DICE : Measurement without data

Guillaume Flament
Abstract

In this article, we propose to build on the DICE model (Nordhaus, 2018) where we introduce a new quantity, commonly called *exergy* (Ayres et al., 2003). Data show that the variation of primary *exergy*, or simply called *exergy* is a good predictor of the Total Factor Productivity variation. That remark allows us to design new economic scenarios that respects the Paris Agreement. These scenarios are of particular interest for future climate stress-tests (cf. ACPR (2020)). Moreover, our model is capable to conciliate different projections by the introduction of environmental constraints. On one hand, if *exergy* increases, we have similar projections with standard economic models. On the other hand, if *exergy* decreases, we obtain projections closer to other Integrated Assessment Models, where economic output declines (Meadows et al., 2013; Capellán-Pérez et al., 2020).

1 Introduction

Since 1988, the Intergovernmental Panel for Climate Change (IPCC) tries to evaluate the impact of human activities on the climate and has very recently launched yet another worrisome report IPCC (2021). Around the same time, Nordhaus (1991) published a pioneer work was, for the first time, a cost benefit analysis where published and discussed the optimal greenhouse gas emissions abatement policy. He then published the first iteration of the Dynamic Integrated model of Climate and Economy (DICE) model (Nordhaus et al., 1992) to take into account the damage of climate change on the economy. His work seems to have met a huge success as he has been granted the economic Nobel prize in 2018. Many economists have expressed criticism about DICE starting with Stern (2013). In this report, Stern states that economic growth shouldn’t be taken for granted and wrote that new models needs to be build to add possible recessions. In DICE, economic declines is impossible due to the exogenous growth of Total Factor Productivity (TFP). To build new models where economic decline is a possible scenario, it is crucial to change the exogeneity hypothesis of the TFP. This quantity is often interpreted as a proxy for technological progress. A review from Isaksson (2007) states that this progress is often led by education, health, infrastructure, and financial developments. In practice, it seems very difficult to quantify all these variables to build a meaningful model of TFP. Some models rely on econometric relationships (Alestra et al., 2021) using proxy variables such as relative price of energy or number of years in the schooling system. Some others rely on pure economic assumptions, such as the catch-up model developed by Aghion et al. (2001); Vandenbussche et al. (2006). Unfortunately, it seems that the relationship between all these variables and TFP are not very strong and these models seem to be unable to provide sufficient ground to introduce endogenous TFP variations in the DICE framework.

Overall, the goal of this article is similar to the one of Alestra et al. (2021). That is, to build an easily tractable and interpretable economic model. The most striking difference between their contribution and Nordhaus (2018) model, is the assumption of endogenous TFP variation. However, Alestra et al. (2021) model lacks crucial properties needed for relevant future Gross Domestic Product (GDP) forecasts. First, the variables used to predict TFP seems to be very hard to project which is a problem given the key role that variable must play in the model. One may wonder to what extent their model is endogenous as the predictor variable cannot be easily estimated. Second, they assume energy consumption will be set by GDP and the energy mix will adapt according to the relative price of each energy year by year. Unfortunately such assumption is very optimistic, because the energy mix doesn’t evolve much from a year to another. And regardless of energy prices, a nuclear, coal or gas power plant is build for several decades. Third the assumption that energy mix evolves according to the relative price undermines the physical difficulty of replacing fossil fuels by other source of energy. Finally, we argue that price is a very poor indicator of energy scarcity, and reflects very poorly the central role it plays in every sector of the economy.

In this article, we focused on the DICE model, however we are aware other IAM models are used in the literature. The NGFS uses GCAM (Calvin et al., 2019), MESSAGE (Krey et al., 2020) and REMIND (Luderer et al., 2015). GCAM and MESSAGE are models that set GDP as an exogenous variable hence inappropriate for the purpose of this article. REMIND, however, has a different specification where it defines a production function that embed final energy. In a way it may seem similar to our model but they tune efficiency parameters to match exogenous GDP growth path : “It assigns an efficiency parameter to each
production factor in the various macroeconomic CES (constant elasticity of substitution) functions. The changes of efficiency parameters over time are tuned such that baseline economic growth and energy intensity improvements match exogenous scenario specifications.” Which basically makes GDP growth exogenous just as all other models. Thus, it means that the NGFS takes into account IPCC predictions with exogenous GDP growth paths. These assumptions are at best misleading and lead to systematic risk as financial institutions are unable to properly measure climate risk. We think these models are inherently dangerous in a stress-test framework.

That are the reasons that motivate us to build another model that addresses these critics. Our goal is to build a statistical model using observable variables to predict future values of TFP. The main advantage of our method is that it uses one observable variable that seems to be a driver of TFP. It also has the appealing property of being very easily estimated from a climate trajectory. Then, using that model we would like to show projections of GDP using our modified version of the DICE model (Nordhaus, 2018). The article will be organized as follows, in the first section, we will briefly describe the DICE model developed by Nordhaus (2018). Then, in the second section, we will describe all the assumptions made to develop our own version with a focus on the TFP modelisation. In the third section, we will discuss two ways to build exergy scenario using exogenous assumptions and endogenous ones. In the fourth section, we will build the climate change module. The aim of this module is to translate climate change into economic damage. In the fifth section, we will derive our full model and compare it to some other propositions such as Nordhaus (2018) and the scenario used by the ACPR (2020) for the french climate stress-test, then the last section focuses on the potential development of our proposed approach.

1.1 The DICE model

The DICE model introduced by Nordhaus et al. (1992) and recently reevaluated by Nordhaus (2014, 2018), is an economic model that tries to describe the interaction between the economy and the environment. Such model answers the following question: what would be the cost of the emissions reduction needed to meet the Paris Agreement? To answer such question, Nordhaus identifies two costs.

The first is the direct cost of climate change. Unfortunately, climate change increases the likelihood and the magnitude of a disastrous event. Recent events such as the heat dome in Canada, wildfire in north Africa or the chronic drought in the west of the USA are only a few observations of what is meant to come. Obviously environmental disasters are very costly for a society, for example, Hurricane Katrina costed more than $100 billions (Shreve and Kelman, 2014) and global warming is expected to kill millions if not billions of people (Mora et al., 2017, 2018). Thus, Nordhaus, using the work of Tol (2009) estimated a damage function in order to quantify these damages. Direct damages are represented by the function \( \Omega \) in the model. That function has been the center of many critics, see for example Weitzman (2010); Stern (2013).

The second cost is the adaptation to a warming climate and the investments required to lower the greenhouse gas emissions of our economy. These investments and decommissions may slow down or reverse growth. First, we need to decommission all internal combustion engines (planes, ships, cars, ...), all electric generators that burns fossil fuels, change all the current cemeteries and steel mills, by 2050 at a global scale. Then we need to invest in a low carbon economy. But one may argue that, to date, green investments are not as cost effective as traditional investments, else they would already be financed and a net-zero economy would be reached without any public intervention. That difference, between traditional and low carbon investments, might introduce a cost of greening the economy. That cost is represented in the model by the \( 1 - \omega(t) \) function, which represents indirect damages.

