Regulation of Selection Technologies*

Work in progress. Please do not circulate

Marie Obidzinski† Marianne Verdier‡

January 6, 2025

Abstract

We analyze how a monopoly chooses the quality of a technology to select its consumers, when it is uncertain that the latter will comply with the legal rules required to buy its product. The firm may decide to exclude a consumer after observing a signal received on her compliance, which accuracy depends on the quality of the technology. The choice of the selection technology also impacts the consumers' incentives to comply with the legal rules. The firm incurs heterogeneous costs of serving compliant and non-compliant consumers, respectively. We explain why the firm's choice of the quality of the technology differs from the social optimum by extending the model of Veiga and Weyl (2016). Then, we use our model to assess whether the implementation of a fraud detection algorithm is welfare-enhancing. We analyze the role of several regulatory instruments to improve social welfare: the regulation of the selection technology, the *ex ante* regulation of the misclassification cost through sanctions, the *ex post* imposition of fines on non-compliant consumers.

Keywords: compliance, selection market, monetary sanctions sanctions, regulation of algorithms.

*We thank the participants to the 40th symposium on Money, Banking and Finance of the GDRE conference (July, 2024) for their comments on the paper.

†Université Paris Panthéon-Assas, CRED EA 7321, 75005 Paris, France. e-mail: marie.obidzinski@u-paris2.fr

‡Université Paris Panthéon-Assas, CRED EA 7321, 75005 Paris, France. e-mail: marianne.verdier@u-paris2.fr

JEL classification: K4.

1 Introduction

In selection markets, firms often incur higher costs of selling to some consumer types which are costly to observe. To segment the market, they sometimes successfully offer menus of contracts with different prices and qualities. Then, consumers self-select themselves by choosing the contract corresponding to their type (Stiglitz, 1977, Mussa and Rosen, 1978). However, in many cases, firms prefer instead to offer a single contract to their consumers, with the use of non-price features (e.g., downpayment) to sort their consumers (Rothschild and Stiglitz, 1976, Veiga and Weyl, 2016). A common instrument to sort consumers is the choice of a *selection technology*, which predicts a classification of consumers. Given the result of the prediction, the firm may decide to exclude some consumers from the market.

The choice of a selection technology is particularly widespread in markets where consumers have to comply with legal rules to buy a product. For example, it is necessary to perceive revenues from legal activities to open a bank account, or firms may sometimes need to comply with environmental standards to receive some funding. Consumer compliance is often costly to enforce, because the firm has to invest in a technology to try and identify the consumer's type. Therefore, a firm may decide to sell to non-compliant consumers, but at a higher cost, either in terms of reputation or regulatory sanctions. When it uses an imperfect selection technology to filter its consumers, the firm may sell at a uniform price, while incurring the costs of making errors, which differ according to the consumer's type. This implies that consumers differ in their imperfectly-contractible profitability to the firm.

When a firm uses a selection technology, it may still partly control who buys its product by changing prices (Akerlof, 1970, Einav and Finkelstein, 2011). Veiga and Weyl (2016) label this effect as *selection by quantity*.[1] However, the firm also relies on the quality of the selection technology to

[1] Unlike Veiga and Weyl (2016), we consider that product quality is exogenous and the choice of the quality of a selection technology is endogenous. The quality of the selection technology also impacts the consumer's decision to buy the product.

expand its ability to sort profitable consumers from non-profitable consumers. The quality of the prediction delivered by the selection technology changes the consumers' marginal willingness-to-pay for the product for a given price according to their decision to comply with the law. This is because consumers may anticipate that, with some probability, the firm will prevent them from purchasing its product. Therefore, in such markets, firms also rely on *sorting by quality* (Veiga and Weyl, 2016), through the endogenous choice of the quality of the selection technology.

Our paper offers an economic framework for analyzing the welfare efficiency of an algorithm, when the firm is able to select consumers by quantity and by the quality of the screening technology. For this purpose, we analyze how a monopolistic firm chooses the quality of its selection technology and why this choice may not be socially optimal. We offer a measure of the welfare effects of the choice of a selection technology that takes into account the firm's trade-off between selecting by quantity and sorting by quality.

Our model can be applied to fraud detection algorithms, which a firm may use to estimate the probability that a buyer is compliant, and refuse to sell her the product. The firm trades off the profits of expanding its sales to non-compliant buyers and incurring the costs of the regulatory sanctions. For instance, a criminal obtains a benefit of engaging in illegal activities. However, the latter cannot open a bank account because money laundering is forbidden. The bank does not observe the consumer's type and needs to screen its consumers with a Know-Your-Customer policy. In case the bank does not perform its due-diligence duty, it has to incur the cost of sanctions, which are decided by the supervisor.

We model a market in which consumers receive some benefit of renouncing to comply with the law. In this market, complying with the law is necessary to buy a higher-quality product or service offered by a seller. However, the monopolistic seller does not observe the consumer's type, and needs to rely on a screening technology, which delivers a signal on the consumer's compliance. The signal is imperfectly informative, but enables a classification of consumers into two categories: compliant and non-compliant. Anticipating this choice, the consumers may either decide to become compliant or non-compliant. Therefore, the selection technology impacts the market segmentation of the consumers' types. After this segmentation, a consumer's type is bi-dimensional, because it is both characterized by its benefit of not-complying with the law and its effective choice of becoming non-compliant. Non-compliant consumers may sometimes have a higher willingness-to-pay for the firm's services, which generates adverse selection. The firm may select its consumers by raising its price or by increasing the quality of the selection technology.

We determine how the firm chooses the quality of its selection technology and why this choice may differ from the social optimum. We identify another effect that extends the framework of Veiga and Weyl (2016). If the quality of the selection technology increases, this changes the marginal probability that a buyer is excluded from the market. However, the firm only takes into account the impact of the marginal variation of the exclusion probability on its cost, without internalizing the social damage. We use our framework to offer a measure of the welfare effects of fraud detection algorithms, which differs from the existing literature. We introduce a loss function for errors that applies more generally to any selection market in which a firm incurs heterogeneous costs of serving different consumer types.

We complete our analysis by analyzing the role of sanctions, which increase the cost of serving non-compliant consumers. The firm passes through the costs of the sanctions into higher prices. Therefore, the regulator sometimes needs to trade off the costs of sanctioning non-compliant firms against the benefits of including compliant consumers in the market. We analyze whether the regulator has an incentive to discipline the firm's behavior with sanctions.

The rest of the paper is organized as follows. Section 2 surveys the literature that is related to our study. Section 3 develops the model and the assumptions. Section 4 determines the firm's choice of the quality of the selection technology and compares it to the first-best. Section 5 illustrates our model with the case of a fraud detection algorithm used by a bank to fight money laundering. Section 6 concludes.

2 Related literature

Our paper offers a model to analyze the regulation of a monopoly, which undertakes a selection activity. The selection technology impacts the consumers' expected utility of buying a higher-quality service with respect to an outside option. Therefore, our framework has also similarities with the model of Spence (1975), which analyzes the regulation of the quality offered by a monopoly. Our paper differs because we assume that product quality is exogenous, and consider instead the endogeneous choice of the quality of a selection technology. The monopoly's selection activity impacts the consumers' ability to buy the service, and indirectly generates social damage. In this context, the role of the regulation consists of providing the monopoly with incentives to select compliant buyers, who generate lower social damage.

Our paper belongs to the literature on selection markets (Einav, Finkelstein and Cullen, 2010, Einav and Finkelstein, 2011, Mahoney and Weyl, 2017) and extends in particular the model of Veiga and Weyl (2016). In a selection market, firms incur different costs of serving heterogeneous consumer types and need to attract the right users to be profitable. This idea has also been developed in several models of the literature of platform markets (i.e., Veiga et al. (2017) for a one-sided platform, or Biancini and Verdier (2023) for a two-sided platform). We differentiate from this literature by considering the choice of the quality of a selection technology, and identify the same sorting effect as in Veiga and Weyl (2016), which is caused by the marginal effect of an increase the quality of the selection technology on consumers' incentives to buy the product.

The issue of consumer selection is related to the broader issue of the increasing role of private firms as gatekeepers.[2] In several sectors, public authorities tend to delegate law enforcement to private firms. The focus is not on the harm that the private firm directly inflicts on society, but on the harm caused by users, whom Spier and Van Loo (2025) calls "bad actors". However, financial intermediaries or technology platforms can influence the proportion of bad actors by investing in detection, which we call in our paper a selection technology. A recent literature has developed on platform regulation through liability or negligence rules (Creti and Verdier, 2014, Hua and Spier,

[2] See also the more general analysis of Van Loo (2020), which highlights the role of large firms as new gatekeepers and in particular their role *vis-à-vis* third parties.

2022, 2023).[3] We contribute to this discussion by focusing on a selection market. We analyze a second-best situation, in which the social planner may impact the firm's selection costs through the choice of a regulatory parameter, such as a sanction.

In a selection market, a firm incurs heterogeneous costs of selecting different consumer types. The notion of selection costs can be related to a literature that incorporates data as an input in firms' production function (see Farboodi and Velkamp, 2022). The investment in quality of a selection technology could be interpreted as the choice to collect more consumer data, which is similar to several other papers of the literature (e.g., Gurkan and Vericourt, 2022). However, we consider that the consumers' decision to become compliant, and therefore, their marginal valuations for the firm's product are endogenous, because they depend on the firm's choice of a selection technology. In the literature, the firm's incentives to collect data may depend on the price discrimination possibilities (Bergemann et al., 2019, Ichibashi, 2020), the individuals' decision to share their personal data (Acemoglu, Makhdoumi et al., 2022), and competition (Jones and Tonetti, 2020). In our paper, consumer data is a private asset, which is used ex ante by a monopoly to screen its consumers. The latter do not exert any effort to hide their personal data, but may decide to buy the service from the competitors when there is a high risk that the firm will exclude them.[4] Therefore, our work can also be seen as a contribution to the literature on data privacy, with the assumption that the consumer's inconvenience cost of data collection for two different market segments endogenously depends on the quality of the technology. A better quality of the selection technology increases the compliant consumers' expected utility of buying the higher-quality service, whereas it increases the non-compliant consumers' expected inconvenience cost of being excluded. This is similar to other papers of the literature on privacy, such as Markovich and Yehezkel (2021), who model the consumers' inconvenience cost of data collection. In our paper, this cost is group-specific and it endogenously depends on the firm's choice of a selection technology. Other papers in the literature on privacy assume that users have heterogeneous preferences for privacy (see Smith, 2014, Lefouilli and Riordan, 2020, Acemoglu, Makhdoumi et al., 2022). Several papers consider that strategic

[3] These last two theoretical papers are complemented by a paper analyzing US legislation and case law on platform liability (see Spier and Van Loo (2025)).

