S}^2}C^2 - C\left(\frac{3\bar{D}}{c\bar{S}^2}(\frac{1}{2}\bar{S}c - C)\right)$$

and that with mining to be

$$\frac{1}{2}p\bar{D} + \frac{n-1}{n}R - \frac{c}{8}\bar{D} - \frac{1}{4}cd - \frac{3}{2}\frac{\bar{D}}{c\bar{S}^2}C^2 + \frac{3}{8}\frac{c}{\bar{D}}d^2 - C\left(\frac{3}{2}\frac{d+\bar{D}}{\bar{S}} - 3C\frac{\bar{D}}{c\bar{S}^2}\right).$$

It is easy to verify that the latter is higher than the former when

$$\frac{n-1}{n}R > \frac{1}{4}cd - \frac{3}{8}\frac{c}{\bar{D}}d^2 + \frac{3}{2}C\frac{d}{\bar{S}}.$$

In summary, when the bitcoin market is adequately large (so that R is high), it can subsidize the grid to be willing to adopt emission reducing pricing schedule to miners.

2.8 Grid Competition

We have so far considered one single monopolistic grid. While this assumption is likely true in the local market, this assumption certainly is not accurate from a global perspective, which is relevant since bitcoin mining is a global business. Furthermore, if we take into account the micro-level operations of a grid, the grid's supply often sums the supply of many individual generators within the grid. Indeed, as Sioshansi (2024) summarizes, there are two typical designs regarding how a grid aggregates the unit decisions from participating generators: The U.S. markets have evolved towards centrally committed designs (other than Texas and California), in which the market operator (MO) collects complex multi-part offers and solves unit-commitment problem to co-ordinate these decisions. Almost all other markets outside of the United States (except for Texas and California) use self-committed designs, in which generators determine unit commitments individually and the MO clears demand against supply based on simple energy offers. In sum, the decision of the grid may not always be well-captured by a single decision maker, but may rather more closely assemble a competitive or oligopolistic market structure. This section extends the analysis to study such extensions.

(TO BE COMPLETED)

3 Discussions and empirical evaluations

Admittedly, the analytical results we have derived in this paper may be counterintuitive at first sight. Therefore, it is helpful to calibrate the model with real parameters to demonstrate the quantitative magnitude of our results. For this calibration exercise, we first take note all the exogenous parameters within the model:

- the unit price of electricity charged to households/industry p, in units of \$/kwh. We take it to be \$0.15/kwh (see https://www.energybot.com/electricity-rates/).

- the quantity of demand of electricity from households/industry $\tilde{D}$, in units of kwh, characterized by mean, standard deviation, or higher moments. According to the EIA (see https://www.eia.gov/todayinenergy/detail.php?id=12711), a representative region typically has a total electricity consumption of 57 - 155 GW per hour. We thus assume it follows a uniform distribution within 0 and 200 GW. Note that the exact level the quantity of demand is not important, as we can always "scale" the economy and interpret other quantities (for example, renewable capacities) relative to non-mining electricity demand.

- the random utilization rate of renewal energy $\tilde{S}$, as a percentage number, characterized by mean, standard deviation, or higher moments. In electricity parlance, this ratio is known as the capacity factor, which is the ratio of actual electrical energy output over a given period of time to the theoretical maximum electrical energy output over that period. Capacity factors varies across different renewable energy sources, ranging from over 90% for nuclear, 32.2% to 34.7% for wind farms in the U.S. (see www.eia.gov/electricity/monthly/), 23% to 45% for hydroelectric dams (see en.wikipedia.org/wiki/Capacity_factor), and 12.0% to 29.1% for photovoltaic (solar) power stations (see en.wikipedia.org/wiki/Capacity_factor). There is also significant inter-temporal variations. For calibration we set $\tilde{S}$ to be a uniform distribution between 0% and 50% (the exact level is not that important for our qualitative arguments, as we can always adjust the capacity building K accordingly).

- the unit cost of electricity generated from fuel energy c, in units of \$/kwh. This number ranges from 3-15 cents per kwh.

- the number of bitcoin miners (mining pools) n. We can pick it to be 30.

- the dollar value of bitcoin mining reward R, in units of \$. Assuming a 10-minute average block time, 6.25 BTC per block, and a $60,000 bitcoin price, then R would be about $6 \times 6.25 \times 60,000 = \$2,250,000$.

However, it appears none of the numbers really matter that much for qualitative confirmation. What really matters is the correlation between $\tilde{D}$ and $\tilde{S}$.

(TO BE COMPLETED)

4 Conclusion

While bitcoin mining has long been argued to be environmentally unfriendly, we point out this conventional wisdom is not necessarily correct. In a simple model, we demonstrate how combining the unique features of bitcoin mining and well-designed demand response program can actually lower total emission. We plan to go beyond the existence proof and finish up deriving the optimal policy under general conditions soon.

Our results have multiple policy implications:

- Banning bitcoin mining, like some jurisdictions have chosen to do (e.g., China) may actually be counter-effective and missing important incentives for renewable adoption;
- To incentivize renewable buildup, a properly-designed demand response program is paramount; offering bitcoin miners the market rate of electricity would not have the desirable effect;
- the emission-reducing pricing to miners may not be in line with the profit-maximizing policy chosen by a monopolistic grid — therefore regulatory intervention may be desirable;
- in order for the incentive scheme to work, we should allow adequate amount of electricity assumption by bitcoin miners as compared to households/industries.

Future research can further extend our analysis. For example, while we demonstrate that there exist reasonable pricing policies (demand response programs) that can allow Bitcoin mining to induce lower carbon emissions, future research can build on our results and solve for the optimal pricing policies (demand response programs). Such extensions can take a mechanism design / contract theory perspective and quantify the maximal extent to which Bitcoin mining may contribute to carbon emission reduction.

Another direction is to assemble actual information about fine-grained details of each grid and their underlying market structure, and conduct an accurate quantification of the potential benefit from Bitcoin mining and the optimal demand response programs. This direction will likely be of less an economics flavor but take an operations research perspective to look for algorithms to solve for large-scale optimizing problems.

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