Overall, Nordhaus considers a standard Cobb-Douglas production function multiplied by two other functions representing direct and indirect damages of the climate change.

\[
Y(t) = (1 - \omega(t)) \frac{A(t)K(t)^{\beta}L(t)^{1-\beta}}{1 + \Omega(t)}. \tag{1}
\]

In Equation (1), \( Y(t) \) represents the output of the economy at time \( t \). Many other production functions exist (see Thompson (2006)) but we won’t discuss them as we argue the most important parameter is the TFP,
regardless of the production function. Thus, we want to focus this paper on building an endogenous TFP model with the simplest and most studied production function. In Equation (1) \( K(t) \) and \( L(t) \) respectively represent capital and labor, \( \Omega \) is the amount of damage caused by climate change while \( \omega \) represents the cost of emissions reduction and adaptation, they have the following definitions:

\[
\begin{align*}
\Omega(t) &= \alpha_1 T(t) + \alpha_2 T(t)^2 \approx 0.0026 \times T(t)^2, \\
\omega(t) &= \theta_1 \mu(t) \theta_2.
\end{align*}
\]

In Equation (2), \( T(t) \) is the mean temperature of earth at time \( t \) while \( \alpha_1 \) and \( \alpha_2 \) are regression parameters, Tol (2009) described one method to estimate them. In Equation (3), \( \mu(t) \) is the reduction of greenhouse gases and \( \theta_1, \theta_2 \) are predefined constants, Nordhaus and Sztorc (2013) state that \( \theta_2 = 2.8 \) and, between 2020 and 2050, \( \theta_1 \) grows from 0.039 to 0.068. If one is interested in estimating economic output growth rate, then the most important driver is total factor productivity \( A(t) \). According to the specification of Nordhaus and Sztorc (2013), this variable is defined exogenously by:

\[
\begin{align*}
A(t) &= A(t-1)(1 + g_A(t)), \\
g_A(t) &= g_A(t-1)(1 + \delta_A), \\
g_A(2015) &= 7.9\% \text{ per 5 years}, \\
\delta_A &= 0.6\% \text{ per 5 years}.
\end{align*}
\]

This model specification leads to a very important economic growth, of about 1.9%/year for the coming century, while allowing this growth to be independent of any other variable. That growth comes mainly from the TFP growth as it grows for about 1.5% per year. The remaining growth comes from population increase and capital accumulation. Overall, it means that the cost of climate change is almost null. More and more contributions in the recent literature consider that this is at best, very optimistic if not completely misleading (Pindyck, 2020).

## 2 Economic model assumptions

### Notation

In the following table and the rest of the article, we use a subscript to specify the country while the time will be between parentheses. When there is no risk of confusion, the country subscript is omitted.

<table>
<thead>
<tr>
<th>Symbole</th>
<th>Signification</th>
</tr>
</thead>
<tbody>
<tr>
<td>( Y_c(t) )</td>
<td>Gross Domestic Product (GDP)</td>
</tr>
<tr>
<td>( A_c(t) )</td>
<td>Total Factor Productivity (TFP)</td>
</tr>
<tr>
<td>( K_c(t) )</td>
<td>Capital stock</td>
</tr>
<tr>
<td>( L_c(t) )</td>
<td>Number of working hours</td>
</tr>
<tr>
<td>( \Omega_c(t) )</td>
<td>Direct damage of climate change</td>
</tr>
<tr>
<td>( \omega_c(t) )</td>
<td>Transition investment on a green economy</td>
</tr>
<tr>
<td>( T(t) )</td>
<td>Temperature variation since the industrial revolution</td>
</tr>
<tr>
<td>( E_c(t) )</td>
<td>Exergy</td>
</tr>
<tr>
<td>( TP_c(t) )</td>
<td>Total primary energy</td>
</tr>
<tr>
<td>( G(t) )</td>
<td>greenhouse gases emissions</td>
</tr>
<tr>
<td>( SCO_2(t) )</td>
<td>additional greenhouse gases in the atmosphere.</td>
</tr>
<tr>
<td>( C )</td>
<td>List of countries</td>
</tr>
</tbody>
</table>

**Table 1: Notations**

In this paper, bold symbols designate vectors. For example, \( Y(t) \) is a vector with the GDP for a set of countries. In the following we will note the logarithm of every quantity, with a lower case. Thus, for example, \( a_c(t) = \log(A_c(t)) \).
2.1 GDP estimation

As we stated in introduction, we modify the DICE model to introduce endogenous TFP growth. We will use a standard production function, a two factors Cobb-Douglas function, such as Nordhaus and Sztorc (2013); Alestra et al. (2021):

\[ Y_c(t) = A_c(t)K_c(t)^\beta L_c(t)^{1-\beta}. \]

Here \( Y_c(t) \) represents the GDP for country \( c \) at time \( t \) without any climate or transition damage (e.g, \( \omega \) and \( \Omega \)). The total factor productivity, capital and labor are respectively represented by \( A_c(t) \), \( L_c(t) \), \( K_c(t) \). The parameter \( \beta \) represents the elasticity between capital and labor. Its usual value, as used by Nordhaus and Sztorc (2013) or Alestra et al. (2021), is \( \beta = 0.3 \). We will keep the damage function introduced by Nordhaus (Equation (2) or a variant), however, as we will discuss in the following sections, we will endogenize the indirect damage into the TFP growth model (see Equation (3)), thus, we will not use the indirect damage function introduced by Nordhaus. It will remove significant unknown as that function is nearly impossible to calibrate as, yet, no observation exists to test its forecasts.

2.2 Capital stock

As Nordhaus and Sztorc (2013) or Solow (1956) described, we use a classical model for capital variation, let \( \delta_K \) be the depreciation of the capital, then the dynamic of \( K(t) \) is

\[ K(t) = (1 - \delta_K)K(t - 1). \]

In the standard settings, \( \delta_K \) is a constant parameter, for example, Nordhaus and Sztorc (2013) assumes this coefficient to be equal to 0.10. However, it would be interesting to modify this hypothesis as climate change will likely increase its value. For example, a very aggressive policy towards carbon emissions reduction would increase the depreciation rate of every industrial buildings required by the fossil fuels industry. It is the stranded assets problem, see for example : Van der Ploeg and Rezai (2020).

2.3 TFP estimation

The variable that leads to sustained growth in the aforementioned model is Total Factor Productivity (TFP), as Ayres et al. (2003); Warr and Ayres (2006); Stern (2015) discussed, growth rate could be very dependant of an observed variable, \( exergy \). To simplify the notation and ease the reading, we will refer to \( primary\ exergy\ as\ exergy \), some others refer to that notion under the name final energy Bercegol and Benisty (2022). Exergy is a thermodynamic quantity which represents the amount of work a given system could do before reaching equilibrium. That quantity could be estimated using the amount of energy used and the efficiency of its transformation into useful work. We argue that useful work is a good predictor of TFP as productivity is driven by \( exergy \) intensive industrial process Cleveland et al. (1984). Hence, the higher \( exergy \) the higher the number of produced goods. All other variable being equal it should equate to more GDP. Thus, in the framework developed by Nordhaus, we argue that, the variations of \( exergy \) should be a good predictor of the variations of the TFP.