[4] Unlike Jones and Tonetti (2020) who model data as a non-rival input generated as a by-product of economic activity, data is a private asset in our model.

consumers exert externalities on each other when they decide to share their data (Garratt and van Oordt, 2021, Acemoglu et al., 2022).[5] A different type of externality arises in our framework between consumers when they decide whether or not to comply with the legal framework. Since consumers pay a uniform price, the probability that a non-compliant consumer participates in the market impacts the compliant consumers' utility of purchasing the higher-quality service.

Artificial Intelligence (AI) is an example of a technology that can be used to select consumers. The role of AI consists in screening a higher volume of data more efficiently than humans and helping them to make a decision (see Cowgill et al. (2021), Goh and Lee, (2019), or Mc Kay, (2020)). An important aspect of AI adoption relates to the interactions between the technology and human judgment in decision-making (Agrawal et al. (2018), Daugherty and Wilson (2018), Mullainathan and Spiess (2017), Kleinberg et al. (2017)). We abstract from studying these issues by considering that the firm always follows the recommendation of the selection technology ex post, but that it may ex ante choose the precision of the signal received. We also assume that all agents are perfectly informed about the selection technology.

We contribute to the scarce theoretical literature focusing on the interactions between AI adoption and product pricing. In a close paper, Gans (2022) analyzes AI adoption by a monopoly facing demand uncertainty. In his paper, the firm chooses its price and quantity ahead of demand, and may make two different types of errors: unsold inventory or missed sales. Our paper differs because we consider that the technology is used to select consumers, with different consequences for the society in terms of errors. Gurkan and Vericourt (2022) model the firm's incentives to price its product in a two-period model to collect some data on its consumers so as to feed new data back to the algorithm. They show that the firm has an incentive to underprice the product in the first-period to collect more data when an increase in the provider's effort has a significant positive impact on accuracy. The empirical literature on the use of AI in selection markets focuses on the analysis of a specific algorithm, or compares the efficiency of several methods or criteria to select consumers (see Fraisse and Laporte (2021), and Hurlin et al. (2022) for credit scoring or Zhang and

[5] A literature in computer science analyzes how strategic consumers can manipulate the information given to a selection technology (i.e., an algorithm) so as to impact the outcome of the classification process, and whether it is possible to design learning processes that are robust to potential data manipulation (See Dong, et al., Hardt et al.).

Trubey (2019) for anti-money laundering). However, the empirical papers do not offer a framework for measuring the interactions between the firm's choice of an algorithm and its strategy in the product market.

Our paper is related to the literature on the regulation of AI. Acemoglu and Leiseman (2024) also consider that the adoption of AI may generate social damage and show in a dynamic setting that firms tend to adopt the technology too fast when it is possible to learn about the potential negative effects of the technology. In contrast, we do not consider multiple sectors, and focus on the adoption of a selection technology in a specific sector with a static game. As in their paper, we also consider that the firm may not choose the socially optimal adoption of the technology and analyze how sanctions and end-user taxes may correct for misuse of the technology.

3 The model

We build a model to study a monopolistic firm's choice of the quality of a selection technology, which impacts consumers' incentives to comply with the legislation. We modify the framework of Veiga and Weyl (2016), and use it to discuss the regulation of a selection technology.[6] These authors build a general model to analyze the choice of product quality by a monopoly when the consumers' types are multi-dimensional.[7] They show that the monopoly distorts the choice of product quality compared to a social planner, when the firm has the opportunity of using non-price product features to sort consumers by quality. We differentiate from their work by considering that product quality is exogenous. However, we assume that the monopoly may sort consumers through the choice of the quality of a selection technology, such as an algorithm. The firm may use the result delivered by the selection technology to refuse to sell to some consumers, when there is a likelihood that they do not comply with the legal rules required to purchase the product.

[6] As shown in Appendix A-2, we need to adapt the model of Veiga and Weyl (2016) because our framework does not exactly satisfy to their assumptions.

[7] The Assumption that consumer types are at least bi-dimensional is essential to obtain their results.

Consumers

A monopoly offers a service of exogenous quality $\Delta \geq 0$ at a price p to a continuum of risk-neutral consumers, who differ across their benefit b of being non-compliant with a legal framework. We assume that b is distributed on $(0, B)$ according to the probability density f, with cumulative distribution F, where $B > 0$. Compliant consumers have a willingness to pay for a service of quality $\Delta \geq 0$ given by $(1+\Delta)y$, whereas non-compliant consumers have a willingness to pay given by $(1+\Delta)(y+b)$, where $y \geq 0$. Consumers make two consecutive decisions:

- They decide whether or not to comply with the legal framework, where $i = c$ if a consumer is compliant or $i = nc$ if she is not compliant. After this decision, the consumer's type is bi-dimensional and we denote it by $\theta = (b, i)$.[8]
- They decide whether or not to buy the service of quality $\Delta > 0$ at a price p from the monopoly or a lower-quality version of quality $\Delta = 0$ from a competitive fringe of firms, selling at their marginal cost, which is equal to zero.

The monopoly may rely on a selection technology of quality S, which is chosen in a subset of $\mathbb{R}_+$ such as an algorithm to screen its consumers. The competitive fringe does not use any selection technology. The quality of the selection technology may be either chosen by the firm or by the regulator. If the firm does not perform any screening activity, it does not exclude any consumer from the market.[9] When they choose from which firm to buy, we assume that consumers are perfectly informed about the quality S of the selection technology.[10]

After the monopoly screens its consumers, it may decide to exclude some of them from the market. If the monopoly excludes a consumer, she is constrained to buy the outside option from the competitive fringe of firms and does not pay the price for the higher-quality service. In addition, we assume that non-compliant consumers may have to pay a fine $F \geq 0$ to the regulator when the

[8]The same sorting effect as Veiga and Weyl (2016) arises only if the consumer's type is bi-dimensional. We use the notation of their paper for the consumer's type.

[9]Unlike in the literature on rational inattention, we abstract from analyzing the role of the firm's ex ante belief on the consumer's identity (see Sims, 2003, and Mackowiak, Matejka, and Wiederholt, 2023, for surveys).

[10]This assumption can easily be relaxed by assuming that only a proportion $\eta \in (0, 1)$ of consumers is informed about the technology.

firm detects them.[11] We simplify the analysis by we assuming that the firm perfectly reports their identity to the regulator. Naturally, the regulator's ability to impose a fine depends on whether non-compliant consumers have deep pockets or are judgment-proof, and whether it is costly to enforce the legislation.[12]

The quality of the selection technology determines the likelihood that a consumer will be able to purchase the higher-quality service.[13] Given the quality S of the selection technology, conditional on the consumer being of type θ, the firm excludes her from the market with probability $e(S,\theta)$, and agrees to sell her the higher-quality service with probability $1-e(S,\theta)$, where $0 \leq e(S,\theta) \leq 1$. With probability $e_c \equiv e(S,b,c)$, the firm excludes a compliant consumer (a false positive consumer) and makes an error of type I, whereas with probability $1-e_{nc} \equiv e(S,b,nc)$, it sells to a non-compliant consumer (a false negative consumer) and makes an error of type II.[14] Note that the function e is very general, and encompasses the case in which the quality of the selection technology has heterogeneous effects on the probabilities to exclude different consumer types.[15]

To simplify the notations, we will use in the mathematical expressions $i=0$ when the consumer is compliant and $i=1$ when she is not compliant and the index $i=c$ when the consumer is compliant, or $i=nc$ when she is not compliant. If she buys the service from the firm, since she has a probability $1-e_i$ of passing the selection process, a consumer of type $\theta=(b,i)$ expects to obtain the utility $u(S,\theta)-(1-e_i)p$, where

$$u(S,\theta) \equiv (1-e_i)(ib+y)(1+\Delta)+e_i(y+i(b-F)). \tag{1}$$

If she buys the outside option, since she does not need to be compliant if there is no selection

[11] As in the literature on the taxation of the monopoly, the regulator faces a trade off between taxing the firm and taxing the end-users.

[12] These cases can be captured in our setting by making some comparative statics with respect to F.

[13] The quality of the selection technology is similar to the quality parameter x of the model of Veiga and Weyl (2016).

[14] The null hypothesis corresponds to the assumption that a consumer is not compliant.

[15] Sometimes, an investment in the performance of the algorithm may increase the probability to exclude non-compliant consumers, and at the same time increase the probability to exclude compliant ones.

technology, a consumer expects to obtain the utility

$$v(\theta) \equiv b + y. \tag{2}$$

For $i \in \{c, nc\}$, we denote by $\theta_i = (b_i, i)$ the type of the consumer who is indifferent between purchasing the firm's service and the outside option, and it is implicitly defined by

$$\tilde{u}(S, \theta_i) - (1 - e_i)p = 0,$$

where $\tilde{u}(S, \theta) \equiv u(S, \theta) - v(\theta)$. From (1) and (2), we have

$$b_i = \frac{ie_iF + (1 - e_i)(p - \Delta y)}{(1 - e_i)\Delta i + i - 1}, \tag{3}$$

with $i = 0$ when the consumer is compliant and $i = 1$ if the consumer is not compliant. Moreover, we denote by b_I the value of the benefit b that leaves the consumer indifferent between being compliant and not being compliant when she consumes the higher-quality service, and it is implicitly defined by $u(S, b_I, nc) - u(S, b_I, c) = 0$, or else:

$$b_I \equiv \frac{(\Delta y - p)(e_{nc} - e_c) + Fe_{nc}}{1 + \Delta(1 - e_{nc})}. \tag{4}$$

We denote by $\mathcal{B}_c$ (resp., $\mathcal{B}_{nc}$) the set of compliant consumers (resp., non-compliant) who prefer to buy the firm's service rather than the outside option if they are not excluded, and by $\mathcal{B} \equiv \mathcal{B}_c \cup \mathcal{B}_{nc}$.[16] The set of consumers of type $i \in \{c, nc\}$ who prefer the outside option rather than the firm's service is denoted by $\mathcal{O}_i$. The set of marginal buyers of type $i \in \{c, nc\}$ is defined as $\mathcal{M}_i \equiv \{b : b = b_i\}$, and the set of marginal buyers is $\mathcal{M} \equiv \mathcal{M}_c \cup \mathcal{M}_{nc}$. As shown in Appendix A, consumer demand for the firm's service depends on the price of the service and the quality of the selection technology:

- If $p > \Delta y$, we have $\mathcal{B}_c = \emptyset$ and $\mathcal{B}_{nc} = (b_{nc}, B)$.

