# Parametric insurance under demand and solvency constraints

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**Abstract**

Parametric insurance is frequently mentioned as a method to reduce protection gaps, especially in the field of emerging risks. Compared to traditional insurance, the idea is to compute the compensation from a parameter (or index) that can be measured soon after the occurrence of the claim. The advantage is that payment is fast, and this might be appealing for the policyholder that needs to quickly receive funds to rebuild after an incident, rather than waiting for an expert to be sent on site. For the insurer, the costs related to claim management are considerably reduced. Moreover, the volatility of the risk is usually controlled since, by design, the parameter is a quantity on which a significant amount of data has been collected, hence its distribution is well estimated statistically speaking. Machine learning techniques play a significant role in optimizing the design of such indexes. An important difference with the concept of Cat Bonds is that the feasibility of such a cover relies on the possibility to mutualize. Mutualization, on the other hand, is achieved only if a sufficiently high number of policyholders accept to subscribe. The purpose of this paper is to introduce a model for the demand in parametric insurance and to provide conditions under which the solvency of the portfolio is achieved. We deduce from these conditions a product that combines parametric and traditional insurance in order to benefit from the best of the two worlds.

**Key words:** Parametric insurance; expected utility; demand in insurance.

**Short title:** Parametric insurance.

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# 1 Introduction

Parametric insurance is often promoted as a solution capable of addressing certain structural weaknesses of traditional insurance (see for example Barnett and Mahul [2007], Carter et al. [2017], Prokopchuk et al. [2018] or Han et al. [2019]). The principle of these coverages lies in the use of an index (or parameter) that can be easily calculated based on information available immediately after the incident. This automated calculation significantly simplifies claim management. Since the compensation is determined without the need for an expert to evaluate the amount, it can be paid out very quickly to the insured. The clarity of the indemnification conditions also reduces the likelihood of legal disputes. On the other hand, the insured must bear a basis risk (see Clement et al. [2018]), as the compensation is not based on the actual loss rather on an approximation of it. Consequently, there is a concern that parametric insurance may have a disappointing aspect, which is seen as a barrier to its development (see Johnson [2021]).

The design of an index that can serve as the basis for insurance coverage is similar to a statistical problem of estimation or prediction: using available variables, the goal is to approximate as closely as possible an unobserved quantity (the loss), see Cesarini et al. [2021]. However, this problem involves a number of constraints due to the need to align with the insured's expectations. For instance, the construction of an index proposed by Conradt et al. [2015], Zhang et al. [2019], or Chen et al. [2023] is based on maximizing the insured's utility rather than relying on a more standard metric commonly used in regression or forecasting.

Indeed, behind the question of meeting the insured's needs lies the issue of demand. Parametric insurance remains an insurance product: while other index-based products, such as Cat Bonds, can achieve balance through diversification strategies inherent to financial instruments, a parametric insurance product relies solely on pooling to withstand adverse outcomes. However, the pooling mechanism requires a sufficiently large number of insured participants. Insufficient demand will weaken the product, beyond simply failing to recover the cost of designing the index.

The purpose of the present paper is to study, through a modeling of the demand for parametric insurance, the viability of such a product when it competes with a traditional insurance product. The aim here is less about designing an optimal index and more about examining the conditions under which such an index becomes acceptable in a situation where the insurer must meet a solvency requirement. Particular attention will be paid to the impact of the loading rate (which can be lower in the case of parametric insurance due to reduced management costs) as a lever for achieving this objective. This analysis will also lead us to propose the construction of a "hybrid" coverage: by hybrid, we mean a combination of traditional insurance with parametric insurance, which supplements it in certain cases to leverage the best of both worlds, with parametric insurance not intervening when the basis risk is too high.

The rest of the paper is organized as follows. In section 2 we set up the notations and formalize the problem of demand in parametric insurance and its link with solvency. Section 3 then shows conditions to determine (under the assumption that the utility of the policyholder is exponential) for a sufficient demand in parametric insurance to exist. These conditions lead to a natural choice for a hybrid coverage, mixing traditional and parametric insurance. A practical example in cyber insurance is then provided in section 4.

# 2 Notation and settings

In this section, we formalize the general framework that we consider to study the demand (section 2.1) and solvency (section 2.2) of a portfolio of parametric insurance products, first in the classical case where policyholders are independent, then introducing (section 2.3) to add some accumulation component that may happen with a simultaneous claim for a large number of policyholders.

## 2.1 Parametric Insurance demand and statement of the problem

We consider a situation where a policyholder has a choice between two insurance products:

- The first one is a "traditional" insurance product, where the loss $Y \geq 0$ of the policyholder in the following year is fully covered.
- The second one is a parametric insurance product, that is based on $\phi(\mathbf{W})$ where $\mathbf{W}$ is a set of covariates that are measured after a claim in order to compute the index.

Typically, $\phi(\mathbf{W})$ will be lower than $Y$, the situation of overcompensation being supposed to be rare in parametric insurance. To simplify, we consider that a policyholder experiences at most one claim, otherwise $\mathbf{W}$ should be understood as the (different) circumstances of all encountered claims. If there is no claim, $Y = 0$ and $\phi(\mathbf{W}) = 0$.

A traditional way to model the demand in insurance relies on the concept of expected utility, as in Cummins and Mahul [2004], Hao et al. [2018] or Eeckhoudt and Kimball [1992]. A refinement of this approach, especially in the context where there is a choice between different products is proposed in Braun and Muermann [2004] or Fujii et al. [2016] and relies on the notion of "regret". In this paper, we concentrate ourselves with the most classical framework, that is expected utility, also because of recent contributions to the conception of index based covers which rely on expected utility maximization, see for example Zhang et al. [2019] or Chen et al. [2023].

Consider a class of utility functions $\mathcal{U} = \{x \in \mathbb{R} \rightarrow U_\alpha(x) : \alpha \in \mathcal{A}\}$, where $\mathcal{A} \subset \mathbb{R}^k$. Every function $U_\alpha$ is supposed to be non decreasing and strictly concave to materialize a property of risk aversion. Each policyholder is associated with a different function, that is with a different value of $\alpha$. The decision to buy parametric insurance compared to traditional one is motivated by the maximization of the expected utility, corresponding of the final output.