Moreover, we don’t want to add \( exergy \) as a substitute for labour or capital: \( Exergy\ can’t\ be\ replaced\ by\ capital\ because\ capital\ needs\ it\ to\ transform\ flow\ of\ natural\ resources\ into\ produced\ goods.\ Labour\ may\ to\ some\ extent\ replace\ a\ part\ of\ exergy\ as\ the\ human\ body\ can\ perform\ physical\ work.\ Unfortunately,\ the\ human\ body\ is\ order\ of\ magnitude\ less\ powerful\ than\ other\ sources\ of\ energy.\ It\ may\ be\ the\ reason\ why\ humans\ historically\ replaced\ their\ body\ (and\ to\ some\ extent\ all\ living\ species)\ by\ external\ machines\ powered\ by\ fossil\ fuels\ (Huber, 2009).\ This\ is\ also\ an\ argument\ to\ dismiss\ the\ proposed\ production\ functions\ that\ directly\ include\ energy\ (Van\ der\ Werf, 2008)\ and\ an\ argument\ to\ include\ exergy\ into\ the\ error\ term.\ That\ development\ is\ also\ explored\ by\ Bercegol\ and\ Benisty\ (2022)\ who\ try\ to\ explain\ past\ TFP\ growth\ using\ exergy,\ though,\ they\ used\ a\ different\ functional\ form,\ and\ we\ propose\ two\ advantages\ over\ their\ method:\ first,\ our\ goal\ is\ to\ build\ forward\ looking\ models.\ Second,\ we\ try\ as\ much\ as\ possible\ to\ provide\ confidence\ interval\ for\ all\ our\ predictions.
Our modelisation also echoes the idea of Santos et al. (2021, 2018) where they tried to model TFP with
the efficiency of exergy. That efficiency represents the amount of real useful work done in various sectors.
For instance the efficiency of internal combustion engines that transform chemical energy onto kinetic energy
or the efficiency of coal power plants that also transform chemical energy into electricity that will finally be
used by machines and transformed to other form of energy (either thermal, kinetic or potential). While our
modelisation seems to be very close to their approach, it still has drawbacks and we propose some variations.
First, we want to be able to forecast long term GDP to match the stress-test use case, but we don’t
know the exact amount of each work will be needed in each sector at a 30 years horizon. Thus it is hard
to compute exergy efficiency. Also, that computation requires the efficiency of each transformation in each
sector which is also unknown in the long run due to technical innovations. Although we could have a rough
idea of such quantity, we don’t want to introduce more unknown variables in our model in order to reduce
long-term variance. That is the reason we wanted to use exergy variations, even though that model might
have a worst goodness of fit, we don’t rely on unknown variables to project TFP which is a great feature for
long term forecasting. However, we think such approaches might be very interesting to develop a sectoral
approach. Primary exergy can be computed using primary energy and conversion factor (for further details :
Brockway et al. (2014)).

2.3.1 Standard auto-regressive model

In this subsection, our data consists of exergy and TFP time series, from 1948 to 2018. Let denote
\((e(t))_{t \in [1,T]}\) the logarithm of the exergy time series and \((a(t))_{t \in [1,T]}\) the TFP one. They are \(I(1)\) according
to the Dickey-Fuller test. We tested whether or not it exists a cointegration relationship.
Let us recall that for this purpose, we have to check if their exists a scalar \(\Gamma_1\), such that : \((a(t) - \Gamma_1 e(t))_{t \in [1,T]}\) is stationary. Thus, to test the existence of such vector we have to estimate the following
model : \(a(t) = \Gamma_0 + \Gamma_1 e(t) + \eta(t)\) and test for the stationarity of \((\eta(t))_{t \in [1,T]}\). The Figure 1 shows the plot of
the residuals of the OLS regression. The fitted versus observed plot are presented in Appendix B.

\[
\forall t \in [1,T] \quad a(t) - a(t-1) = \alpha_0 + \alpha_1 (a(t-1) - a(t-2)) + \beta_0 (e(t) - e(t-1)) + \varepsilon_t. \quad (4)
\]

The number of lags for \(a(t) - a(t-1)\), was selected by minimizing the BIC and the results could be checked
in Table 2.

Figure 1: Residuals \(\hat{\eta}(t) = a(t) - \hat{\Gamma}_0 + \hat{\Gamma}_1 e(t)\) on US data from 1948 to 2018.
Dep. Variable: A  
Model: VAR  
Method: OLS  

<table>
<thead>
<tr>
<th>Log-Likelihood:</th>
<th>356.239</th>
<th>R2: 0.4626</th>
</tr>
</thead>
<tbody>
<tr>
<td>Df Residuals:</td>
<td>69</td>
<td>BIC: -15.4899</td>
</tr>
</tbody>
</table>

| coef | std err | t | P>|t| |
|-------|---------|----|------|
| const | 0.007686 | 0.001959 | 3.924 | 0.000 |
| Δa(t−1) | -0.173345 | 0.094394 | -1.836 | 0.066 |
| Δe(t) | 0.422000 | 0.056792 | 7.431 | 0.000 |

Table 2: ARX(1) coefficients estimations

We could plot in the Figure 2 the fitted values and the observed variation of the TFP and the residuals in the Figure 3:

![Figure 2](image1.png)  
**Figure 2**: Observed and fitted values according to Equation (4).

![Figure 3](image2.png)  
**Figure 3**: Residuals (Dickey Fuller p−value < 0.001)

The Ljung-Box test has a p-value of 0.77 at lag 10, The Jarque-Bera test has a p-value of 0.13. Also, it seems that our modelisation get a higher $R^2$ than most of the TFP modelisation found in the literature, see for instance Alestra et al. (2021). Using our ARX(1) modelisation, we have a new interpretation of Total Factor Productivity. On the one hand, as explained by Cleveland et al. (1984), TFP improvement might come from technological improvement that offset the declining concentration of mine ore and the declining energy return on investment (cf Hall (2011)). The pace of technological advancement is set by the constant term in this model, for example, in the recent years, information technology may have improved efficiency while reducing exergy consumption. On the other hand TFP improvement could also come from the augmentation of the amount of exergy used in the economic system. It seems to be a quite natural interpretation of factor productivity, as an augmentation of productivity of existing machines mainly comes from an augmentation of energy consumed by those machines. Nevertheless, we must admit we only used US data to build that model, which means we don’t measure the importance of imports. Even if the US economy is very diversified we still don’t measure possible specificity.

Let us note that our model is not able to measure the impact of the outsourcing of polluting industrial processes as it only consider locally used exergy. Industrial outsourcing became widely used in the last decades in western economies. Bercegol and Benisty (2022) also briefly discussed that problem. We think this aspect minimises the impact of exergy on the US TFP. US economy relies massively on the service sector and imports more goods from other country than ever before Ott et al. (1987); Buera and Kaboski (2012).
possible reason of the decoupling observed on Figure 1 may come from the outsourcing of its manufacturing industry at the benefit of the service sector.