[16] The consumer's benefit of being non-compliant b is equivalent to τ in Veiga and Weyl (2016).

- If $\underline{p} \leq p \leq \Delta y$, with
$$\underline{p} \equiv \Delta y - \frac{F e_{nc}}{(1+\Delta(1-e_c))(1-e_{nc})},$$
we have $b_c \leq b_I \leq b_{nc}$. The market is not covered, with $\mathcal{B}_c = (0, b_c)$ and $\mathcal{B}_{nc} = (b_{nc}, B)$.

- If $p < \underline{p}$, we have $b_{nc} \leq b_I \leq b_c$, and the market is covered, with $\mathcal{B}_c = (0, b_I)$ and $\mathcal{B}_{nc} = (b_I, B)$.

Given the quality of the selection technology S and the price p for the higher-quality service, the firm expects to receive a demand
$$D_i(S,p) \equiv \int_{\mathcal{B}_i} f(b)db,$$
from consumers of category $i \in \{c, nc\}$. The total demand for the firm's product is
$$Q(S,p) \equiv D_c(S,p) + D_{nc}(S,p) = \sum_{i \in \{c,nc\}} \int_{\mathcal{B}_i} f(b)db.$$

The costs of the quality of the selection technology

The monopoly incurs a marginal cost $c(S, \theta, r)$ of using the selection technology of quality S to screen a consumer of type θ. The parameter r represents the potential impact of the regulation on the firm's selection costs. For example, the regulator may choose to impose a sanction $r > 0$ on the firm for each misclassification error.

The firm's objective

If the firm chooses the selection technology, we assume that its objective consists of maximizing its profit with respect to S and p. Since the total quantity of buyers Q is strictly decreasing in p, there exists a differentiable inverse demand function $P(S,q)$ such that $Q(S, P(S,q)) = q$.[17] The firm's profit is given by:
$$\tilde{\pi} \equiv \sum_{i \in \{c,nc\}} \pi^i(S,q), \tag{5}$$

[17] It is equivalent to maximize the firm's profit with respect to (S,p) or (S,q).

where $\pi^i(S,q,r) \equiv \int\limits_{\mathcal{B}_i} (1-e_i)(P(S,q) - c(S,b,i,r))f(b)db$.

The social planner's objective

We define the gross contribution to welfare of an individual of type $\theta = (b,i)$ who consumes the higher-quality service if she is not excluded as $w(\theta)$ (respectively, $o(\theta)$ if she consumes the outside option). In particular, these contributions include the social damage generated by the consumption of non-compliant consumers. However, we neither include in w nor in o the consumers' benefit of renouncing to comply with the legal framework.[18] The social planner maximizes total social welfare given by

$$W \equiv \sum_{i\in\{c,nc\}} \int\limits_{\mathcal{B}_i} (1-e_i)(w(\theta) - c(S,\theta,r))f(b)db$$

$$+ \sum_{i\in\{c,nc\}} \int\limits_{\mathcal{B}_i} e_i o(\theta) f(b)db + \sum_{i\in\{c,nc\}} \int\limits_{\mathcal{O}_i} o(\theta)f(b)db \quad (6)$$

either with respect to S and p in the first-best scenario, or with respect to r in a second-best scenario. Social welfare can be rewritten as

$$W = \sum_{i\in\{c,nc\}} \int\limits_{\mathcal{B}_i} (1-e_i)(w(\theta) - c(S,\theta,r) - o(\theta))f(b)db + \mathbb{E}(o(\theta)). \quad (7)$$

An illustration

In the paper, we illustrate our results by considering the example of a bank, which uses an algorithm to detect money laundering. The bank has to sort honest consumers from criminals, and incurs the expected costs of the regulatory sanctions if it does not perform its due-diligence duty.[19] For this illustration, we assume that b is uniformly distributed on $(0,B)$. The monopoly's marginal cost of

[18] This benefit could be included in our framework with a weight λ in the social planner's objective, which would not affect the intuitions of our results. We focus on the special case in which $\lambda = 0$.

[19] The AML in financial institutions is built on two pillars: vigilance at the beginning of a business relationship (including the K.Y.C duties) and constant vigilance throughout the business relationship. In this market, the bank's choice of a selection technology impacts the consumers' incentives to commit a crime.

selecting a consumer of type $\theta = (b, i)$ for a quality of the selection technology S is given by:

$$c(S, \theta, r) = k(S) + ir,$$

where k represents the cost of classifying a consumer into a given risk class and $r > 0$ is the regulatory sanction for misclassification errors. The firm incurs asymmetric marginal misclassification costs for errors, that depend on the consumer's type. In this example, the marginal misclassification cost of a missed sales to a compliant consumer is zero.[20] A branch of the literature on machine learning compares various algorithms in terms of accuracy and misclassification costs, acknowledging that in some contexts, such as the detection of fraud or medical diagnosis, certain kinds of errors are more costly than other (e.g., Turney, (1995) and (2000) or Drummond and Holte, (2000)).[21] In the terminology of this computer science literature, we model a complex classification cost matrix.[22]

Finally, we assume that the consumption of the service by a non-compliant consumer generates a fixed social damage, which equals $h > 0$ with the outside option and $H \geq h$ with the firm's service, respectively. Compliant consumers do not generate any social damage. Therefore, we have that $w(S, b, 1) = y(1 + \Delta) - H$, $w(S, b, 0) = y(1 + \Delta)$ and $o(S, \theta) = y - h$, respectively.

[20]The firm could consider the expected costs of having to compensate compliant consumers if the latter try to obtain some damage after the firm refuses to sell them the high-quality service. However, the cost of suing the firm may be too high for small retail consumers, and therefore, for the sake of simplicity, we consider that the cost of misclassifying a compliant consumer is zero, and certainly lower than the misclassification cost of non-compliant consumers.

[21]Several machine learning algorithms consider the costs of the tests that are necessary to classify an individual into a category, while other consider the costs of classification errors. Turney (1995) considers both. His work belongs to a branch of the literature that designs classifiers which are cost-sensitive in themselves.

[22]We assume that these marginal costs are constant, though they could depend on the particular case of an individual (see the discussion in Fawcett and Provost, (1996) and (1997), for fraud detection algorithms). A classification cost matrix is said to be complex when there are such misclassification cost asymmetries. The classification cost matrix differs from the confusion matrix, which reports the true positive rate, the true negative rate, the false negative rate and the false positive rate, respectively.

Additional notations

Similar to Veiga and Weyl (2016), we define the marginal impact of a price increase on the set of marginal buyers of type $i \in \{c, nc\}$ by

$$M_c(S,p) = -\frac{\partial b_c}{\partial p} f(b_c) > 0,$$

and

$$M_{nc}(S,p) = \frac{\partial b_{nc}}{\partial p} f(b_{nc}) > 0,$$

respectively. We use opposite signs because b_c is decreasing with the price p, whereas b_{nc} is increasing with the price p (see Eq.(3)). Then, using the notations of their paper, we have $M(S,p) = -\partial Q/\partial p$, and $M(S,p) = M_c(S,p) + M_{nc}(S,p)$. The marginal consumer surplus is

$$MS \equiv \frac{Q}{M}.$$

We also define two additional expectation operators that will be useful for our analysis. Since one component of the consumer's type is a discrete variable in our setting, for an arbitrary smooth function $z(S,b,i,r)$, the expectation conditional on the set of buyers $\mathcal{B}$ is given by:

$$\mathbb{E}\left[z(S,\theta,r)\,|\mathcal{B}\right] = \frac{1}{Q} \sum_{i \in \{c,nc\}} \int_{\mathcal{B}_i} z(S,b,i,r) f(b) db.$$

We also define the expectation of any $z(S,\theta,r)$ conditional on the set of marginal consumers:

$$\mathbb{E}\left[z(S,\theta,r)\,|\mathcal{M}\right] \equiv \frac{1}{M} \sum_{i \in \{c,nc\}} (-1)^{(i+1)} \frac{z(S,b_i,i)}{\frac{\partial \tilde{u}(S,b_i,i)}{\partial b}} f(b_i),$$

with $i=0$ or equivalently $i=c$ and $i=1$ or $i=nc$, and M given above.

We use Newton's notation to denote partial derivatives with respect to S, namely,

$$z'(S,b,i) = \frac{\partial z(S,b,i)}{\partial S}.$$

Timing of the game

We consider the following game:

t=0 Nature chooses the magnitude of the private non-compliance benefit b obtained by a consumer.

t=2 Either the firm or the regulator chooses the quality of the selection technology S and the price p for the higher-quality service.

t=3 Each individual learns his private benefit of being non-compliant b and after observing p and S, decides whether or not to comply with the existing legal framework.

t=4 Know-your-consumer stage:

- (a) Each individual decides whether or not to consume the firm's product.
- (b) The firm decides whether or not to exclude the consumer from the market.
- (c) If the firm accepts to sell to the consumer, the consumer pays the price p and obtains the additional value of consuming the service. Otherwise, she consumes the outside option at no cost.

t=5 Law enforcement stage:

- (a) The regulator audits the firm's selection process. It may fine the firm if the latter has failed to detect non-compliant consumers.
- (b) The judge punishes the non-compliant consumers which have been detected by the firm.

4 Regulation of Selection Technologies

In this section, we determine the firm's private choice of the quality of its selection technology. Then, we compare it to the first-best, and to a second-best situation, in which the social planner is able to impact the firm's selection cost. We also determine whether the provision of the higher-quality service is socially optimal.

4.1 Profit Maximization

The monopoly chooses the quality S of the selection technology and the price p for the higher-quality service. Proposition 1 extends the result of Veiga and Weyl (2016) to our setting, in which the monopoly chooses the quality of a selection technology.