- In the case of parametric insurance, the policyholders pays a premium $\pi_\phi$, encounters a loss $Y$ and receives a compensation $\phi(\mathbf{W})$. The corresponding expected utility is

$$\mathfrak{U}_\phi(\alpha) = E\left[U_\alpha(\phi(\mathbf{W}) - Y - \pi_\phi)\right].$$

- In the case of traditional insurance, the difference stands in the fact that the price is $\pi_Y$, but the compensation is $Y$. However, we want to materialize the fact that this compensation is usually paid with a longer delay than with parametric insurance, which may be a problem for the policyholder who requires liquidity to repair the damages. Therefore we consider that the compensation will be discounted by a factor $\exp(-\tau)$ for some $\tau \geq 0$. This leads to the following expected utility for this solution,

$$\mathfrak{U}_{Y,\tau}(\alpha) = E\left[U_\alpha\left(\{\exp(-\tau) - 1\}Y - \pi_Y\right)\right].$$

Let us note that we did not consider the initial capital of the policyholder. This can be taken into account via incorporating this aspect in the parameter $\alpha$ (that may be multivariate). Moreover, let us assume that we consider a function that is defined on $\mathbb{R}$ to allow the possibility to consider negative value for the fortune of the policyholder (in which case, a debt is created).

Then, a policyholder with parameter $\alpha \in \mathbb{R}^k$ will chose to rely on parametric insurance only if

$$\mathfrak{U}_\phi(\alpha) - \mathfrak{U}_{Y,\tau}(\alpha) > 0. \tag{2.1}$$

Here, we assume to simplify that the customer buys one of the two contracts. Alternatively, one could easily consider the option where a third choice of not buying any insurance protection is possible.

## 2.2 Effect on mutualisation

To be viable, a parametric insurance product requires to be subscribed by a sufficient number of policyholders. Consider a target population of potential customers is of large size $N$, the number of policyholders buying the parametric contract will be approximately

$$n = N \int \mathbf{1}_{\mathfrak{U}_\phi(\alpha) - \mathfrak{U}_{Y,\tau}(\alpha) > 0} d\mu(\alpha), \tag{2.2}$$

where $\mu$ is the distribution of $\alpha$ among the population.

The question is then to know is this number is large enough to build a portfolio that is economically viable. Considering that $(Y_i)_{1\leq i\leq n}$ are the losses of the $n$ policyholders, the loss of the insurance company is

$$L_n(\pi_\phi) = \sum_{i=1}^{n} \phi(\mathbf{W}_i) - n\pi_\phi.$$

The size of the portfolio should be large enough to ensure that ruin during the next year is avoided with a sufficiently high probability, that is we want

$$\mathbb{P}\left(L_n(\pi_\phi) \geq 0\right) \leq \varepsilon, \tag{2.3}$$

where $\varepsilon > 0$ is close to zero.

If we assume that the policyholders are independent and identically distributed, the Central Limit Theorem applies, and

$$n^{-1/2}\left\{\sum_{i=1}^{n} \phi(\mathbf{W}_i) - \pi_\phi^*\right\} \Longrightarrow \mathcal{N}\left(0, \sigma_\phi^2\right),$$

where $\sigma_\phi^2 = Var(\phi(\mathbf{W}))$ and $\pi_\phi^* = E[\phi(\mathbf{W})]$. From this distributional convergence, one can deduce the approximation

$$\mathbb{P}\left(L_n(\pi_\phi) \geq 0\right) \approx S\left(\frac{n^{1/2}\theta\pi_\phi^*}{\sigma_\phi}\right),$$

where $S$ is the survival function of a $\mathcal{N}(0,1)$ variable, and $\pi_\phi = (1+\theta)\pi_\phi^*$. Hence (2.2) approximately rewrites

$$\frac{n^{1/2}\theta\pi_\phi^*}{\sigma_\phi} \geq S^{-1}(\varepsilon). \tag{2.4}$$

Of course, increasing $\theta$ does not necessarily lead to an improvement of the solvency, since $n$ decreases with $\theta$ as demand is reduced.

We will keep this Gaussian approximation in the following, but let us observe that this requires the variance of $\phi(\mathbf{W})$ to be finite, which may not be the case for heavy-tail distributions. If $\phi(\mathbf{W})$ is heavy tail, and if $\varepsilon$ is small compared to $\varepsilon$, other kind of approximations based on Generalized Pareto distributions may be used, see for example Mikosch and Nagaev [1998] for more details. However, heavy tail variables are not our main focus in the present paper, since incompatible with the exponential utility approach developed in section 2 (which requires the loss to have a finite Laplace transform).

On the other hand, the assumption of independence between policyholders is more restrictive. For example, in the case of crop insurance (which is probably one of the most famous use case of parametric products, see for example Cesarini et al. [2021] or Barnett and Mahul [2007]), weather events may strike a significant part of the portfolio simultaneously. A better view of the dependence between policyholders can help to take this aspect into account. We propose, in the following section, a simplified way to proceed via the introduction of a shock on the i.i.d. model that materializes the presence of catastrophic events.

### 2.3 A simplified way to include accumulation phenomena

An accumulation phenomenon occurs when a significant number of policyholders encounter claims in a short period of time. In climate related risk, this is essentially linked to the proximity between policyholders: insured that live in the same area are affected by similar weather conditions. In other cases, the dependence may not solely rely on geographic proximity, like in cyber insurance. In such cases, it may be hopeless to obtain a clear map of the links between the policyholders. Moreover, following this path of modeling the dependence between policyholders would introduce an additional difficulty in our context where we take demand into account: even in the case of geographic dependence, this would require to model a link between the distribution $\mu$ (describing the behaviors of the policyholders) and the localization of the potential customer. Calibrating the model would then require an important amount of data that may be difficult to get.

Consequently, we consider a simplified case where the accumulation episode materializes via an additional loss which is heavy tail. The total loss of the portfolio is then

$$L_n(\pi_\phi) = A_n + \sum_{i=1}^{n} \delta_i \phi(\mathbf{W}_i) - n\pi_\phi,$$

where $\delta_i = 0$ if policyholder $i$ was part of an accumulation episode, and $A_n$ represents the amount related to accumulation episodes.

For $A_n$, we consider a Generalized Pareto distribution, that is

$$\mathbb{P}(A_n \geq t) = \frac{1}{\left(1 + \frac{\gamma t}{ns}\right)^{1/\gamma}},$$

with $\gamma < 1$ and $s > 0$. Here we consider a Generalized Pareto where the scale parameter is proportional to $n$ : this is the idea that the cost of the accumulation episode is proportional to the size of the portfolio.