If we assume such relationship holds true at a global scale, then, given the predictions on future values of exergy, we should be able to compute future values of the Total Factor Productivity. These projections can be of special interest in the case of climate stress-test, but in order to build meaningful stress-test scenario one important feature is the associated uncertainty. In the Bayesian paradigm, adopted in the following sections, uncertainty will be measured through the credibility intervals. Also the Bayesian paradigm allows us to share information between countries that may have similar economies.

2.3.2 Data

Before diving into our proposed Bayesian models, let us describe the data we used. It consists of time series ranging from 1971 to 2018 for seven countries: United States, United Kingdom, France, Germany, Japan, Canada and Italy. It consists of the biggest economies by GDP. We purposely excluded India, China and Brazil for which we could only gather exergy data from 1990 to 2018. Also, we must warn the reader, our data consisted only of renewables sources which includes among other things, hydroelectricity, solar and wind. Unfortunately we wanted to have a more granular view of those sources thus we made the assumptions that hydro powers where already fully developed, in the studied countries, from the year 2000 and from that date, all new renewables comes from wind and solar. We might overestimate a little the actual real amount of wind and solar, but anyway that assumptions doesn’t introduce huge error as renewables only account for a fraction of a percent of final energy consumption in these countries. We discovered in Bercegol and Benisty (2022) different data (Malanima, 2020). It may be very interesting to re-estimate our models with more observations to check whether our model holds true for a longer time series.

2.3.3 Bayesian auto-regressive model

The generalization of the previous model to any other country might be dubious because the economic structure of the US is probably not exactly the same as other countries. But we think they are to some extent similar. Also, as stated in Section 2.3.2 we could only gather data from 1971 to 2018 for other countries. Thus, we think a Bayesian approach might be appropriate. An other advantage of such approach is the ability to easily build credibility intervals. A very similar Bayesian model, used in a different scientific field, could be find in McMillan et al. (2005) and Sahu et al. (2007); Sahu (2012). They try to predict the ozone level at a given station using past values of ozone level and current values of meteorological data. We have the same data structure. It gives us confidence that our Bayesian approach will probably be identifiable and converge to a unique solution. We want to forecast values of TFP using past values of TFP and current values of exergy. Let assume $C$ is the set of considered countries, then, for every country $c \in C$ we could denote the logarithm of total factor productivity $a_c(t)$ and the logarithm of exergy $e_c(t)$ for any $t \in [1, T]$. Let assume model (4) still holds and each country has true value $\alpha_{0,c}, \alpha_{1,c}$ and $\beta_{0,c}$ then :

$$\Delta a_c(t) = \alpha_{0,c} + \alpha_{1,c} \Delta a(t-1) + \beta_{0,c} \Delta e_c(t) + \epsilon_{c,t}. \quad (5)$$

Where $\Delta$ is the difference operator, that is, for instance : $\Delta a_c(t) = a_c(t) - a_c(t-1)$. We can also rewrite the above equations using vector notation :

$$\Delta a(t) = \alpha_0 + \alpha_1 \Delta a(t-1) + \beta_0 \Delta e(t) + \epsilon_1.$$

We could assume any country has a true value similar to other developed economy. The underlying idea is that any western economy rely on energy to produce goods and the technology used in these countries might be to some extent similar. Thus the exergy requirement to produce goods should be similar. Hence, we assume we have some priors on the distribution of the parameters and then knowing these priors, we could draw the regression coefficients of our model. Figure 4 is an illustration of that idea.
Figure 4: Hierarchical Bayesian model; densities assumptions are given in Appendix D.

To check the convergence, we estimated our model with several priors settings ranging from very informative to more uninformative ones. We summarized our setup in Table 3.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Prior 1</th>
<th>Prior 2</th>
<th>Prior 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \mu_{\alpha_0} )</td>
<td>( N(0, 1) )</td>
<td>( N(0, 100) )</td>
<td>( N(0, 0.25) )</td>
</tr>
<tr>
<td>( \mu_{\alpha_1} )</td>
<td>( N(0, 1) )</td>
<td>( N(0, 100) )</td>
<td>( N(0.05, 0.25) )</td>
</tr>
<tr>
<td>( \mu_{\beta_0} )</td>
<td>( N(0, 1) )</td>
<td>( N(0, 100) )</td>
<td>( N(0.1, 0.25) )</td>
</tr>
<tr>
<td>( \sigma )</td>
<td>( C(1) )</td>
<td>( C(10) )</td>
<td>( C(1) )</td>
</tr>
<tr>
<td>( \sigma_{\alpha_0} )</td>
<td>( C(10) )</td>
<td>( C(10) )</td>
<td>( C(1) )</td>
</tr>
<tr>
<td>( \sigma_{\alpha_1} )</td>
<td>( C(10) )</td>
<td>( C(10) )</td>
<td>( C(1) )</td>
</tr>
<tr>
<td>( \sigma_{\beta_0} )</td>
<td>( C(10) )</td>
<td>( C(10) )</td>
<td>( C(1) )</td>
</tr>
<tr>
<td>Tuning steps</td>
<td>5000</td>
<td>5000</td>
<td>5000</td>
</tr>
</tbody>
</table>

Table 3: Parameters and priors for model (5)

Only one experiment, corresponding to Prior 2, will be shown below. The plots from all described experiments are provided. In Table 3, \( C(\beta) \) corresponds to a half-Cauchy density of parameter \( \beta \), \( f_{C(\beta)} \), and \( N(\mu, \sigma^2) \) to a Gaussian density, \( f_{N(\mu, \sigma^2)} \), of mean \( \mu \) and variance \( \sigma^2 \). These densities are defined as:

\[
\forall x \in \mathbb{R} : f_{C(\beta)}(x) = \frac{2}{\pi \beta \left( 1 + \left( \frac{x}{\beta} \right)^2 \right)} 2x > 0 \quad \text{and} \quad f_{N(\mu, \sigma^2)}(x) = \frac{1}{\sigma \sqrt{2\pi}} \exp \left( -\frac{1}{2} \left( \frac{x - \mu}{\sigma} \right)^2 \right).
\]

With the priors described in Table 4, the posterior densities for all parameters and the total factor productivity variation posterior are shown in Figure 5. We implemented the model thanks to Salvatier et al. (2016) and the PyMC3 python package. Our code may be found here.

The first striking result of Figure 5 is that exergy variations coefficient is significantly greater than 0 for almost all countries (but France). The other quite interesting result is that every coefficient are similar for every country with a mean value near 0.2. It may indicates that every countries have the same dependence upon exergy. The lag coefficient is not significant for all countries. But the constant term which might account for exogenous technological improvement is greater than 0 for every country but 2. In this model,

\(^1\)soon to be added

\(^2\)soon to be added
Figure 5: Left panel: Two estimates according to model (5) (dotted and plain lines) for each posterior density of: $\alpha_0, \alpha_1, \beta_0$ and $\sigma$. Each color corresponds to one country (see the appendix for a List of countries), represented by a plain and a dotted line. Middle panel: sampled values used to estimate posterior densities. Right panel: Posterior means, 0.1 and 0.9 quantiles for each country, according to model (5) The $R^2$ is 0.31 and the mean squared error (mse) is $10^{-3}$. The $R^2$ and the mse have been computed according to Equation (11) and Equation (12).
that exogenous improvement only account for a few tenth of a percent. Thus, the exogenous TFP variation supposed by Nordhaus (2018) is explain in this model by two components: the first one is indeed an exogenous element that drive TFP upward, it may be interpreted as technological advancement. And the second component to explain the TFP variation is *exergy* variation. It may come from the necessity to increase *exergy* use to improve the productivity, in terms of produced goods, of already existing assets. The Figure 5 shows the predictions of our model on historic TFP variations.