Proposition 1. *The per-consumer marginal effect of an increase in the quality of the selection technology on profit is:*

$$\frac{\partial \tilde{\pi}}{\partial S}\frac{1}{q} = -\mathbb{E}\left[(1-e)c' \,\middle|\mathcal{B}\right] + \mathbb{E}\left[\tilde{u}' + e'p \,\middle|\mathcal{M}\right]\mathbb{E}\left[1-e\,|\mathcal{B}\right] + \frac{\mathrm{Cov}\left[\tilde{u}' + e'p, (1-e)(P-c)\,|\mathcal{M}\right]}{MS} - \mathbb{E}\left[e'(P-c)\,\middle|\mathcal{B}\right].$$

Proof. See Appendix B. □

An increase in the quality of the selection technology has four effects on the firm's profit. The first three effects are similar to Veiga and Weyl (2016), with the nuance that the firm is able to exclude some consumers from the market. The last effect is specific to our setting.

- 1) **Direct cost effect**: when the firm increases the quality of the selection technology, it loses the average increase in the cost of all buyers *who are not excluded*, which results from an increase in provided quality, $\mathbb{E}\left[(1-e)c'\,|\mathcal{B}\right]$. Veiga and Weyl (2016) obtain the same effect with $e = 0$.

- 2) **Private Spence term**: the firm raises the price by $\mathbb{E}\left[\tilde{u}' + e'p\,|\mathcal{M}\right]\mathbb{E}\left[1-e\,|\mathcal{B}\right]$ when it increases the quality S of the selection technology, because to hold fixed q, the price must offset the average benefit that marginal consumers derive from the additional quality. The average benefit takes into account the average probability that buyers are not excluded. Veiga and Weyl (2016) obtain the same effect with $\mathbb{E}\left[1-e\,|\mathcal{B}\right] = 1$ and $e' = 0$.

- 3) **Private sorting effect**: a higher quality of the selection technology has a sorting effect on the firm's profit, which depends on whether the marginal consumers who are most strongly attracted by a better-quality for the selection technology are those with a high or a low c. Veiga and Weyl (2016) obtain the same effect, because if $e = 0$, we have

$\text{Cov}\left[\tilde{u}' + e'p, (1-e)(P-c)\,|\mathcal{M}\right] = -\text{Cov}\left[\tilde{u}', c\,|\mathcal{M}\right]$. We refer the reader to Veiga and Weyl (2016) for the analysis of the distinction between selection and sorting in the literature (Akerlof, 1970, Einav and Finkelstein, 2011).

- 4) **Private exclusion effect**: when the firm increases the quality of the selection technology, this impacts the marginal probability that a buyer is excluded from the market. If the probability that a consumer is excluded increases in average, this represents a cost for the firm, $\mathbb{E}\left[e'(P-c)\,|\mathcal{B}\right]$. This term cancels out when the probability to exclude a consumer is independent of the quality of the selection technology.

If there is an interior solution when the firm maximizes its profit, the expression of Proposition 1 is equal to zero.

4.2 Welfare Maximization

4.2.1 First Best

We now analyze a first-best scenario, in which a social planner is able to choose the quality S of the selection technology and the price p for the higher-quality service. Proposition 2 extends Veiga and Weyl (2016) to our setting, and shows that the social exclusion effect may differ from the private exclusion effect, when the probability to exclude a consumer depends on the quality of the selection technology.

Proposition 2. *The per-consumer marginal effect of an increase in the quality of the selection technology on the additional welfare is:*

$$\frac{\partial W}{\partial S}\frac{1}{q} = -\mathbb{E}\left[(1-e)c'\,\middle|\mathcal{B}\right] + \frac{\text{Cov}\left[\tilde{u}' + e'p, (1-e)(w-c-o)\,|\mathcal{M}\right]}{MS} - \mathbb{E}\left[e'(w-c-o)\,\middle|\mathcal{B}\right].$$

Proof. See Appendix B. □

An increase in the quality of the selection technology has three effects on social welfare. The first two effects are similar to Veiga and Weyl (2016), with the nuance that the firm is able to exclude some consumers from the market. The third effect is specific to our setting. A fourth effect is present in Veiga and Weyl (2016) but absent in our setting, because we assume that product quality is exogenous.

1. **Direct cost effect**: when the firm increases quality, it loses the average increase in the cost of all buyers *who are not excluded*, which results from an increase in provided quality, $\mathbb{E}\left[(1-e)c' \,|\mathcal{B}\right]$. Veiga and Weyl (2016) obtain the same effect with $e = 0$.

2. **Social sorting effect**: a higher quality of the selection technology has a sorting effect on social welfare, which depends on whether the marginal consumers who are most strongly attracted by a better-quality for the selection technology are those who generate a higher social benefit of being served by the firm. Veiga and Weyl (2016) obtain the same effect, because if $e = 0$, we have $\text{Cov}\left[\tilde{u}' + e'p, (1-e)(w-c)\,|\mathcal{M}\right] = -\text{Cov}\left[\tilde{u}', c\,|\mathcal{M}\right]$. In our setting, the contribution of marginal consumers to welfare differs from their contribution to profit. The difference between the private sorting effect and the social sorting effect generates the sorting distortion.

3. **Social exclusion effect**: when the social planner increases the quality of the selection technology, this impacts the marginal probability that a buyer is excluded from the market. If the probability that a consumer is excluded increases in average, this represents a change in the average social welfare, $\mathbb{E}\left[e'(w-c-o)\,|\mathcal{B}\right]$. This term cancels out when the probability to exclude a consumer is independent of the quality of the selection technology. The difference between the private exclusion effect and the social exclusion effect generates the exclusion distortion.

4. **Social Spence term**: the social planner internalizes the preferences of all buyers, while the monopolist only internalizes the preferences of the marginal buyers when it raises its price. Veiga and Weyl (2016) obtain this effect which is absent in our setting because product quality is exogenous, whereas the firm chooses the quality of the selection technology. This implies that $\mathbb{E}\left[w' \,|\mathcal{B}\right]$=0.

Finally, note that if the social planner does not control the firm's choice of the price for the higher-quality service, it takes into account the indirect effect of the quality of the selection technology on the firm's price at the next stage.

4.2.2 The welfare-maximizing provision of the higher-quality service

Given the choice of the welfare-maximizing selection technology S^w and the price of the service, a natural question that arises is whether the provision of the higher-quality service by the monopolistic intermediary with a selection technology is welfare-enhancing. If the intermediary does not offer the higher-quality service, all consumers are non-compliant and purchase the outside option. In this case, social welfare is equal to $\mathbb{E}(o(b,1))$.[23] Proposition 3 derives the condition such that the intermediary's provision of the higher-quality service is socially optimal.

Proposition 3. *The social planner prefers that the intermediary offers the higher-quality service with a selection technology S^w and the first-best price p^w which satisfies to*

$$\mathbb{E}\left[(1-e)^2(w-c-o)\,\middle|\mathcal{M}\right]=0,$$

if and only if and only if

$$\sum_{i\in\{c,nc\}}\int_{\mathcal{B}_i^w}(w(\theta)-c(S^w,\theta,i)-o(\theta))f(b)db+\mathbb{E}(o(\theta))-\mathbb{E}(o(b,1))>0.$$

Proof. From (7) and the Appendix. □

When a private intermediary chooses both the price of the higher-quality service p^m and the quality of the selection technology S^m, its decision to sell the higher-quality service with a selection technology may differ from the first-best. First, we explained after Proposition 2 why the intermediary distorts the choice of the quality of the selection technology compared to the first-best. Second, the set of marginal buyers, which depends on the price and the quality of the selection technology (i.e., $\mathcal{B}_i^m$), also differs from the first-best (i.e., $\mathcal{B}_i^w$). Third, the intermediary does not perfectly internalize the social benefits and costs of its choices. Therefore, the intermediary's decision to offer the higher-quality service with a selection technology is not socially optimal. In particular, there

[23] Recall that $o(b,1)$ represents the gross contribution of non-compliant consumers to social welfare.

is under-provision of the higher-quality service if and only if the provision of the higher-quality service is socially optimal and:

$$\tilde{\pi}(S^m, p^m) = \sum_{i \in \{c,nc\}} \int_{\mathcal{B}_i^m} (1 - e(S^m, \theta))(p^m - c(S^m, \theta, r)) f(b) db \leq 0. \tag{8}$$

4.2.3 Comparing the welfare effects of selection technologies

In several situations, the consumption of the higher-quality service by non-compliant consumers generates negative welfare effects. In this case, the social planner prefers that the intermediary offers the higher-quality service if and only if the average contribution of compliant consumers to social welfare compensates for the average negative effect of the consumption of non-compliant consumers. The best situation for the society would be that the entire population of consumers purchases the higher-quality service and is compliant. However, the imperfection of the selection technology implies a welfare loss for the society, which we give in Proposition 4.

Proposition 4. *With respect to a situation in which the entire population of consumers is compliant and purchases the higher-quality service, the adoption of an imperfect selection technology with the socially optimal quality S^w and the first-best price p^w generates a welfare loss given by:*

$$L(S^w, p^w) = \int_{\mathcal{B}_{nc}^w} (w(\theta) - c(S^w, \theta, 1) - o(\theta)) f(b) db$$

$$+\mathbb{E}(w(b,0) - c(S^w, b, 0, r) - o(b, 0)) - \int_{\mathcal{B}_c^w} (w(\theta) - c(S^w, \theta, 0) - o(\theta)) f(b) db. \tag{9}$$

The loss function measures how the imperfection of the selection technology impacts social welfare when there is both selection by quantity (price) and sorting by quality (through the choice of the quality of the selection technology). It is the sum of two different errors. The first type of error corresponds to the social losses caused by the consumption of the higher-quality service by non-compliant consumers (the set $\mathcal{B}_{nc}^w$), because the firm erroneously accepts to include them (see the first line of Eq.(9)). This error corresponds to the social cost of the False Positive Rate (FPR) and is also called error of type I in selection models.