In this case, the probability of ruin is bounded by

$$\mathbb{P}\left(L_n(\pi_\phi) \geq 0\right) \leq \mathbb{P}\left(A_n - \frac{n\theta}{a} \geq 0\right) + \mathbb{P}\left(\sum_{i=1}^{n} \delta_i \phi(\mathbf{W}_i) - n\left(1 + \left\{1 - \frac{1}{a}\right\}\right) n\theta\pi_\phi^* \geq 0\right),$$

for all $a > 1$. Using the same Gaussian approximation as in section 2.2, the right-hand side is approximately

$$\frac{1}{\left(1 + \frac{\gamma\theta}{as}\right)^{1/\gamma}} + S\left((a-1)\frac{n^{1/2}\theta\pi_\phi^*}{a\sigma_\phi}\right).$$

To make this quantity less than the tolerance $\varepsilon$, we need

$$\frac{n^{1/2}\theta\pi_\phi^*}{\sigma_\phi} \geq \frac{a}{(a-1)} S^{-1}\left(\varepsilon - \frac{1}{\left(1 + \frac{\gamma\theta}{as}\right)^{1/\gamma}}\right), \tag{2.5}$$

if

$$1 < a < \frac{\gamma\theta\varepsilon^\gamma}{s(1-\varepsilon^\gamma)},$$

which imposes $\theta$ to be large enough to absorb the accumulation episode.

Logically, dealing with an additional accumulation risk increases the number of required policyholders to achieve mutualization. Moreover, a constraint appears on the loading factor $\theta$ which should by enough. Again, since the achievable $n$ tends to decrease when the loading factor increases due to a lower demand, this number may become impossible to reach in some cases.

In the following section, we discuss conditions on the demand for (2.4) and (2.5) to hold in the special case where the utility function is exponential: the simpler form of the utility allows to obtain simple constraints on the measure $\mu$.

# 3 Sufficient conditions for the viability of a parametric insurance product under exponential utility

In this section, we consider the particular case where the utility function is exponential. This allows to simplify considerably the formulation of the problem, and to provide in section 3.1 sufficient conditions for a parametric product to be preferable compared to a traditional one. Then, we consider in section 3.2 the consequences on the solvency of the portfolio. Section 3.3 introduces a way to combine traditional and parametric insurance to optimize the attractiveness of the product.

## 3.1 Exponential utility

In this section, we consider $U_\alpha(x) = -\alpha\exp(-\alpha x)$. The parameter $\alpha$ can be interpreted as a materialization of risk aversion, in the sense that a policyholder with high value of $\alpha$ will tend to accept an higher premium in exchange of a insurance protection against the risk. In this case, the condition (2.1) can be simplified. We introduce the Laplace transform and conditional Laplace transforms of $Y$,

$$\begin{aligned}\Psi_Y(\alpha) &= E\left[\exp(\alpha Y)\right],\\ \psi_Y(\alpha|\mathbf{w}) &= E\left[\exp(\alpha Y)|\mathbf{W}=\mathbf{w}\right].\end{aligned}$$

We assume that $\Psi_Y(\alpha) < \infty$ for all $\alpha$ in the support of $\mu$. Then,

$$\begin{aligned}\mathfrak{U}_\phi(\alpha) - \mathfrak{U}_{Y,\tau}(\alpha) > 0 \quad &\Longleftrightarrow \quad -\alpha\left\{E\left[\psi_Y(\alpha|\mathbf{W})\exp\left(-\alpha\{\phi(\mathbf{W})-\pi_\phi\}\right)\right] - \Psi_Y(\alpha')\exp(\alpha\pi_Y)\right\} > 0,\\ &\Longleftrightarrow \quad E\left[\psi_Y(\alpha|\mathbf{W})\exp\left(-\alpha\phi(\mathbf{W})\right)\right] < \Psi_Y(\alpha')\exp\left(\alpha(\pi_Y-\pi_\phi)\right), \qquad (3.1)\end{aligned}$$

where $\alpha' = (1 - \exp(-\tau))\alpha$.

From this expression, we see that condition (2.1) is essentially a matter of bounding the difference $m_Y(\alpha|\mathbf{W})) - \phi(\mathbf{W})$, where $m_Y(\alpha|\mathbf{w}) = \log \psi_Y(\alpha|\mathbf{w})/\alpha$. The difference should be small enough compared to the difference of prices $\pi_Y - \pi_\phi$, and the presence of $\Psi_Y(\alpha')$ allows this difference to go higher when $\tau$ increases.

Therefore, a first idea could be to take $\phi(\mathbf{w}) = m_Y(\alpha|\mathbf{w})$ as an indemnity function. But this solution would not be efficient, in the sense that this pay-off would lead to a too high price: from Jensen's inequality, the pure premium would then be

$$E\left[m_Y(\alpha|\mathbf{w})\right] > E[Y].$$

Hence, except if we are very close to equality (which would happen only if $\alpha$ is close to zero and/or $Y|\mathbf{W}$ has a variance close to zero), it would become very difficult to offer a premium $\pi_\phi$ smaller that $\pi_Y$, and in some cases even impossible with a loading factor $\theta_\phi > 0$. In addition to the problem of high prices, this type of contract would typically lead to a too important compensation in many cases. This may collide with some legal constraints depending on insurance regulations[1].

To be compatible with this operational necessity to keep a low price, we consider a pay-off $\phi(\mathbf{w}) = \phi_\beta(\mathbf{w}) = \beta E[Y|\mathbf{W} = \mathbf{w}]$, with $\beta \leq 1$. This choice also allows to control the probability of over-compensating for a claim. From Chernoff inequality, this probability is

$$\mathbb{P}\left(Y - \beta E[Y|\mathbf{W}] < 0|\mathbf{W} = \mathbf{w}\right) \leq \psi_Y(\rho|\mathbf{w}) \exp\left(-\rho(1-\beta)E[Y|\mathbf{W} = \mathbf{w}]\right), \tag{3.2}$$

for all $\rho > 0$ such that $\psi_Y(\rho|\mathbf{w}) < \infty$. Another approach to define the pay-off is to perform utility maximization as in Zhang et al. [2019] of Chen et al. [2023]. In such an approach, the idea is to maximize a trade-off between an important value of the compensation $\phi(\mathbf{W})$ and an affordable price. However, let us note that, with this approach, $E[\phi(\mathbf{W})]$ may be superior to $E[Y]$ is some situations. Therefore we here prefer to consider a particular shape of pay-off, determining which difference of price between the classical and the parametric product is acceptable by the customer. If we determine a situation where $\phi(\mathbf{W}) = \beta E[Y|\mathbf{W}]$, associated to a loading factor $\theta$, is an acceptable parametric product, an additional optimization can be performed if the constraint on $E[\phi(\mathbf{W})] \leq E[Y]$ is not an issue.