According to this result, we observe huge differences with the trajectory proposed by Nordhaus (2018), for instance, in the DICE model, the growth rate is near 1.9% per year. That trajectory is compatible with our modelisation if and only if we suppose an increase of *exergy* use by about 2% a year. Unfortunately, that growth seems to be in contradiction with the Paris Agreement as we will explain in Section 3. On the other hand, if one tries to reduce greenhouse gas emissions (to meet the Paris Agreement) then we have to reduce *exergy*. Hence we have to reduce economic output. It is the main difference between our model and the classical DICE, we add what may be interpreted as ecological constraint. This constraint is similar to the constraints proposed by Meadows et al. (2013).

2.3.4 Bayesian auto-regressive model with an additional exogenous variable.

That precedent modelisation still lacks one important feature: indeed, a country that imports a lot of manufactured goods and exports a lot of services might have a relatively low correlation simply because our data doesn’t consider such cases. Let’s simply add one other variables, which is the variation of world *exergy*. Precisely, it is the variation of the sum of *exergy* from our listed countries. That variable might be a proxy that account for imports to a given country. Similarly to the previous model, you could find its illustration in Figure 6.

\[ \Delta a_c(t) = \alpha_{0,c} + \alpha_{1,c} \Delta a(t-1) + \beta_{0,c} \Delta e_c(t) + \beta_{1,c} \Delta \left( \sum_{c \in C} e_c(t) \right). \]  

(6)

Figure 6: Proposed hierarchical Bayesian model, the priors are given in Appendix E
Only one experiment (Prior 7) will be shown below but all plots from all described experiment could be checked\(^3\). Prior 4, 7 and 8 seems to have converged to similar solutions, while Prior 5 and 6 clearly diverged.

The constant, lag and the variance posterior densities (Figure 7) are similar to the previous posterior densities (Figure 5), hence our interpretation remains the same for these coefficients. Even though the estimated densities showed in Figure 5 are quite inline with the maximum likelihood estimations showed in table 2 the estimated densities for this model (Figure 7) are quite different for the two *exergy* coefficients. The densities of the coefficients $\beta_0,c$ seems to be centered around a positive value but a lower one than obtained in the previous model. It means that the variations of the *exergy* of a country $c$ doesn’t explain a lot of its TFP variation. However, the variation of the *exergy* of the world explains a lot of the variations of the TFP for other countries especially for Italy. It may reflect interdependences and the importance of importation for western economies. It would be of great interest to include China in the same model because they manufacture a lot of goods for the western economies, unfortunately we lack their data. That model seems to have a better interpolation of the observed data and will be used for the further analyses.

\(^{3}\)soon to be added

---

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Prior 4</th>
<th>Prior 5</th>
<th>Prior 6</th>
<th>Prior 7</th>
<th>Prior 8</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mu_{\alpha_0}$</td>
<td>$N(0,1)$</td>
<td>$N(0,100)$</td>
<td>$N(0,1)$</td>
<td>$N(0,100)$</td>
<td>$N(0,1)$</td>
</tr>
<tr>
<td>$\mu_{\alpha_1}$</td>
<td>$N(0.05,1)$</td>
<td>$N(0,100)$</td>
<td>$N(0,1)$</td>
<td>$N(0,100)$</td>
<td>$N(0,1)$</td>
</tr>
<tr>
<td>$\mu_{\beta_0}$</td>
<td>$N(0,1)$</td>
<td>$N(0,100)$</td>
<td>$N(0,1)$</td>
<td>$N(0,100)$</td>
<td>$N(0,1)$</td>
</tr>
<tr>
<td>$\mu_{\beta_1}$</td>
<td>$N(0.1, 0.25)$</td>
<td>$N(0,100)$</td>
<td>$N(0,1)$</td>
<td>$N(0,100)$</td>
<td>$N(0,1)$</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>$C(0.1)$</td>
<td>$C(10)$</td>
<td>$C(1)$</td>
<td>$C(10)$</td>
<td>$C(1)$</td>
</tr>
<tr>
<td>$\sigma_{\alpha_0}$</td>
<td>$C(1)$</td>
<td>$C(10)$</td>
<td>$C(1)$</td>
<td>$C(10)$</td>
<td>$C(1)$</td>
</tr>
<tr>
<td>$\sigma_{\alpha_1}$</td>
<td>$C(1)$</td>
<td>$C(10)$</td>
<td>$C(1)$</td>
<td>$C(10)$</td>
<td>$C(1)$</td>
</tr>
<tr>
<td>$\sigma_{\beta_0}$</td>
<td>$C(1)$</td>
<td>$C(10)$</td>
<td>$C(1)$</td>
<td>$C(10)$</td>
<td>$C(1)$</td>
</tr>
<tr>
<td>$\sigma_{\beta_1}$</td>
<td>$C(1)$</td>
<td>$C(10)$</td>
<td>$C(1)$</td>
<td>$C(10)$</td>
<td>$C(1)$</td>
</tr>
<tr>
<td>Tuning steps</td>
<td>5000</td>
<td>5000</td>
<td>5000</td>
<td>20000</td>
<td>20000</td>
</tr>
</tbody>
</table>

Table 4: Tested priors of model (6).
Figure 7: Left panel: $\alpha_0, \alpha_1, \beta_0, \beta_1$ and $\sigma$ Posterior density. Each color corresponds to one country (see a list in List of countries). 2 chains have been generated represented by a plain and a dotted line. The sampled values are also shown. Right panel: Posterior means and 0.1 and 0.9 quantiles for each country. The $R^2$ is 0.31 and the mean squared error (mse) is $10^{-3}$. The $R^2$ and the mse have been computed according to: Equation (12).
2.4 Forecasted values during the Covid-19 crisis and the recover

Also, we must check the relevance of our model to forecast real TFP values. We have been living a very unique period in modern history, indeed, during the covid-19 crisis, it was the first time humanity reduced its greenhouse gas emissions at such rates. Of course, it didn’t last long. But, it is a very unique chance for us to compare our model with the real data that are being published. As the greenhouse gas reduction only comes from a slowdown in the use of fossil fuel, and not a drastic change in the way we produce energy, it may directly be translated to *exergy* variations. We can’t provide more extreme test as, historically, no such variations have been observed. To our knowledge no other TFP model is able to make similar predictions. Table 5 presents the forecasted total factor productivity under the firsts waves of Covid-19, while Table 6 presents the forecasted total factor productivity after the 2021 recovery for different scenarios.