The second type of error is due to the fact that there is a positive probability that some consumers are not compliant, which depends on the monopolist's price and the selection technology (see the second line of Eq.(9)). If no consumer is compliant, that is, if $D_c(S, p) = 0$, the opportunity cost of not selling to compliant consumers is the maximal social benefit of offering the service to compliant consumers and avoiding the social damage, that is, $\mathbb{E}(w(b, 0) - c(S^w, b, 0, r) - o(b, 0))$. If there is a set of B_c^w compliant consumers who buy the higher-quality service, the expected marginal social benefit of including them is $w(b, 0) - c(S^w, b, 0, r) - o(b, 0)$, which reduces the marginal social cost of having non-compliant consumers in the economy. This error is not identical to an error of type II in selection models, that is, the social cost of the False Negative Rate (FNR). Our model differs because the monopoly may use the selection technology to sort consumers by quality and the price to select consumers by quantity, which reduces the social damage.

The loss function of Proposition 4 has some similarities with the literature on algorithms, which analyzes the social planner's regulation of selection decisions. Rambachan et al. (2021) model the social preferences over the screening decisions as the sum of social weights multiplied by the expected average outcome of interest given a selection rule. The loss function that we introduce is more general for algorithms which are used in selection markets, because the proportion of consumers belonging to a given group is endogenous and both depends on the firm's choice of a price for the higher-quality service and the choice of the quality of the selection technology. Moreover, our framework allows for heterogeneous individual contributions to social welfare (i.e., the social weights are not constant).

This loss function also offers an economic framework for comparing different selection technologies. In particular, different algorithms may generate different probabilities of excluding non-compliant and compliant consumers, respectively, and imply different incentives for the intermediaries to invest in the performance of the algorithm, depending on the classification and misclassification costs (e.g., by training the algorithm on richer data sets, or choosing to train the algorithm more often). Suppose that an intermediary needs to choose between implementing an algorithm A_i with a quality S_{A_i}, with a price $p^m_{A_i}$ for the higher-quality service, then, the algorithm A_1 is socially preferable to A_2 if and only if $L(S_{A_1}, p^m_{A_1}) < L(S_{A_2}, p^m_{A_2})$.

4.3 Choice of the sanction for misclassification errors

We now focus on the more realistic case in which the social planner is neither able to choose the quality of the selection technology nor the price of the product. However, before the firm makes its choices, the social planner may impact the cost of selecting different consumer types through the choice of the parameter r. In particular, this happens when the social planner is able to impose a sanction $r > 0$ to the firm for misclassification errors.

We denote by $p^m(r)$ and $S^m(r)$ the price and the quality of the selection technology that maximize the firm's profit for a given r, respectively, and by p_r^m and S_r^m the marginal impact of an increase in r on the price of the service and the quality of the selection technology, respectively. As shown in the Appendix, the signs of p_r^m and S_r^m may be either positive or negative.

Proposition 5 gives the per-consumer marginal effect of an increase in the regulatory parameter r.

Proposition 5. *The per-consumer marginal effect of an increase in the regulatory parameter r on the additional welfare is:*

$$\frac{\partial W}{\partial r}\frac{1}{q} = -\mathbb{E}\left[(1-e)(c' S_r^m + \frac{\partial c}{\partial r})\,\middle|\mathcal{B}\right] + \frac{1}{MS}\mathbb{E}\left[(\tilde{u}' + e'p)(1-e)(w-c-o)S_r^m\,\middle|\mathcal{M}\right]$$

$$-\mathbb{E}\left[e'(w-c-o)S_r^m\,\middle|\mathcal{B}\right] + \frac{1}{MS}\mathbb{E}\left[(1-e)^2(w-c-o)p_r^m\,\middle|\mathcal{M}\right].$$

The social planner may indirectly impact the choice of the quality of the selection technology by choosing a regulatory parameter r that impacts the cost of selecting consumers, such as a sanction. However, the selection cost may be passed through to consumers into higher prices. Therefore, the social planner needs to take into account how the regulation jointly impacts the price of the service and the quality of the selection technology.

An increase in the regulatory parameter has four effects on social welfare. First, there is a direct cost effect: an increase in r changes the marginal cost of serving consumers who are not excluded. This effect is the resultant of the direct impact of r on the marginal cost of selecting consumers and the indirect impact of r on the intermediary's choice of the quality of the selection technology.

Second, there is an indirect impact of r on the social sorting effect. Third, there is an indirect impact of r on the social exclusion effect. Fourth, an increase in r impacts the intermediary's choice of a price for the service, which generates a marginal social benefit and a marginal social cost.

5 Illustration: the regulation of a fraud detection algorithm

In this section, we illustrate our setting by considering the special example of the regulation of a fraud detection algorithm. The monopoly may select consumers by changing the price of the service, or the quality of the selection technology. There are three possible families of selection technologies, which imply the participation of either one or two consumer categories:

- **a de-risking technology (dr):** a selection technology, which implies that there is no demand from non-compliant consumers at any price chosen by the intermediary,
- **a no-exclusion technology (ne):** no selection of consumers, which implies that the firm only sells to non-compliant consumers, because all consumers prefer to obtain the benefit of being non-compliant as there is no risk of being excluded,
- **an imperfect blocking technology (ib):** a selection technology, which may generate a positive demand from both compliant and non-compliant consumers.

To obtain more precise results, we consider the special case of the example of a bank given in the presentation of the model. We start by considering the case in which the probability to exclude a non-compliant consumer is increasing with the quality S of the selection technology, while the probability to exclude a compliant consumer is decreasing with S.

5.1 The maximum profit with a de-risking selection technology

If the selection technology is such that there is no demand from non-compliant consumers for any price p of the high-quality service, the monopolist chooses the price that maximizes the profits obtained from compliant consumers. Suppose that the firm adopts a 'de-risking' selection technology

$S \geq S_F$, with $D_{nc}(S_F, 0) = 0$. If $S = S_F$, the probability to exclude non-compliant consumers is given by the equality:

$$e_{nc}(S_F) = \frac{\Delta(y + B)}{F + \Delta(B + y)}.$$

If there is an interior solution, the profit-maximizing price is given by $p^{dr} = (\Delta y + k(S^{dr}))/2$ and with the profit-maximizing quality of the selection technology S^{dr}, the firm makes profit:

$$\pi^{dr} = (1 - e_c(S^{dr}))^2 \frac{(\Delta y - k(S^{dr}))^2}{4B},$$

provided that $k(S^{dr}) \leq \Delta y$, and makes zero profit if $k(S^{dr}) > \Delta y$. The firm chooses the quality S^{dr} such that the marginal benefit of having a lower exclusion of compliant consumers is equal to the marginal cost of selecting consumers:

$$-(e_c)'(S^{dr})(\Delta y - k(S^{dr}))/2 - (1 - e_c(S^{dr}))k'(S^{dr}) = 0.$$

The firm's profit with the de-risking technology does not depend on the misclassification cost, because the firm only sells to compliant consumers.

5.2 The firm's maximum profit with an imperfect blocking selection technology

Suppose that the firm adopts an imperfect blocking selection technology, such that there may be a positive demand both from compliant and non-compliant consumers, respectively. Then, it must be that $b_{nc}(S, 0) < B$, otherwise, there are only compliant consumers who buy the higher-quality service and this contradicts the fact that the firm adopts an imperfect blocking selection technology.

The monopolist may also use its price to select its consumers, because a price increase is a means to exclude some consumer categories. Since non-compliant consumers have a higher willingness-to-pay for the firm's product than the compliant ones, the firm trades off between only selling to non-compliant consumers or to both consumer types.

Proposition 5 gives the conditions such that the firm may sometimes exclude compliant consumers from the market.

Proposition 6. *If the firm excludes compliant consumers, it prefers to choose the minimal quality of the selection technology $S = 0$. The firm's incentives to exclude compliant consumers by choosing a price such that $p^{nc} > \Delta y$ with an imperfect blocking selection technology depend on the regulation of r:*

i) if $r \leq \underline{r}(0) \equiv \Delta(y - B)$, the firm always prefers to sell to both consumer types.

ii) if $r \geq \overline{r}(0) \equiv \Delta(y + B)$, the firm always weakly prefers to sell to both consumer types, because if it only sells to non-compliant consumers, it chooses a price equal to r, and makes zero profit.

iii) if $\underline{r}(0) < r < \overline{r}(0)$, the firm trades off between excluding compliant consumers from the market and selling to both consumer types. If the firm excludes compliant consumers, with $S = 0$, it chooses a price

$$p^{nc}(0) = \frac{\Delta(y + B) + r}{2},$$

and makes profit

$$\pi^{nc}(p^{nc}(0)) = \frac{(\Delta(y + B) - r)^2}{4B\Delta}.$$

Proof. See Appendix C-1. If the firm excludes compliant consumers, it chooses a price

$$p^{nc}(S) = \frac{\Delta(y + B - b_F) + r + k}{2},$$

with

$$b_F = \frac{Fe_{nc}}{\Delta(1 - e_{nc})}.$$

Since $S = 0$, this implies that

$$p^{nc}(0) = \frac{\Delta(y + B) + r}{2}.$$

Condition i) means that $p^{nc}(0) \leq \Delta y$, which implies that the price is low enough, such that compliant consumers prefer to buy the service from the intermediary instead of consuming the outside option. Condition ii) means that $p^{nc}(0) \leq r$ such that the firm does not make a positive margin when it only sells to non-compliant consumers. Condition iii) means that $p^{nc}(0) > \Delta y$ and

$p^{nc} > r$, such that the firm makes a positive margin of charging a price that implies the exclusion of compliant consumers. We have $\underline{r}(0) < \overline{r}(0)$ because $B > 0$. □

When the cost of misclassifying non-compliant consumers is very high, only selling to non-compliant consumers is too costly. The firm has no incentives to enter the market in this case if it only sells to non-compliant consumers because its margin is negative. Since the firm is unable to distinguish compliant consumers from non-compliant ones once they pass the screening process, it sells them at a uniform price. Therefore, it passes through the misclassification cost into higher prices both to compliant and non-compliant consumers. When the misclassification cost becomes very low, the firm's profit-maximizing price is so low that compliant consumers also have an incentive to buy the service. Therefore, the firm may sometimes exclude compliant consumers only for intermediary values of the misclassification cost.

It follows that it is always possible for the regulator to provide the firm with incentives to prefer the de-risking strategy to the no-exclusion strategy by raising the sanction:

$$r \geq \Delta(y+B) - \sqrt{\Delta}(1-e_c(S^{dr}))(\Delta y - k(S^{dr})).$$

If the sanction is lower than this threshold value, the firm always prefers not to exclude consumers, rather than choosing a de-risking strategy.