The following Proposition 3.1 shows that the parametric insurance product is chosen by a policyholder with risk aversion $\alpha$ provided that the loading factor $\theta$ is small enough and that a constraint on the conditional Laplace transform at point $\alpha$ holds.

[1]For example, according to the French legislation, this could be interpreted as "enrichment without cause", although the existence of a claim may be a protection against this argument, see for example S. Bros, L'assurance paramétrique en assurance de dommages, bjda.fr 2023, Dossier n° 6.

**Proposition 3.1** *Assume that*

$$\sup_{\mathbf{w}\in\mathcal{W}} \frac{m_Y(\alpha|\mathbf{w}) - \phi_\beta(\mathbf{w})}{E[Y]} < 1 - \beta + \theta_Y. \tag{3.3}$$

*Let*

$$\eta = 1 - \beta + \theta_Y - \left\{ \sup_{\mathbf{w}\in\mathcal{W}} \frac{m_Y(\alpha|\mathbf{w}) - \phi_\beta(\mathbf{w})}{E[Y]} \right\}.$$

*Then, for $\tau \geq 0$, Condition (2.1) holds if*

$$\theta \leq \frac{\eta}{\beta}.$$

*Hence, in this case, there exists a parametric product with a positive loading factor that is preferable for a policyholder with risk aversion $\alpha$.*

The proof is given in section 6.1.

In this result, Condition (3.3) is key and needs to be examined more closely.

By Jensen's inequality, $m_Y(\alpha|\mathbf{w}) \geq E[Y|\mathbf{W} = \mathbf{w}]$, so

$$m_Y(\alpha|\mathbf{w}) - \phi_\beta(\mathbf{w}) \geq (1-\beta)E\left[Y|\mathbf{W} = \mathbf{w}\right].$$

Taking the expectation, we see that

$$\frac{E\left[m_Y(\alpha|\mathbf{w}) - \phi_\beta(\mathbf{w})\right]}{E[Y]} \geq (1-\beta). \tag{3.4}$$

Hence, a necessary condition for (3.3) is that the difference between the left-hand side and right-hand side of (3.4) is not too high (less than $\theta_Y$). This difference tends to become lower when $\alpha$ tends to zero, confirming the intuition that risk aversion plays against the parametric insurance product. Moreover, a smaller value of $Var(Y|\mathbf{W} = \mathbf{w})$ will also reduce the gap between $m_Y(\alpha|\mathbf{w}) - \phi_\beta(\mathbf{w})$ and its lower bound $(1-\beta)E[Y|\mathbf{W} = \mathbf{w}]$. This directly refers to the ability to efficiently predict $Y$ from the available information $\mathbf{W}$, from which is computed the index.

## 3.2 Consequences on the solvency of the portfolio

Let us note that Proposition 3.1 only provides a sufficient condition for Condition (2.1) to hold. It is valid for all values of $\tau$, including $\tau = 0$, and is then sufficient to obtain a lower bound for the demand, since higher values of $\tau$ will increase the disadvantage of the traditional insurance product. In case of $\tau > 0$, there is room for weakening (3.3) and/or the condition on $\theta$.

From Proposition 3.1, we easily gate the following corollary.

**Corollary 3.2** *Assume that (3.3) holds for some $\alpha_0 > 0$. Then, if $\theta \leq \eta\beta^{-1}$, Condition (2.1) holds for all values of $\alpha \in (0, \alpha_0 + h_\beta(\tau)]$ with*

$$h_\beta(\tau) = F^{-1}\left(F(\alpha_0)\exp(-\tau)\exp(-\alpha[\pi_Y - \pi_{\phi_\beta}])\right) - \alpha_0,$$

*where* $F(\alpha) = \Psi'(\alpha) = E[Y \exp(\alpha Y)]$, *and*

$$n \geq N\mu\left((0, \alpha_0 + h_\beta(\tau)]\right). \tag{3.5}$$

This result is a direct consequence of the more general result of Lemma 6.1.

**Proposition 3.3** *Under the framework of section 2.2 (i.i.d. policyholders and no accumulation phenomenon) and the conditions of Corollary 3.2, the condition (2.3) holds for* $\phi_\beta$ *provided that* $\theta \leq \eta\beta^{-1}$ *and*

$$\eta \geq \frac{\sigma\beta S^{-1}(\varepsilon)}{N^{1/2}E[Y]\mu\left((0, \alpha_0 + h_\beta(\tau)]\right)^{1/2}}, \tag{3.6}$$

*where*

$$\sigma^2 = Var\left(E[Y|\mathbf{W}]\right),$$

*which can also be rewritten as*

$$\sup_{\mathbf{w}\in\mathcal{W}} \frac{m_Y(\alpha|\mathbf{w}) - \phi_\beta(\mathbf{w})}{E[Y]} \leq 1 - \beta + \theta_Y - \frac{\sigma\beta S^{-1}(\varepsilon)}{N^{1/2}E[Y]\mu\left((0, \alpha_0 + h_\beta(\tau)]\right)^{1/2}}.$$

In case of including the possibility of accumulation episodes as in section 2.3, the result of Proposition 3.4 is slightly modified.

**Proposition 3.4** *Under the framework of section 2.3 (probability of an accumulation phenomenon described by a Generalized Pareto distribution) and the conditions of Corollary 3.2, the condition (2.3) holds for* $\phi_\beta$ *provided that* $\theta \leq \eta\beta^{-1}$ *and that, for some* $\varepsilon' < \varepsilon$ *and* $a > 1$,

$$\eta \geq \max\left(\frac{\sigma\beta a S^{-1}(\varepsilon - \varepsilon')}{(a-1)N^{1/2}E[Y]\mu\left((0, \alpha_0 + h_\beta(\tau)]\right)^{1/2}}, \frac{a\beta s(1 - \varepsilon'^{\gamma})}{\gamma\varepsilon'^{\gamma}}\right) \tag{3.7}$$

*which can also be rewritten as*

$$\sup_{\mathbf{w}\in\mathcal{W}} \frac{m_Y(\alpha|\mathbf{w}) - \phi_\beta(\mathbf{w})}{E[Y]} \leq 1-\beta+\theta_Y-\max\left(\frac{\sigma\beta a S^{-1}(\varepsilon - \varepsilon')}{(a-1)N^{1/2}E[Y]\mu\left((0, \alpha_0 + h_\beta(\tau)]\right)^{1/2}}, \frac{a\beta s(1 - \varepsilon'^{\gamma})}{\gamma\varepsilon'^{\gamma}}\right).$$

The proofs of these two results are given in Section 6.2.