<table>
<thead>
<tr>
<th>Country</th>
<th>Greenhouse gas variation (%)</th>
<th>Estimated <em>exergy</em> reduction in (%)</th>
<th>Model prediction (%)</th>
<th>observed GDP variation (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>United States</td>
<td>−10.9</td>
<td>−10.9</td>
<td>−2.0 (−2.6, −1.4)</td>
<td>−2.3</td>
</tr>
<tr>
<td>France</td>
<td>−14.3</td>
<td>−14.3</td>
<td>−1.8 (−2.7, −0.8)</td>
<td>−4.2</td>
</tr>
<tr>
<td>Canada</td>
<td>−9.6</td>
<td>−9.6</td>
<td>−2.1 (−2.8, −1.5)</td>
<td>−5.6</td>
</tr>
<tr>
<td>Italy</td>
<td>−10.7</td>
<td>−10.7</td>
<td>−2.7 (−3.6, −2)</td>
<td>−5.9</td>
</tr>
<tr>
<td>Germany</td>
<td>−9.7</td>
<td>−9.7</td>
<td>−1 (−1.6, −0.4)</td>
<td>−1.4</td>
</tr>
<tr>
<td>Japan</td>
<td>−6.5</td>
<td>−6.5</td>
<td>−0.9 (−1.3, −0.5)</td>
<td>−1.7</td>
</tr>
<tr>
<td>United Kingdom</td>
<td>−11.2</td>
<td>−11.2</td>
<td>−1.7 (−2.4, −1.1)</td>
<td>−4</td>
</tr>
</tbody>
</table>

Table 5: Models (5) and (6) predictions during the Covid-19 crisis. For each country, the mean predictions are given. The 0.1 and 0.9 quantiles are shown between parenthesis.

<table>
<thead>
<tr>
<th>Country</th>
<th>prediction (%) for 5% <em>exergy</em> growth</th>
<th>prediction (%) for 7% <em>exergy</em> growth</th>
</tr>
</thead>
<tbody>
<tr>
<td>United States</td>
<td>1.5(1.2, 1.8)</td>
<td>1.9(1.5, 2.4)</td>
</tr>
<tr>
<td>France</td>
<td>1.3(0.8, 1.6)</td>
<td>1.6(1.0, 2.0)</td>
</tr>
<tr>
<td>Canada</td>
<td>0.8(0.4, 1.1)</td>
<td>1.2(0.7, 1.6)</td>
</tr>
<tr>
<td>Italy</td>
<td>0.99(0.6, 1.4)</td>
<td>1.4(1.0, 2.0)</td>
</tr>
<tr>
<td>Germany</td>
<td>1.8(1.4, 2.2)</td>
<td>2.2(1.7, 2.7)</td>
</tr>
<tr>
<td>Japan</td>
<td>1.2(0.9, 1.5)</td>
<td>1.6(1.2, 2.0)</td>
</tr>
<tr>
<td>United Kingdom</td>
<td>1.6(1.1, 2.0)</td>
<td>2.0(1.5, 2.5)</td>
</tr>
</tbody>
</table>

Table 6: Models (5) and (6) predictions for 2021 under different *exergy* scenarios. For each country, the mean predictions are given. The 0.1 and 0.9 quantiles are shown between parenthesis.

To our best knowledge, our model is the only one able to explain, at least partially and for a wide range of countries, the huge variations seen during the covid-19 crisis. Our model is able to do such predictions even though no huge variations are observed in the training data. We think our model is adapted in the context of climate stress-test as it provides a natural framework to compute credibility intervals, it is also an adapted framework to understand the crucial link between the output of an economy and the energy required to transform natural resources flux. Finally, it is an other way of interpreting the total factor productivity by
directly integrating the quantity that makes that productivity possible. One may argue that the traditional interpretation of technical progress is enough. Otherwise, it would mean technical progress was lost during the Covid crisis.

3 Estimation of future exergy availability and climate change damage on real GDP

3.1 Model for future exergy availability

Exergy values are computed using the methodology presented by Brockway et al. (2014), thus we use the same energy sources as the International Energy Agency (IEA): Coal, Oil, Natural gas, nuclear, hydroelectricity, biomass, solar and wind. To produce exergy scenarios, we have to produce scenarios for each of these energy sources. We could consider exogenous path, it could be interpreted as a government policy that set a track to be followed. Or we could consider a track according to the future relative price of each energy source. That second methodology has the appealing property of directly taking into account a carbon tax which is thought to be an efficient way to reduce carbon emissions, see for instance: Gillingham and Stock (2018). Under that second idea, we could reverse the question. One may wonder what would be the ideal carbon tax in order to reach some level of greenhouse gas emission reduction. Answering that question in an economic model where exergy has its fair share is key for policy makers. Unfortunately, the scenario developed by Nordhaus seems unsuitable from that perspective. The economic growth depends mainly on exogenous assumptions, hence, the economic impact of such taxes are very small.

If we are able to draw a path for that carbon tax, we could easily incorporate it into the relative price of each sources, that idea closely match the approach of Alestra et al. (2021), based on elasticities between energy input estimated by Stern (2012) or Papageorgiou et al. (2017).

3.1.1 Exogenous trajectory

As we use the Brockway et al. (2014) methodology, we need the quantity produced by each energy source to compute exergy. Future values of energy input could be set according to regulatory objectives, for example, limitating global warming to 2°C as proposed by the Paris Agreement. It states:

*Holding the increase in the global average temperature to well below 2°C above pre-industrial levels and pursuing efforts to limit the temperature increase to 1.5°C above pre-industrial levels, recognizing that this would significantly reduce the risks and impacts of climate change.*

To ensure we are on track to meet that Agreement, we have to lower our carbon emissions by approximately 4% per year from now on until we reach net zero. Of course, the energy sector is not the only greenhouse gas emitter, but if we assume every economic sector (even agriculture) have to decrease at the same pace then we know that coal, oil and natural gas consumption have to be reduce by 4% a year. Now if we assume an increase by 1% per year of nuclear and hydro-electricity and a 5% per year increase of new renewables, then we could compute a scenario for exergy. These projections are completely unrealistic as they completely ignore the difficulty of such industrial, political and social efforts. They are of great interest to see our model best case transition to a net zero economy while keeping global warming below 2°C. The different TFP paths resulting from that scenario are represented in Figure 11 and Figure 12. These figures respectively represents the result of model described in section 2.3.3 and 2.3.4.