However, the regulator may prefer that the firm sells to both consumer types (instead of the de-risking strategy), depending on the social damage created by non-compliant consumers. In that case, if possible, the regulator should find a level of sanction such that the firm makes a higher profit of selling to both consumer types. Proposition 6 gives the firm's maximum profit of selling to both consumer types.

Proposition 7. *If the firm sells to both consumer types with a technology of quality S^{ib}, it chooses a price*

$$p^c(S^{ib}) = \frac{\Delta y + k(S^{ib})}{2} + \frac{\Delta(B - b_F^{ib}) + r}{2(\Delta(1-e_c^{ib})+1)},$$

and makes profit

$$\tilde{\pi}(p^c(S^{ib})) = \frac{(1-e_{nc}^{ib})((B-b_F^{ib}+y)\Delta - r - k(S^{ib}))^2}{4B\Delta}$$

$$+\frac{(1-e_c^{ib})^2}{4B(1-e_{nc}^{ib}+\Delta(1-e_c^{ib})^2)}(-(1-e_{nc}^{ib})(r+\Delta(B-b_F^{ib}))^2+(1-e_{nc}^{ib}+\Delta(1-e_c^{ib})^2)(\Delta y - k(S^{ib}))^2).$$

Proof. See Appendix C-2. □

The sanction has a complex impact on the firm's profit of selling to both consumer types. To understand whether it may provide the firm with incentives to choose an imperfect blocking strategy, we start by considering that the quality of the selection technology is exogenous and equal to zero. In this case, from Proposition 6, we see that the firm prefers the imperfect blocking strategy to the no-exclusion strategy only if the sanction r is low enough. Therefore, the regulator may only provide the firm with incentives to choose an imperfect blocking strategy by lowering the value of the sanction. This is because the firm passes through the cost of the sanction into higher prices, which generates a higher exclusion of compliant consumers.

6 Conclusion

In this paper, we analyzed why a monopoly chooses a quality for a selection technology that differs from the social optimum. Veiga and Weyl (2016) identified three distortions when the monopoly chooses the quality of a product in a selection market: a direct cost effect, a sorting effect, and the Spence distortion. We showed that an additional distortion arises when a monopoly chooses the quality of a selection technology, that is, the inefficient decision to exclude some consumers from the market. We also offered a measure of the welfare effect of the choice of a fraud detection algorithm in a selection market.

References

Agrawal, A., Gans, J., and Goldfarb, A. (2018). Prediction, Judgment, and Complexity: A Theory of Decision-Making and Artificial Intelligence, pages 89–110. University of Chicago Press.

Akerlof, G. A. (1970). 4. the market for 'lemons': quality uncertainty and the market mechanism. Market Failure or Success, 66.

Biancini, S. and Verdier, M. (2023). Bank-platform competition in the credit market. International Journal of Industrial Organization, 91:103029.

Cowgill, B., Davis, J., Montagnes, B. P., and Perkowski, P. (2021). Matching for strategic organizations: Theory and empirics from internal labor markets. Available at SSRN.

Creti, A. and Verdier, M. (2014). Fraud, investments and liability regimes in payment platforms. International journal of industrial organization, 35:84–93.

Daugherty, P. R. and Wilson, H. J. (2018). Human+ machine: Reimagining work in the age of AI. Harvard Business Press.

Einav, L. and Finkelstein, A. (2011). Selection in insurance markets: Theory and empirics in pictures. Journal of Economic perspectives, 25(1):115–138.

Fraisse, H. and Laporte, M. (2021). Return on investment on ai: The case of capital requirement.

Hua, X. and Spier, K. E. (2022). Holding platforms liable. HKUST Business School Research Paper, (2021-048).

Hua, X. and Spier, K. E. (2023). Platform safety: Strict liability versus negligence. Available at SSRN.

Hurlin, C., Pérignon, C., and Saurin, S. (2022). The fairness of credit scoring models. arXiv preprint arXiv:2205.10200.

Kleinberg, J., Lakkaraju, H., Leskovec, J., Ludwig, J., and Mullainathan, S. (2017). Human Decisions and Machine Predictions*. The Quarterly Journal of Economics, 133(1):237–293.

Mullainathan, S. and Spiess, J. (2017). Machine learning: An applied econometric approach. Journal of Economic Perspectives, 31(2):87–106.

Rambachan, A., Kleinberg, J., Mullainathan, S., and Ludwig, J. (2021). An economic approach to regulating algorithms. Technical report, National Bureau of Economic Research.

Spier, K. E. and Van Loo, R. (2025). Foundations for plateform liability. Notre-Dame Law Review. *Forthcoming*.

Van Loo, R. (2020). The new gatekeepers. Virginia Law Review, 106(2):467–522.

Veiga, A. and Weyl, E. G. (2016). Product design in selection markets. The Quarterly Journal of Economics, 131(2):1007–1056.

Veiga, A., Weyl, E. G., and White, A. (2017). Multidimensional platform design. American Economic Review, 107(5):191–195.

Zhang, Y. and Trubey, P. (2019). Machine learning and sampling scheme: An empirical study of money laundering detection. Computational Economics, 54(3):1043–1063.

Appendix

Appendix A: consumer demand for the service and some preliminary results

Appendix A-1: consumer demand for the service

In this preliminary Appendix, we determine consumer demand for the service according to the price charged by the monopoly. In order to achieve it, we need to determine different thresholds on the benefit to be non-compliant b. Let us denote b_c the type of agent indifferent between being a compliant consumer of the service and being a non-compliant consumer of the outside option, b_{nc} the type of the agent being a non-compliant consumer of the service and a non-compliant consumer of the outside option.

A compliant consumer who buys the service from the firm obtains utility:

$$u_c = (1 - e_c)(y(1 + \Delta) - p) + e_c y$$

The firm agrees to sell the service with probability $1 - e_c$. The agent consumes instead the outside option with probability e_c. A non-compliant consumer buys the outside option at no cost and obtains utility $u_o = b + y$. The indifferent compliant consumer between buying the service and becoming instead a non-compliant consumer who buys the outside option ($u_c = u_o$) is given by

$$b_c \equiv (\Delta y - p)(1 - e_c).$$

Intermediary threshold b_{ne}. If the firm does not exclude any consumer from the market, all consumers are non-compliant and obtain $u_o = b + y$ with certainty. If they buy the service from the firm, they obtain an additional utility $(b + y)\Delta$ but pay the price p. A consumer prefers to buy the service if and only if

$$b \geq b_{ne} \equiv (p/\Delta) - y$$

and prefers the outside option otherwise.

If the firm excludes some consumers, a non-compliant consumer who buys the service from the firm obtains the expected utility:

$$u_{nc} = ((b + y)(1 + \Delta) - p)(1 - e_{nc}) + e_{nc}(b + y - F).$$

Indeed, the firm agrees to sell with probability $1 - e_{nc}$, and otherwise, with probability e_{nc}, the firm detects the non-compliant consumer, who is constrained to buy the outside option and incur the cost of the fine F. If a non-compliant consumer renounces buying the higher-quality service, he obtains $u_o = y + b$. Therefore, a non-compliant consumer prefers to buy the service ($u_{nc} \geq u_o$) if and only if

$$b \geq b_{nc} \equiv \frac{p}{\Delta} + b_F - y,$$

where

$$b_F = \frac{F e_{nc}}{\Delta(1 - e_{nc})}$$

represents the additional benefit that must be given to the non-compliant consumer, so that the latter adopts the same consumption behavior as if there were no risk of being sanctioned. We have:

$$b_{nc} = b_{ne} + b_F.$$

A consumer who buys the service prefers to be non-compliant than compliant ($u_{nc} \geq u_c$) if and only if

$$b \geq b_I \equiv \frac{(\Delta y - p)\hat{e}}{1 + \Delta(1 - e_{nc})} + \frac{\Delta(1 - e_{nc})b_F}{1 + \Delta(1 - e_{nc})},$$

where $\hat{e} \equiv e_{nc} - e_c > 0$. The letter I stands for incentives, because the firm's detection technology provides the consumer with incentives to become compliant when the latter obtain sufficiently low benefits of being non compliant. Note that we have $b_I \leq b_c$ if and only if

$$(1 + \Delta(1 - e_{nc}))(b_I - b_c) = -b_c \frac{(1 + \Delta(1 - e_c))(1 - e_{nc})}{(1 - e_c)} + \Delta(1 - e_{nc})b_F \leq 0.$$

Moreover, we have $b_{nc} \leq b_c$ if and only if $p \leq \underline{p}$, where

$$\underline{p} \equiv \Delta y - \frac{\Delta b_F}{1 + \Delta(1 - e_c)}.$$

In addition, we have $b_I \leq b_{nc}$ if and only if $p \geq \underline{p}$ and $b_{nc} \leq b_c$ if and only if $p \leq \underline{p}$.

We distinguish therefore between different cases.

- **Case VHP (very high price):** If $p > \Delta y$, the firm excludes compliant consumers from the market.
- **Case HP (high price):** If $\underline{p} \leq p \leq \Delta y$, we have $b_c \leq b_I \leq b_{nc}$.
- **Case LP (low price):** If $p < \underline{p}$, we have $b_{nc} \leq b_I \leq b_c$, and the market is covered.

In case HP, consumers with low types (i.e., $b < \max(b_c, 0)$) buy the product and are compliant. Consumers with higher types (i.e., $b \geq \min(b_{nc}, B)$) buy the product and are non-compliant. The market is not covered because consumers such that $\max(b_c, 0) < b < \min(b_{nc}, B)$ prefer the outside option.

In case LP, the market is covered. Consumers with low types (i.e., $b < \max(b_I, 0)$) buy the service and are compliant. Consumers with higher types (i.e., $b \geq \min(b_I, B)$) buy the service and are non-compliant.

Appendix A-2: extending the model of Veiga and Weyl (2016) to our setting

We adapt the definitions given by Veiga and Weyl (2016) to our setting by considering the case in which the market is not covered. In our setting, the consumer's type $\Theta = (b, i)$ is bi-dimensional (whereas they consider a finite arbitrary number of dimensions). There are three differences in our setting. First, one of the dimensions of the consumer's type (i.e., i) is a discrete variable. Second, the value of purchasing the outside option depends on the consumer's type. This implies that b_c and b_{nc} do not have the same monotonicity with respect to price variations (unlike the parameter $\tilde{\tau}$ of their paper, which is strictly increasing with p under their assumptions). In our paper, b_c is decreasing with the price p, whereas b_{nc} is increasing with the price p. Third, the firm may exclude some consumers from the market, which implies that it considers the expected margin associated with a selection technology. Therefore, the quality of the selection technology has an additional effect on the exclusion of consumers, which depends on their types.