Again, according to Proposition 3.3 and 3.4, solvency can be achieved as long as $(m_Y(\alpha|\mathbf{w}) - \phi_\beta(\mathbf{w}))/E[Y]$ is sufficiently small, which, as we mentioned earlier, can be interpreted as the ability of $\mathbf{W}$ to catch sufficient information on $Y$. Let us note that, in this condition, the uniformity with respect to $\mathbf{w}$ is an important weak point: one expects to have situations where it is harder to approximate $Y$ from $\mathbf{W}$, leading to an increase of $Var(Y|\mathbf{W} = \mathbf{w})$. This pleads to introduce, in the next section, an hybrid product, mixing traditional and parametric insurance, where the use of parametric insurance is restricted to the most favorable type of events.

### 3.3 Hybrid product

Let

$$\mathcal{W}_\alpha(\mathfrak{e}, \beta) = \{\mathbf{w} \in \mathcal{W} : m_Y(\alpha|\mathbf{w}) - \phi_\beta(\mathbf{w}) \leq \mathfrak{e}\}.$$

We define the following pay-off,

$$\mathfrak{h}^{\mathfrak{e}}_{\alpha,\beta}(Y, \mathbf{W}) = Y \mathbf{1}_{\mathbf{W} \in \overline{\mathcal{W}_{\alpha,\beta}(\mathfrak{e})}} + \phi_\beta(\mathbf{W}) \mathbf{1}_{\mathbf{W} \in \mathcal{W}_{\alpha,\beta}(\mathfrak{e})}, \tag{3.8}$$

where $\overline{\mathcal{A}}$ is the complementary set of the set $\mathcal{A}$.

The idea is that we use parametric insurance only in cases where we expect this solution to be reliable. From Proposition 3.1, we saw that the unfavorable situations are when $m_Y(\alpha|\mathbf{w}) - \phi_\beta(\mathbf{w})$ is large, which motivates the introduction of $\mathcal{W}_{\alpha,\beta}(\mathfrak{e})$.

The premium $\pi_{\mathfrak{h}}$ associated with this product is

$$\pi_{\mathfrak{h}} = (1 + \theta_Y) E\left[Y|\overline{\mathcal{W}_{\alpha,\beta}(\mathfrak{e})}\right](1 - p_{\mathfrak{e}}(\alpha, \beta)) + (1 + \theta) E\left[Y|\mathcal{W}_{\alpha,\beta}(\mathfrak{e})\right] p_{\mathfrak{e}}(\alpha, \beta),$$

where $p_{\mathfrak{e}}(\alpha, \beta) = \mathbb{P}(\mathbf{W} \in \mathcal{W}_{\alpha,\beta}(\mathfrak{e}))$. We here apply the same loading factor $\theta_Y$ as for the traditional contract for cases where exact compensation is offered, the lower loading factor $\theta$ being applied only on the parametric part.

Proposition 3.5 provides condition for the hybrid product $\mathfrak{h}^{\mathfrak{e}}_{\alpha,\beta}$ to be chosen instead of the traditional contract.

**Proposition 3.5** *If*

$$\eta_{\mathfrak{e}}(\alpha, \beta) = 1 - \beta + \theta_Y - \frac{\mathfrak{e}}{E\left[Y|\mathcal{W}_{\mathfrak{e}}(\alpha, \beta)\right] p_{\mathfrak{e}}(\alpha, \beta)}. \tag{3.9}$$

*Then, if $\theta \leq \eta_{\mathfrak{e}}\beta^{-1}$, the policyholder with risk aversion less than $\alpha$ prefers the contract defined by the pay-off $\mathfrak{h}^{\mathfrak{e}}_{\alpha,\beta}$ for all $\tau \geq 0$.*

From the fact that (3.9) should be non negative, we see that the set $\mathcal{W}_\alpha(\mathfrak{e}, \beta)$ should not be too small, otherwise the probability $p_{\mathfrak{e}}(\alpha, \beta)$ could make the left-hand side larger than $1 - \beta + \theta_Y$. On the other hand, one could be tempted to take a low value for $\mathfrak{e}$ to control the difference between $m_Y(\alpha|\mathbf{w})$ and $\phi_\beta(\mathbf{w})$, but this mechanically tends to make $\mathcal{W}_{\mathfrak{e}}(\alpha, \beta)$ shrink. Let us also note that a too important decrease of $\mathfrak{e}$ introduces more constraints on $\theta$ : a decrease of $\eta_{\mathfrak{e}}(\alpha, \beta)$ makes condition (3.9) easier to achieve, but the loading factor $\theta$ then should be smaller.

**Proof.** Similarly to the case of the purely parametric product, the situation $\mathfrak{U}_{\mathfrak{h}^{\mathfrak{e}}_{\alpha,\beta}}(\alpha) - \mathfrak{U}_{Y,\tau}(\alpha) > 0$ is implied by

$$E\left[\exp(Y - \mathfrak{h}^{\mathfrak{e}}_{\alpha,\beta}(Y, \mathbf{W}))\right] \leq \exp(\alpha[\pi_Y - \pi_{\mathfrak{h}}]). \tag{3.10}$$

The left-hand side rewrites

$$(1 - p_{\mathfrak{e}}(\alpha, \beta)) + E\left[\exp\left(\alpha\left\{m_Y(\alpha|\mathbf{W}) - \phi_\beta(\mathbf{W})\right\}\right) \mathbf{1}_{\mathbf{W} \in \mathcal{W}_{\alpha,\beta}(\mathfrak{e})}\right] \leq (1 - p_{\mathfrak{e}}(\alpha, \beta)) + \exp(\alpha\mathfrak{e}) p_{\mathfrak{e}}(\alpha, \beta) \leq \exp(\alpha\mathfrak{e}).$$

Moreover,

$$\pi_Y - \pi_{\mathfrak{h}} = (1 + \theta_Y - \beta - \beta\theta)E[Y|\mathcal{W}_{\alpha,\beta}(\mathfrak{e})]p_{\mathfrak{e}}(\alpha, \beta).$$

Hence, a sufficient condition for (3.10) is

$$\frac{\mathfrak{e}}{E\left[Y|\mathcal{W}_{\alpha,\beta}(\mathfrak{e})\right]p_{\mathfrak{e}}(\alpha,\beta)} \leq 1 - \beta + \theta_Y - \beta\theta,$$

which means that $\theta$ should be less than $\eta_{\mathfrak{e}}(\alpha,\beta)\beta^{-1}$. As for the proof of Proposition 3.1, $\mathfrak{U}_{\mathfrak{h}^{\mathfrak{e}}_{\alpha,\beta}}(\alpha) - \mathfrak{U}_{Y,\tau}(\alpha) > 0$ implies that $\mathfrak{U}_{\mathfrak{h}^{\mathfrak{e}}_{\alpha,\beta}}(\tilde{\alpha}) - \mathfrak{U}_{Y,\tau}(\tilde{\alpha}) > 0$ for any $\tilde{\alpha} \leq \alpha$. ■

# 4 Illustration with real data

We here illustrate in a simplified example in the field of cyber insurance a way to calibrate the model and to deduce in which case parametric insurance is relevant. In section 4.1, we describe the database and the choices that have been made for the different parameters required in our models. Section 4.2 is devoted to the conception of the hybrid product of section 3.3.