As we stated earlier, we could design an exergy scenario to follow the growth trajectory of the traditional DICE model. Unfortunately such scenario assumes an increase of greenhouse gas emissions by 2% per year which is not remotely close to meet the Paris Agreement. The most pessimistic scenario under the Paris Agreement scenario result in a net loss of total factor productivity (see Figure 11 and Figure 12). An other scenario where nuclear power plants are phased out and biomass couldn’t be scaled due to usage conflict could also be examined, it is the scenario described in Table 10 and presented in Figure 13 and Figure 14.
3.1.2 Endogenous trajectory

If we follow the idea developed by Alestra et al. (2021) we could compute the quantity of each energy sources used at time \( t \) from its price and a substitution coefficient between different energy sources. If we assume \( b_i(t) \) is the benefits to produce a given quantity of energy from the source \( i \) at time \( t \), and if \( TP_{i,c}(t) \) if the total primary energy for the source \( i \) in country \( c \), then we could simulate an energy mix trajectory using the following equation, if we assume \( \delta \) is the depreciation rate, then :

\[
TP_{i,c}(t+1) = TP_{i,c}(t)(1 + \Delta \Omega_{i,c}(t+1)),
\]

\[
\Delta \Omega_{i,c}(t+1) = \Delta \Omega_{i,c}(t) \sum_j \Omega_{j,c}(t) \sigma_{i,j}(b_i(t) - b_j(t)).
\]

Obviously \( b_i(t) \) depends on a lot of parameters. If \( p_{p,i}(t) \) is the price to produce one unit of energy from a source \( i \) and its price on the market is \( p_{m,i}(t) \), then the benefit to invest in a power plant using source \( i \) is :

\[
b_i(t_0) = \int_{t=t_0}^{\infty} (p_{p,i}(t) - p_{m,i}(t)) \exp(-rt) \, dt.
\]

(7)

In Equation (7) it is very easy to include a carbon tax, if we assume such tax is \( C_{i,t} \) for source \( i \) at time \( t \), then, the price to produce one additional unit of energy then becomes :

\[p_{p,i}(t) + C_{i,t}\]

This formalisation as the appeal property of directly integrating a carbon tax into the scenario. Unfortunately, we still can’t model whether that tax will be passed to consumer or not and lead to inflation, profitability loss or purchasing power loss. Such answer requires a very detailed modelisation of energy source substitutability for each economic sector that path will be let for further studied.

3.2 Damage function

One could use the standard damage function used by Nordhaus (2018), it is equivalent to the one used by Alestra et al. (2021), if \( \Omega_c(t) \) is the damage on the real GDP at time \( t \) in the country \( c \), then that function could be written with the following form :

\[
\Omega_c(t) = \sum_i \alpha_i T(t)^i.
\]

In the traditional settings an elevation of 6°C would result in a damage of 8% of GDP. We may use different damage function such as the one presented by Weitzman (2012). In this paper, Weitzman (2012) modifies the damage function to measure a form of uncertainty. The science of climate change rely on models that are very accurate but still have a few unknowns that are taken into account by the IPCC. Weitzman (2012) illustrates it with the following example : a doubling \( co_2 \) concentration might lead to an increase between 2°C and 4.5°C and the most likely temperature increase is 3°C. Such variance is key in order to make a meaningful scenario used in risk management or in a stress-testing framework, such method will be discussed in Section 4.2.

4 Climate scenario

4.1 Energy consumption to greenhouse gases emissions

Each energy source emits an amount of greenhouse gases. It ranges from the direct emissions caused by fossil fuel and the indirect ones caused by the infrastructures required to produce energy. If we note \( \eta_i \) the
amount of greenhouse gases emissions per unit of production from the total life cycle of a power plant then, the total emissions at time $t$ from the energy sector, noted $G(t)$ could be computed from the total energy consumption:

$$G(t) = \sum_i \eta_i \sum_{c \in C} TP_{i,c}(t).$$

$$G_c(t) = \sum_i \eta_i TP_{i,c}(t).$$

Values for $(\eta_i)$ could be taken from the IPCC III (2014) report. Thus, our climate scenario is closely linked to the energy scenario we defined in the previous section, this is the same approach used in Alestra et al. (2021). This modelisation neglects greenhouse gases emissions from non energy sources, such as livestock or cement manufacture. An other simpler model used in our computation assume a growth of $\gamma(t)$ for every fossil fuel sources, if we assume every economic sector reduce at the same pace, then greenhouse gas emissions could be estimated by:

$$G(t) = (1 + \gamma(t) )G(t - 1).$$

Of course that assumption doesn’t allow us to disintegrate the trajectory of different energy sources, it would be interesting to develop our model in this area, however the simpler version is easier to compute, understand, interpret and also it reduces the number of unknown parameters in our model and thus, the variance of the projections.

### 4.2 From greenhouse gases stock to global warming

We use the standard model estimated by Hsiang and Kopp (2018). They state that the relation between global warming and greenhouse gases concentration is linear. Thus, a rather simple and accurate modelisation is:

$$T(t) = \tau SCO_2(t).$$

The usual value of $\tau$ is 0.0008. But we would like to go a bit further. Unfortunately, given a stock of greenhouse gases in the atmosphere, the temperature increases can’t be exactly determined. As IPCC (2007) stated, a doubling of greenhouse gas concentration will likely cause an increase of 3°C, and it will unlikely lead to an increase below 2°C and over 4.5°C. Weitzman (2010) tries to fit distribution according to some intervals given by the IPCC. If we use what he calls thin-tailed density (Gaussian density), we get tail probabilities described in Table 7, computed for a given time $t^*$. The time $t^*$ represents the moment the greenhouse gas concentration reaches equilibrium, it doesn’t necessarily is the time when fossil fuels stop been burned, indeed some feedback loop might be involved in this phenomenon hence prolonging the accumulation of greenhouse gas in the atmosphere.

<table>
<thead>
<tr>
<th>Probability</th>
<th>$SCO_2(t^*)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mathbb{P}(T(t^*) &gt; 2)$</td>
<td>27%</td>
</tr>
<tr>
<td>$\mathbb{P}(T(t^*) &gt; 2.5)$</td>
<td>9%</td>
</tr>
<tr>
<td>$\mathbb{P}(T(t^*) &gt; 3)$</td>
<td>2%</td>
</tr>
<tr>
<td>$\mathbb{P}(T(t^*) &gt; 3.5)$</td>
<td>0.4%</td>
</tr>
<tr>
<td>$\mathbb{P}(T(t^*) &gt; 4)$</td>
<td>0.05%</td>
</tr>
</tbody>
</table>

Table 7: Temperature increase probability under different greenhouse gas concentration scenarios.

Given the previous densities, we can estimated the probability of a damage to be greater than some given threshold, we could consider 2 damage functions:

$$\mathbb{P}(D(t^*) > \text{threshold})$$
\[ \Omega_N(T) = \frac{1}{1 + 0.0026T(t)^2}, \quad (8) \]
\[ \Omega_W(t) = \frac{1}{1 + 0.00239T(t)^2 + 0.000156T(t)^7}, \quad (9) \]

Where Equation (8) corresponds to the damage function proposed by Nordhaus and Sztorc (2013), while Equation (9) corresponds to the one proposed by Weitzman (2010) with a slight modification for computational reasons. As the global warming now becomes a random variable, the damage also becomes a random variable. Exactly as Table 7 we could compute damages tail probabilities presented in Table 8.