Our setting will imply exactly the same selection and sorting effects as in their paper, if we adapt some of their definitions to our framework.

Similar to their paper, from the definition of the marginal buyer of type i given by

$$\tilde{u}(S, b_i, i) - (1 - e_i)p = 0,$$

from the implicit function theorem, we have

$$\frac{\partial b_i}{\partial S} = -\frac{(\tilde{u}'(S, b_i, i) + e'(S, b_i, i)p)}{\frac{\partial \tilde{u}(S, b_i, i)}{\partial b}}, \tag{10}$$

and

$$\frac{\partial b_i}{\partial p} = \frac{1 - e_i}{\frac{\partial \tilde{u}(S, b_i, i)}{\partial b}}.$$

In our particular setting, we have:

$$\frac{\partial \tilde{u}(S, b_i, i)}{\partial b} = (i\Delta(1 - e_i) + i - 1),$$

and

$$\tilde{u}'(S, b_i, i) = -(\Delta(ib + y) + iF)e'(S, b_i, i).$$

Since

$$Q = \sum_{i \in \{c, nc\}} \int_{\mathcal{B}_i} f(b) db,$$

and from (10), differentiating Q with respect to S gives:

$$\frac{\partial Q}{\partial S} = \frac{\partial b_c}{\partial S} f(b_c) - \frac{\partial b_{nc}}{\partial S} f(b_{nc}) = M\mathbb{E}\left[\tilde{u}' + e'p \,\middle|\, \mathcal{M}\right].$$

Then, applying the implicit function theorem to the equation $Q(S, P(S, q)) = q$ that defines the inverse demand function yields

$$\frac{\partial P}{\partial S} = \mathbb{E}\left[\tilde{u}' + e'p \,\middle|\, \mathcal{M}\right].$$

The proof of the results follows Appendix A-1 of Veiga and Weyl (2016). We tackle the maximization of social welfare and profit simultaneously by defining:

$$Z(S, q, r) = \sum_{i \in \{c, nc\}} \int_{\mathcal{B}_i} z(S, q, r) f(b) db.$$

Following Veiga and Weyl (2016), if S is uni-dimensional, the FOC with respect to S is given by:

$$q\mathbb{E}\left[\frac{\partial z}{\partial S} \,\middle|\, \mathcal{B}\right] + M\text{Cov}\left[\tilde{u}' + e'p, z \,\middle|\, \mathcal{M}\right] = 0.$$

Profit maximization considers $z(S, q) = (1 - e(S, \theta))(P(S, q) - c(S, \theta, r))$, whereas welfare maximization considers $z(S, q, r) = (1 - e(S, \theta))(w(\theta) - c(S, \theta, r) - o(\theta))$. Compared to Veiga and Weyl (2016), the function z is multiplied by $(1 - e(S, \theta))$, which represents the probability that the firm does not exclude the buyers of type θ from the market. Veiga and Weyl (2016) consider the special case in which the function e is null, that is, $e = 0$, with $e' = 0$. Moreover, in their setting, the firm chooses product quality, which implies that w may depend on the quality, whereas we consider the quality of the selection technology.

Profit Maximization:

Profit maximization with respect to S:

When the firm maximizes its profit with respect to S, we have:

$$\frac{\partial z}{\partial S} = -(P(S,q) - c(S,\theta,r))e' + (1 - e(S,\theta))(\frac{\partial P}{\partial S} - c').$$

Since

$$\frac{\partial P}{\partial S} = \mathbb{E}\left[\tilde{u}' + e'p \,\middle|\mathcal{M}\right],$$

the FOC of the firm's profit-maximization with respect to S is given by:

$$-q\mathbb{E}\left[(1-e)c' \,\middle|\mathcal{B}\right] + q\mathbb{E}\left[\tilde{u}' + e'p \,\middle|\mathcal{M}\right]\mathbb{E}\left[1 - e\,|\mathcal{B}\right] + M\text{Cov}\left[\tilde{u}' + e'p, (1-e)(P-c)\,|\mathcal{M}\right] - q\mathbb{E}\left[e'(P-c)\,\middle|\mathcal{B}\right] = 0,$$

with $q = D_c + D_{nc}$. Replacing for $MS = q/M$ the marginal consumer surplus, after simplification by $q > 0$, the FOC of profit-maximization is given by:

$$-\mathbb{E}\left[(1-e)c' \,\middle|\mathcal{B}\right] + \mathbb{E}\left[\tilde{u}' + e'p \,\middle|\mathcal{M}\right]\mathbb{E}\left[1 - e\,|\mathcal{B}\right] + \frac{\text{Cov}\left[\tilde{u}' + e'p, (1-e)(P-c)\,|\mathcal{M}\right]}{MS} - \mathbb{E}\left[e'(P-c)\,\middle|\mathcal{B}\right] = 0.$$

Profit maximization with respect to p:

From the definitions of M_c and M_{nc}, solving for the first-order condition of the firm's profit maximization with respect to p gives:

$$-\sum_{i\in\{c,nc\}}(1-e_i)(M_i(p - c(S,b_i,i,r)) + D_i(S,p)) = 0.$$

Using the definition of M_i, this implies that:

$$-M\mathbb{E}\left[(1-e)^2(p-c)\,\middle|\mathcal{M}\right] + \mathbb{E}\left[(1-e)\,|\mathcal{B}\right] = 0.$$

If $e = 0$, we have $\mathbb{E}\left[(1-e)\,|\mathcal{B}\right] = Q$ and $\mathbb{E}\left[(1-e)^2(p-c)\,\middle|\mathcal{M}\right] = p - \mathbb{E}\left[c\,|\mathcal{M}\right]$. Therefore, we find the same result as Veiga and Weyl (2016), that is, the FOC condition of profit maximization with respect to q (or p) is given by

$$p - \mathbb{E}\left[c\,|\mathcal{M}\right] - \frac{Q}{M} = 0.$$

Welfare Maximization:

Welfare maximization with respect to S:

When the firm maximizes social welfare with respect to S, we have:

$$\frac{\partial z}{\partial S} = -(w(S,\theta) - c(S,\theta,r) - o(\theta))e' + (1 - e(S,\theta))(w' - c').$$

The FOC of social welfare maximization with respect to S is given by

$$-q\mathbb{E}\left[(1-e)c'\,\middle|\mathcal{B}\right] + M\mathrm{Cov}\left[\tilde{u}' + e'p, (1-e)(w-c-o)\,|\mathcal{M}\right] - q\mathbb{E}\left[e'(w-c-o)\,\middle|\mathcal{B}\right] = 0.$$

A division of this expression by $q > 0$ and the replacement of q/M with MS gives the result of Proposition 2.

Welfare maximization with respect to p:

The FOC of social welfare maximization with respect to p is given by

$$\mathbb{E}\left[(1-e)^2(w-c-o)\,\middle|\mathcal{M}\right] = 0.$$

If $e = 0$, as Veiga and Weyl (2016), the FOC of welfare maximization with respect to p is given by

$\mathbb{E}\left[w - c \,|\mathcal{M}\right] = 0.$

Second Best:

The social planner maximizes W with respect to r, anticipating that the monopoly will choose the price $p^m(r)$ for the product and the quality $S^m(r)$ for the selection technology. We denote by

$$S_r^m \equiv (S^m)'(r)$$

and

$$p_r^m \equiv (p^m)'(r).$$

Solving for the first-order condition of welfare-maximization with respect to r gives:

$$\frac{\partial W}{\partial r} = \sum_{i \in \{c,nc\}} \int_{\mathcal{B}_i} (-(e_i)'(w(\theta) - c(S^*, \theta, r) - o(\theta)) - c'(1 - e_i))S_r^m - (1 - e_i)\frac{\partial c}{\partial r})f(b)db$$

$$+\frac{\partial b_c}{\partial r}\,(1 - e_c)(w - c - o)|_{b_c} - \frac{\partial b_{nc}}{\partial r}\,(1 - e_{nc})(w - c - o)|_{b_{nc}} = 0$$

Since

$$\frac{\partial b_i}{\partial r} = \frac{\partial b_i}{\partial p}p_r^m + \frac{\partial b_i}{\partial S}S_r^m,$$

we have

$$\frac{\partial b_i}{\partial r} = \frac{1 - e_i}{\frac{\partial \tilde{u}(S, b_i, i)}{\partial b}}p_r^m - \frac{(\tilde{u}'(S, b_i, i) + e'(S, b_i, i)p)}{\frac{\partial \tilde{u}(S, b_i, i)}{\partial b}}S_r^m.$$

This implies that

$$\frac{\partial W}{\partial r} = M\mathbb{E}\left[(\tilde{u}' + e'p)(1 - e)(w - c - o)S_r^m \,\middle|\mathcal{M}\right] + M\mathbb{E}\left[(1 - e)^2(w - c - o)p_r^m \,\middle|\mathcal{M}\right]$$

$$-q\mathbb{E}\left[e'(w - c - o)S_r^m \,\middle|\mathcal{B}\right] - q\mathbb{E}\left[(1 - e)(c'S_r^m + \frac{\partial c}{\partial r})\,\middle|\mathcal{B}\right].$$

After a division by $q > 0$, replacing q/M with MS, we obtain that the per-consumer marginal effect of an increase in the regulatory parameter r on the additional welfare is:

$$\frac{\partial W}{\partial r}\frac{1}{q} = \frac{1}{MS}\mathbb{E}\left[(\tilde{u}' + e'p)(1-e)(w-c-o)S_r^m \,\middle|\mathcal{M}\right] + \frac{1}{MS}\mathbb{E}\left[(1-e)^2(w-c-o)p_r^m \,\middle|\mathcal{M}\right].$$

$$-\mathbb{E}\left[e'(w-c-o)S_r^m \,\middle|\mathcal{B}\right] - \mathbb{E}\left[(1-e)(c'S_r^m + \frac{\partial c}{\partial r})\,\middle|\mathcal{B}\right].$$

Appendix C: The monopoly's optimal pricing strategy for a given quality of the selection technology

Appendix D: The monopoly's optimal pricing strategy with a joint choice of the quality of the selection technology S and the price p

We determine whether the conditions under which the firm does not exclude compliant consumers from the market.