## 4.1 Description of the context and of the database

**Business interruption and cyber insurance.**

We consider, for this illustration, a database of losses generated by business interruption caused by a cyber incidents. Business interruption is indeed an important consequence of cyber attacks, that can even be more expensive than the loss or leak of digital assets. The increased digitalization of the economy makes industries vulnerable when they can not rely anymore on their information systems. The hacking of Colonial Pipeline shows that even non-tech companies can be seriously damaged by such events.

The losses generated by such attacks can be relatively hard to evaluate, also because of some relatively long term consequences, and sometimes even immaterial consequences like loss of reputation[2]. Although the way to cover immaterial damages is far beyond the scope of the present paper, this aspect also explains the increasing appetence for parametric insurance products in the field of cyber[3]. Moreover, a fast compensation mechanism is also key to rebuild after the claim.

Since the time of business interruption is obviously correlated with the severity, it is quite natural to use this quantity, instantly measurable after the event, as the core element to build a parametric product. The link between the duration of business interruption and the economic loss is also a key element in evaluating the consequences of a cyber attack in several studies like

[2]See Hiscox's Cyber Readiness Report, `https://www.hiscoxgroup.com/cyber-readiness`

[3]The French General Direction of Treasury, in a report of 2022, mentioned parametric insurance as one of the possible way to increase coverage of cyber risk, see `https://www.tresor.economie.gouv.fr/Articles/2022/09/07/remise-du-rapport-sur-le-developpement-de-l-assurance-du-risque-cyber`

| Variable | Mean | Minimum | Maximum | Standard deviation |
|---|---|---|---|---|
| $Y$ (Keuros) | 10.9 | 0.73 | 565 | 47 |
| $T$ (days) | 3.5 | 0.34 | 6.2 | 0.83 |
| $Y,\ \delta = 1$ | 7.1 | 0.73 | 516 | 31.7 |
| $T,\ \delta = 1$ | 3.36 | 0.34 | 5.7 | 0.78 |
| $Y,\ \delta = 0$ | 48.5 | 2.9 | 565 | 115.4 |
| $T,\ \delta = 0$ | 4.6 | 3.1 | 6.2 | 0.46 |

Table 1: Descriptive statistics for the database.

Tam et al. [2023]. Of course, the effect of a given duration of service interruption is strongly dependent from the sector of activity of the victim and of its size. In this simplified example, we only consider a portfolio composed of policyholders with the same profile, that is in the same sector of activity, same operating country, and size (turnover between 10 to 50 million euros).

**The database.**

To learn about the link between the loss $Y$ and the time of business interruption, we consider a database contains 1000 incidents. It has been rescaled to be consistent with the market trends we consider for the premium, and that are described below.

For each claim, we have at our disposal:

- the amount of the claim $Y$;
- the time of business interruption $T$;
- an indicator function $\delta \in \{0, 1\}$ that is equal to 1 if the victim managed to trigger some back-up plan, which can reduce the impact of the business interruption.

Some descriptive statistics are given in Table 1 below.

**Premium amount.**

We consider in this simplified framework that each policyholder pays the same premium, which is given by the average insurance premium for year 2022 according to the report LUCY (Light Upon Cyber Insurance) on the French market conducted by AMRAE (Association pour le Management des Risques et l'Assurance de l'Entreprise)[4]. This premium is, according to the report, $\pi_Y = 9,163$ euros. To consider a reasonable value for the loading factor $\theta_Y$, we consider the loss ratios observed on the market between 2020 and 2022 for the companies of "medium size" (following the terminology of the report) that we consider. These loss ratios are 45% (2020), 36% (2021) and 100% (2022), with an average of approximately 60%. We then make

[4] See the AMRAE report LUCY, https://www.amrae.fr/bibliotheque-de-amrae/lucy-light-upon-cyber-insurance-2024-edition

the (strong) assumption that $\pi_Y$ is calibrated to generate an average result which is 40% higher than the pure premium, that is $\theta_Y = 0.4$.

We insist on the fact that this loading factor does not pretend to accurately reflect the practices of the insurance market: first of all, there may be important differences from one customer to another, and the evolution of the loss ratios are not necessarily anticipated: the current state of cyber insurance is quite instable (as shown in the same LUCY report, the amount of premiums considerably evolve from one year to another, showing a constant reevaluation of the risk by the insurers, that react to previous results).

**Frequency of claims.**

The database we consider only gives us information about the losses, but not about the frequency, since we have no indication about the exposure. This gives us information about the pay-off in case of claim, but not about its probability of occurrence.

To consider a plausible number for the frequency, we set the probability of having an incident to $p = 0.06$. This value is, again inspired from the report LUCY (492 medium size companies in the sample used for the report, and 30 claims). Again, we recall that, in our framework, we considered the case that a policyholder does not experiences more than 1 claim per year for simplification.

**Demand and risk aversion.**

We consider a target population of size of $N = 500$, which corresponds approximately to the number of policyholders in the perimeter of the LUCY study (medium-size companies).

For this application, we stay with an exponential utility function, and need to give a measure $\mu$ to describe the distribution of risk aversion among potential policyholders. In the absence of a rigorous market study that allows to measure price elasticity, modeling the measure $\mu$ is the most disputable point. Again, our purpose is only to illustrate the methodology described in the paper, and not to provide a reliable estimation of the demand in cyber insurance.

We consider here that $\mu$ is a shifted exponential distribution, that is

$$d\mu(t) = \lambda \exp(-\lambda(t - \alpha_-))\mathbf{1}_{t \geq \alpha_-}.$$

To determine the value of $\alpha_-$, we observe that the target population of policyholders who already subscribed an insurance contract accepted a price $\pi_Y$. Since their preferences are described via an exponential utility, this is possible only if their risk aversion $\alpha$ is high enough. If we do not take into account the potential discount factor $\tau$ at this stage, this means that

$$\frac{\log \Psi_Y(\alpha_-)}{\alpha_-} = \pi_Y.$$

Estimating empirically $\alpha \to \Psi_Y(\alpha)$ from the database (see Figure 1), we get $\alpha_- = 0.0108$.

Next, to consider a proper value for $\lambda$, we make the assumption that half of the population of the policyholders is ready to accept an increase of 40% of the premium. This choice is arbitrary,

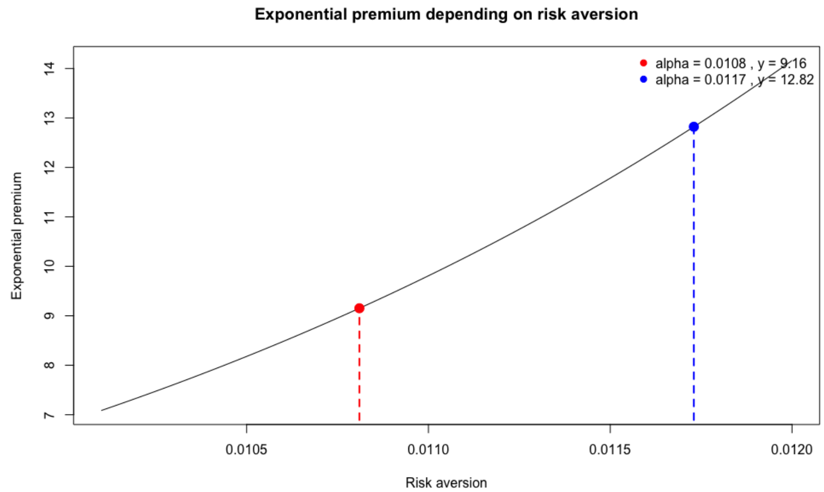



Figure 1: Estimation of $\alpha \to \log \Psi_Y(\alpha)/\alpha$ (exponential premium). The left point corresponds to the value of risk aversion corresponding to an exponential premium equal to $\pi_Y$. The right point (blue) corresponds to the case where $\pi_Y$ increases by 40%.

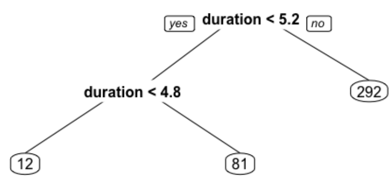
but is motivated by the fact the LUCY report noted an increase of 84% of the collected premiums over 2022, for an increase of 53% of the number of policyholders in the perimeter of the study (while deductible increase and insurance capacity stays stable). This seems to indicate that all of the current policyholders were ready to accept an increase of approximately 20% of their premium. The proportion 40% that is taken to set the value of $\lambda$ is then based on twice this number. This leads to $\lambda = 433$.

## 4.2 Estimation of the conditional expectation and conditional Laplace transform

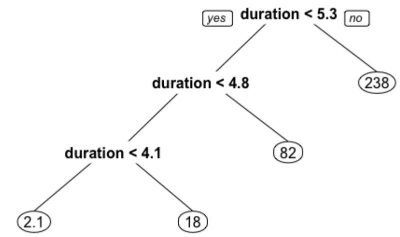
To estimate the conditional distribution of $Y$, we rely on regression trees (see for example Breiman [2017] or Loh [2014]). This choice is motivated by simplicity, the will to have simple and intelligible rules to produce a compensation, and a tractable and simplified way to give an example of design of the "hybrid" product that we propose.

Since the efficiency of the back-up plan plays a particular role in the severity, we arbitrary distinguish between the claims that are associated with a back-up plan that failed (or that was not triggered, $\delta = 0$) and the ones where this solution managed to reduce the impact of the incident. This produces two regression trees that are reported in Figure 2.

The covariates at our disposal after a claim are $\mathbf{W} = (\delta, X)$ where $X = l$ if $T$ is in the $l-$th leaf of the tree. We consider three values of $\beta$, namely $\beta = 0.75$, $\beta = 0.9$ and $\beta = 1$. For each leaf and each tree, we compute $\Delta(\delta, l) = (m_Y(\alpha_0|\delta, l) - \phi_\beta(\delta, l))/\{E[Y|\delta, X = l]p_{\mathfrak{c}}(\alpha_0, \beta)\}$. The




(a) Back-up plan failed ($\delta = 0$).




(b) Back-up plan was at least partially successful ($\delta = 1$).

Figure 2: Regression trees for the distribution of $Y$ depending on the success of the backup plan.

results are reported in Tables 2 and 3.

The values obtained promote the use of a parametric product for small durations of business interruption, depending on the values of $\theta_Y$ and the size of the target population. A more careful analysis of the claims in the left leaves of each tree would lead to a better structured product: in the fit of the regression trees, the shape of the tree is strongly influences by the largest observations, while the majority of cases fall in the left-hand side leaves.

| $X$ | $\delta = 0, \beta = 0.75$ | $\delta = 0, \beta = 0.9$ | $\delta = 0, \beta = 1$ |
|---|---|---|---|
| $l = 1$ | 0.71 | 0.55 | 0.44 |
| $l = 2$ | 2.55 | 2.39 | 2.28 |
| $l = 3$ | 0.82 | 0.66 | 0.55 |

Table 2: Value of $\Delta(0, l)$ for the different leaves of the tree of Figure 2 (b) (the leaves are numbered from left to right).

| $X$ | $\delta = 1, \beta = 0.75$ | $\delta = 1, \beta = 0.9$ | $\delta = 1, \beta = 1$ |
|---|---|---|---|
| $l = 1$ | 0.30 | 0.15 | 0.04 |
| $l = 2$ | 0.79 | 0.63 | 0.52 |
| $l = 3$ | 3.1 | 2.9 | 2.8 |
| $l = 4$ | 1.90 | 1.43 | 1.12 |

Table 3: Value of $\Delta(1, l)$ for the different leaves of the tree of Figure 2 (b) (the leaves are numbered from left to right).

# 5 Conclusion

In this paper, we proposed a framework to discuss the opportunity of introduction of a parametric insurance product in competition with a more traditional contract. The product should be attractive enough to achieve a sufficient number of policyholders to achieve solvency of the portfolio. The conditions we obtain suggest the introduction of an hybrid product, where parametric insurance is considered only in some specific situations to accelerate compensation and reduce the premium. Beyond the concept of index based insurance, this result can also be used for claim management purpose in traditional insurance contracts, removing the cost of experts when dealing with some specific claims. Let us note that we do not cover here one of the appealing aspects of parametric insurance, which is to propose coverage for claims that are not covered by traditional policies: the scope of the present paper is only to discuss the introduction of parametric insurance in a context where traditional insurance is already present. The question of modeling the demand in situations where a traditional policy is not available will be considered elsewhere, and would require a more delicate approach to calibrate the utility function.