C-1: The profit of only selling to non-compliant consumers:

If the firm charges $p \geq \Delta y$, no compliant consumer prefers to buy the product and there are only non-compliant consumers who wish to buy the product. A non-compliant consumer prefers to buy the product if and only if $b \geq b_{nc}$. There is a positive demand from non-compliant consumers if and only if $b_{nc} \leq B$. If $p \geq r + k$, the firm's profit is given by:

$$\pi^{nc}(p) = \frac{1}{B}(p - r - k)(B - b_{nc}),$$

where $b_{nc} = \frac{p}{\Delta} + b_F - y$. The firm chooses $S \in (0, \overline{S})$ and $p \in (r + k(S), \Delta y)$ to maximize its profit.

If there is an interior solution, the profit-maximizing price is:

$$p^{nc}(S) = \frac{\Delta(y + B - b_F) + r + k(S)}{2}.$$

Since b_{nc} and k are both increasing with S, it is optimal when $p \geq \Delta y$ to choose the minimal quality for the selection technology $S = 0$. There is an interior solution if and only if: $p^{nc} > \Delta y$, $p^{nc} \geq r + k$, and $b_{nc} \in (0, B)$, that is,

$$0 \leq \frac{p^{nc}}{\Delta} + b_F - y \leq B.$$

The first inequality $p^{nc} > \Delta y$ can be rewritten as:

$$r \geq \Delta(y - B + b_F) - k.$$

The inequality $0 \leq \frac{p^{nc}}{\Delta} + b_F - y$ is implied by $p^{nc} > \Delta y$ and $b_F \geq 0$. The inequalities $p^{nc} \geq r + k$ and $\frac{p^{nc}}{\Delta} + b_F - y \leq B$ are both equivalent to:

$$r + k \leq \Delta(y + B - b_F).$$

Combining these different inequalities, we obtain that:

$$\Delta(y - B + b_F) - k \leq r \leq \Delta(y + B - b_F) - k.$$

We denote by $\underline{r}(S) \equiv \Delta(y - B + b_F) - k$ and $\overline{r}(S) \equiv \Delta(y + B - b_F) - k$, respectively, and conclude that there is an interior solution for the choice of the profit-maximizing price for a given quality S of the selection technology if and only if:

$$\underline{r}(S) < r < \overline{r}(S).$$

With the profit-maximizing quality of the selection technology $S = 0$, since $b_F(0) = k(0) = 0$, this implies that there is an interior solution if and only if:

$$\Delta(y - B) < r < \Delta(y + B).$$

The left-hand side of the inequality means that the firm does not lower its price too much, such that it is not profitable to include compliant consumers. If $r < \underline{r}(S)$, the firm has no incentives to only sell to non-compliant consumers, because it makes a higher profit by lowering its price and selling to both consumer types. The right-hand side of the inequality means that the firm makes a positive profit of selling to non-compliant consumers. If $r > \overline{r}(S)$, the firm has no incentives to only sell to non-compliant consumers because it does not make any positive profit in this case.

If $S = 0$, the firm chooses a price

$$p^{nc}(0) = \frac{\Delta(y+B)+r}{2},$$

and makes profit

$$\pi^{nc}(p^{nc}(0)) = \frac{(\overline{r}(0)-r)^2}{4B\Delta},$$

that is,

$$\pi^{nc}(p^{nc}(0)) = \frac{(\Delta(y+B)-r)^2}{4B\Delta}.$$

C-2: The profit of selling to both consumer types:

We showed in Appendix C-1 that if $\underline{r}(0) \leq r \leq \overline{r}(0)$, the firm chooses $S = 0$ and makes a positive profit if it chooses a price such that only non-compliant consumers prefer to buy the product. Therefore, we need to determine whether the firm makes a higher profit if it chooses a price such that compliant consumers also prefer to buy the product. For this purpose, we determine the firm's maximum profit when it sells to both consumer types if $\Delta(y-B) < r < \Delta(y+B)$.

Covered market:

Suppose that the firm does not exclude compliant consumers. If it chooses $p \leq \underline{p}$, the market is covered. If the firm chooses a very low price (that is, if $0 \leq p \leq \underline{p}$), its profit is given by

$$\pi^{VLP}(p) = p - k - r\frac{B-b_I}{B}.$$

If $e_{nc} - e_c = 0$ or if $e_{nc} - e_c > 0$ and the misclassification cost is such that $r < \frac{B(1+\Delta(1-e_{nc}))}{(e_{nc}-e_c)}$, the firm's profit is increasing with p and the firm chooses the maximum price such that the market is covered, that is, $p = \underline{p}$, and makes profit $\pi^{VLP}(\underline{p})$. If $r > \frac{B(1+\Delta(1-e_{nc}))}{e_{nc}-e_c}$, the firm's profit is decreasing with p. Therefore, the firm chooses the minimum price such that its profit is non negative, when this price exists. Therefore, if $p \leq \underline{p}$, the firm's maximum profit is $\max(\pi^{VLP}(\underline{p}), 0)$. If the firm chooses the price $p = \underline{p}$, we denote the choice of the profit-maximizing quality of the selection technology by S^c and the firm makes a profit:

$$\pi^{VLP}(\underline{p}(S^c)) = \underline{p}(S^c) - k - r\frac{B - b_I(\underline{p}(S^c))}{B}.$$

Uncovered market:

Suppose that the firm chooses $p > \underline{p}$ and $p < \Delta y$, then the market is not covered. There is a positive demand from non-compliant consumers if and only if $B - b_{nc} > 0$ or else:

$$p \leq \overline{p} \equiv (B + y - b_F)\Delta = \overline{r}(S) + k.$$

If the firm makes a positive profit of excluding compliant consumers, we showed that it must be that $B \geq b_F(S)$.[24] We analyze this case, which is relevant for comparing the profits of excluding compliant consumers, and the profit of including both consumer types.

If $B < b_F(S)$, we showed that there is a corner solution such that the firm chooses $p^{nc} = \Delta y$.

If $B \geq b_F(S)$, we have $\min(\overline{p}, \Delta y) = \Delta y$, and there is a positive demand of non-compliant consumers for the higher-quality service. Indeed, since $B \geq b_F(S)$ and $\overline{p} \geq \Delta y \geq p$, if the firm chooses p such that $\Delta y \geq p$, we have that

$$B - b_F + y - p/\Delta \geq 0,$$

which implies that $b_{nc} \leq B$.

The firm's profit is given by Eq.(5), that is, we have:

$$\tilde{\pi}(p) = \pi^c(p) + \pi^{nc}(p).$$

[24] This inequality stems from the condition $\overline{c} \geq \underline{c}$.

Note that there are $b_{nc} - b_c$ non-compliant consumers who buy the outside option.

The firm chooses $p \in (\underline{p}, \Delta y)$ and $S \in (0, \overline{S})$ to maximize its profit. If there is an interior solution to the firm's profit-maximization problem, for a given quality S of the selection technology, we denote its profit-maximizing price by p^c, and we have

$$p^c = \frac{\Delta y + k}{2} + \frac{\Delta(B - b_F) + r}{2(\Delta(1 - e_c)^2 + 1 - e_{nc})}.$$

We have $p^c \geq \underline{p}$ if and only if:

$$r \geq \hat{r}(S) \equiv (\Delta y - k)(1 - e_{nc} + \Delta(1 - e_c)^2) - \Delta b_F \frac{(2\Delta(1 - e_c)^2 + 2(1 - e_{nc}) - 1 - \Delta(1 - e_c))}{(1 + \Delta(1 - e_c))}.$$

We have $p^c \leq \Delta y$ if and only if

$$r \leq \Delta(y - B + b_F) - k + (k - \Delta y)(e_{nc} - \Delta(1 - e_c)^2).$$

Replacing for $\underline{r}(S) = \Delta(y - B + b_F) - k$, this condition can be rewritten as

$$r \leq \underline{r}(S) + (k - \Delta y)(e_{nc} - \Delta(1 - e_c)^2).$$

Therefore, there is an interior solution if and only if $p^c \geq \underline{p}$ and $p^c \leq \Delta y$. This happens if and only if

$$r \in (\hat{r}(S), \underline{r}(S) + (k - \Delta y)(e_{nc} - \Delta(1 - e_c)^2).$$

If $r > \underline{r}(S) + (k - \Delta y)(e_{nc} - \Delta(1 - e_c)^2)$ and $r \leq \overline{r}(S)$, the firm prefers to only sell to non-compliant consumers. If $r \leq \hat{r}(S)$, there is a corner solution and the firm chooses $p = \underline{p}$.

We denote by S^{ib} the profit-maximizing quality of the selection technology when the firm sells to both consumer types, by e_j^{ib} the probability that the firm excludes a consumer of type j with the profit-maximizing quality of the selection technology, and by $b_F^{ib} = F e_{nc}^{ib} / \Delta(1 - e_{nc}^{ib})$. If $r \leq \hat{r}(S^{ib})$, there is a corner solution and the firm chooses $p = \underline{p}$. If $r \in (\hat{r}(S^{ib}), \underline{r}(S^{ib}) + (k(S^{ib}) - \Delta y)(e_{nc} -$

$\Delta(1-e_c)^2)$, there is an interior solution, and the firm chooses $p^c(S^{ib})$ and S^{ib}.

If the firm sells to both consumer types, it makes profit $\tilde{\pi}(p^c(S^{ib})) = \pi^c(p^c(S^{ib})) + \pi^{nc}(p^c(S^{ib}))$, that is, we have:

$$\tilde{\pi}(p^c(S^{ib})) = \frac{(1-e^{ib}_{nc})((B-b^{ib}_F+y)\Delta - r - k(S^{ib}))^2}{4B\Delta}$$

$$+\frac{(1-e^{ib}_c)^2}{4B(1-e^{ib}_{nc}+\Delta(1-e^{ib}_c)^2)}(-(1-e^{ib}_{nc})(r+\Delta(B-b^{ib}_F))^2 + (1-e^{ib}_{nc}+\Delta(1-e^{ib}_c)^2)(\Delta y - k(S^{ib}))^2).$$

Two questions arise: 1) is the social planner able to provide the firm with incentives to choose the imperfect blocking strategy? 2) how does the quality of the selection technology varies with r?