# 6 Appendix

## 6.1 Proof of Proposition 3.1

Showing the result for $\tau = 0$ is sufficient, since a higher value of $\tau$ reduces the expected utility of the traditional insurance contract.

If $\tau = 0$, condition (2.1) is

$$E\left[\exp(\alpha[m_Y(\alpha|\mathbf{W}) - \phi_\beta(\mathbf{W})])\right] \leq \exp\left(\alpha[1 + \theta_Y - \beta - \beta\theta]E[Y]\right).$$

So taking the logarithm, we get

$$\log E\left[\exp(\alpha[m_Y(\alpha|\mathbf{W}) - \phi_\beta(\mathbf{W})])\right] \leq \alpha[1 + \theta_Y - \beta - \beta\theta]E[Y].$$

Finally,

$$\theta\beta \le 1-\beta+\theta_Y - \frac{\log E\left[\exp(\alpha[m_Y(\alpha|\mathbf{W})-\phi_\beta(\mathbf{W})])\right]}{\alpha E[Y]}. \tag{6.1}$$

Since $\theta\beta$ needs to be strictly positive, this requires

$$\frac{\log E\left[\exp(\alpha[m_Y(\alpha|\mathbf{W})-\phi_\beta(\mathbf{W})])\right]}{\alpha E[Y]} < 1-\beta+\theta_Y.$$

Since

$$\log E\left[\exp(\alpha[m_Y(\alpha|\mathbf{W})-\phi_\beta(\mathbf{W})])\right] < \sup_{\mathbf{w}\in\mathcal{W}} \frac{m_Y(\alpha|\mathbf{w})-\phi_\beta(\mathbf{w})}{E[Y]},$$

it follows from (3.3) that this condition holds. Then, taking $\theta \le \eta\beta^{-1}$ ensures that (6.1), hence (2.1), holds.

## 6.2 Proof of Proposition 3.3 and 3.4

**Proof of Proposition 3.3.**

We have $\pi^*_{\phi_\beta} = \beta E[Y]$, and

$$\sigma^2_{\phi_\beta} = \beta^2 Var\left(E[Y|\mathbf{W}]\right) = \beta^2\sigma^2.$$

From Corollary 3.2, which requires that $\theta \le \eta\beta^{-1}$,

$$n \ge n_0 = N\mu\left((0,\alpha_0+h_\beta(\tau)]\right).$$

Condition (2.3) holds if

$$\frac{n_0^{1/2}\theta\pi^*_{\phi_\beta}}{\sigma_{\phi_\beta}} \ge S^{-1}(\varepsilon).$$

This rewrites

$$\theta \ge \frac{\sigma S^{-1}(\varepsilon)}{N^{1/2}E[Y]\mu\left((0,\alpha_0+h_\beta(\tau)]\right)^{1/2}}.$$

To be compatible with the upper bound on $\theta$, we need condition (3.6).

**Proof of Proposition 3.4.**

The proof is similar to the proof of Proposition 3.3, but with (2.4) replaced by (2.5). If

$$\theta \ge \frac{a\sigma(1-\varepsilon'^\gamma)}{\gamma\varepsilon'^\gamma},$$

we have

$$S^{-1}\left(\varepsilon - \frac{1}{\left(1+\frac{\gamma\theta}{a\sigma}\right)^{1/\gamma}}\right) \le S^{-1}(\varepsilon-\varepsilon').$$

Again, we need the upper and lower bounds on $\theta$ to be compatible, which leads to the result of the Proposition.

### 6.3 Risk aversion heredity property

**Lemma 6.1** *Assume that condition (2.1) holds for* $\phi$, $\alpha_0 > 0$ *and with* $\tau = 0$. *Let* $F(\alpha) = \Psi'(\alpha) = E\left[Y \exp(\alpha Y)\right]$. *Then, (2.1) holds for all* $\tau$ *and* $\alpha \in (0, \alpha_0 + h(\tau)]$ *with*

$$h(\tau) = F^{-1}\left(F(\alpha_0)\exp(-\tau)\exp(-\alpha[\pi_Y - \pi_\phi])\right) - \alpha_0.$$

**Proof.** Since $\Psi(\alpha') \geq 1 = \Psi(0)$, if (2.1) holds for $\tau = 0$, it also holds for all $\tau > 0$. Hence we show the result in two steps. First, we show that condition (2.1) holds for $\tau = 0$ and all $\alpha \leq \alpha_0$. Then we study the case $\alpha_0 < \alpha \leq \alpha_0 + h(\tau)$.

**First case:** $\alpha \leq \alpha_0$.

Let $Z = Y - \phi(\mathbf{W}) - \pi_Y + \pi_\phi$. Condition (3.1) rewrites

$$E\left[\exp(\alpha Z)\right] \leq \Psi(\alpha').$$

We have, as a consequence of Jensen's inequality,

$$\frac{\log E\left[\exp(\alpha Z)\right]}{\alpha} \leq \frac{\log E\left[\exp(\alpha_0 Z)\right]}{\alpha_0}.$$

Hence

$$E\left[\exp(\alpha Z)\right] \leq 1.$$

**Second case:** $\alpha_0 < \alpha \leq \alpha_0 + h(\tau)$.

Let $\alpha = \alpha_0 + x$ for $0 < x \leq h$, and $\alpha' = \alpha(1 - \exp(-\tau))$. From a Taylor expansion,

$$\Psi(\alpha') \geq \Psi(\alpha_0') + x(1 - \exp(-\tau))E\left[Y \exp\left(\alpha_0' Y\right)\right].$$

On the other hand,

$$E\left[\exp(\alpha Z)\right] \leq E\left[\exp(\alpha_0 Z)\right] + x\exp(-\alpha\{\pi_Y - \pi_\phi\})E\left[Y \exp(\alpha Y)\right].$$

Since $E[\exp(\alpha_0 Z)] \leq 1 \leq \Psi(\alpha_0')$,, condition (2.1) holds if

$$\frac{F(\alpha)}{F(\alpha_0)} \leq \exp(-\tau)\exp(-\alpha[\pi_Y - \pi_\phi]).$$

By definition, this condition holds for $h \leq h(\tau)$. ■

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