<table>
<thead>
<tr>
<th>Probability</th>
<th>SCO2(t*)</th>
<th>400 (≈ RCP 2.6)</th>
<th>450</th>
<th>500</th>
<th>550</th>
<th>600 (≈ RCP 4.5)</th>
<th>650</th>
<th>700 (≈ RCP 6.0)</th>
</tr>
</thead>
<tbody>
<tr>
<td>P(1 - \Omega_N(T(t^*)) &gt; 1%)</td>
<td>28%</td>
<td>53%</td>
<td>67%</td>
<td>75%</td>
<td>80%</td>
<td>83%</td>
<td>85%</td>
<td></td>
</tr>
<tr>
<td>P(1 - \Omega_W(t(t^*)) &gt; 1%)</td>
<td>43%</td>
<td>65%</td>
<td>76%</td>
<td>81%</td>
<td>85%</td>
<td>87%</td>
<td>88%</td>
<td></td>
</tr>
<tr>
<td>P(1 - \Omega_N(T(t^*)) &gt; 5%)</td>
<td>10^{-4}%</td>
<td>0.6%</td>
<td>5%</td>
<td>13%</td>
<td>22%</td>
<td>31%</td>
<td>39%</td>
<td></td>
</tr>
<tr>
<td>P(1 - \Omega_W(t(t^*)) &gt; 5%)</td>
<td>10%</td>
<td>33%</td>
<td>50%</td>
<td>62%</td>
<td>69%</td>
<td>74%</td>
<td>78%</td>
<td></td>
</tr>
<tr>
<td>P(1 - \Omega_N(T(t^*)) &gt; 10%)</td>
<td>10^{-3}%</td>
<td>10^{-2}%</td>
<td>0.04%</td>
<td>0.5%</td>
<td>2%</td>
<td>5%</td>
<td>9%</td>
<td></td>
</tr>
<tr>
<td>P(1 - \Omega_W(t(t^*)) &gt; 10%)</td>
<td>0.4%</td>
<td>20%</td>
<td>38%</td>
<td>51%</td>
<td>60%</td>
<td>67%</td>
<td>71%</td>
<td></td>
</tr>
<tr>
<td>P(1 - \Omega_N(T(t^*)) &gt; 20%)</td>
<td>10^{-2}%</td>
<td>10^{-1}%</td>
<td>10^{-1}%</td>
<td>10^{-2}%</td>
<td>10^{-2}%</td>
<td>0.02%</td>
<td>0.1%</td>
<td></td>
</tr>
<tr>
<td>P(1 - \Omega_W(t(t^*)) &gt; 20%)</td>
<td>0.7%</td>
<td>9%</td>
<td>24%</td>
<td>38%</td>
<td>49%</td>
<td>56%</td>
<td>63%</td>
<td></td>
</tr>
</tbody>
</table>

Table 8: Damage probability under different greenhouse gas concentration scenarios.

For example, in Table 8 the probability of the damage to be greater than 1% is 43% for the Weitzman damage function with a greenhouse gas concentration of 400ppm. As expected, big damages have very low probability to happen using the Nordhaus damage function. Nevertheless, as we have no observation it is very hard to calibrate such functions and the computed probabilities have to be interpreted as indication rather than precise estimates, a recent discussion about the construction of such function have been presented by Barnett et al. (2021).

5 Real GDP forecast

We computed real output (Figure 8) by combining several assumptions made in this article. We tested both Nordhaus (Equation (8)) and Weitzman (Equation (9)) damage function. We used prior 3 and 4 as described in Table 3 and Table 4. The population growth match the assumption of Nordhaus (2018) and the capital stock model is described in Section 2.2. We also computed a second simulation with a pessimistic case of climate warming (Figure 9). The exergy scenario used are presented in Table 9. Figure 8 and Figure 9 show global economic output if one assumes that TFP variation will follow the US model. That illustration tries to compare our findings with the assumptions of Nordhaus (2018). Of course, one could compute the same model for all the considered countries with the TFP paths shown in Figure 11, Figure 12, Figure 13 or Figure 14.
Figure 8: Left panel : Simulated output paths using priors 3 from Equation (5) and Nordhaus damage function, Right panel : Simulated output paths using priors 4 from Equation (6) and Nordhaus damage function.

Figure 9: Left panel : Simulated output paths using priors 3 from Equation (5) and damage function with a 0.1 quantile (Table 7) temperature increase, Right panel : Simulated output paths using priors 4 from Equation (6) and damage function with a 0.1 quantile (Table 7).

Our projections are very different from the ones proposed by Nordhaus (Nordhaus, 2018) and the ACPR (2020). They respectively obtained around 200 trillions and 150 trillions $2010 in 2050. It changes drastically the consequences and the policy required to handle the social consequences of GDP growth slowdown. Each trajectories represent one draw from the posterior estimated by a model presented in Section 2.3.3 or Section 2.3.4. If we assume such predictions holds true for any country, worldwide, then it raises the question of welfare redistribution. Under a pessimistic scenario, where GDP declines or stays roughly constant, we can’t ask for the same decline in western countries and in the Sub-Saharan countries. In the first case it would require a mild comfort reduction, in the second case, it might result in a catastrophic humanitarian crisis.

We must stress that, even if one thinks climate change won’t cause as much risk as transition towards a low carbon economy, the amount of fossil fuels on earth is a given quantity. One can’t burn that resource recklessly for eternity, it must come to an end. A low carbon economy will happen regardless of political or social will. The only remaining question about that transition should be the timing and the pace.
Moreover, one must keep in mind that our scenarios are best case scenarios and undermines major technological uncertainties. For instance we assumed the transition from fossil fuels to electricity to be instantaneous for any industrial process. We assumed the physical property (disponibility, density, ...) of low carbon electricity sources to be identical to fossil fuel sources. That is obviously very optimistic. We also assumed climate change to have no impact on population growth and we also assumed perfect and complete collaborations between all countries. These assumptions are far from being satisfactory. Such scenarios are a first step in the right direction to consider ecological constraints in the economic literature. It may be seen as a first step to bridge the gap between the original work of Meadows et al. (2013) and the original work of Nordhaus et al. (1992).

5.1 Impact of primary exergy decline on nominal GDP

Besides real GDP it is relevant to explore other economic indicators, in a stress-test framework, it is, for instance, interesting to consider inflation or unemployment. Let \( M_t, V_t \) and \( P_t \) be the money supply, the velocity of money and the price at time \( t \), respectively. These variables are related by the well known equation of exchange \( M_t V_t = P_t Y_t \). According to most of our scenarios and for most of the regions, following the Paris Agreement leads to a reduction of produced goods. In other words, after the transition starts at time \( T \), we have a decline of \( Y_t \) which can be defined as \( \delta Y_t = \frac{Y_{t+1} - Y_t}{Y_t} \). Similarly we could defined \( \delta P_t \). Then, we have the following relation :

\[
\frac{M_{t+1}V_{t+1}}{M_tV_t} = \frac{P_{t+1}Y_{t+1}}{P_tY_t},
\]

\[
\ln\left(\frac{M_{t+1}}{M_t}\right) + \ln\left(\frac{V_{t+1}}{V_t}\right) = \ln\left(\frac{P_{t+1}}{P_t}\right) + \ln\left(\frac{Y_{t+1}}{Y_t}\right),
\]

\[
\ln(1 + \delta M_t) + \ln(1 + \delta V_t) = \ln(1 + \delta P_t) + \ln(1 + \delta Y_t).
\]

Thus :

\[
\delta M_t + \delta V_t - \delta Y_t \approx \delta P_t.
\]

If we assume Equation (10) holds true then we know that \( \delta P_t \) must be positive if \( M_t \) and \( V_t \) stays constants. If the money supply increases, for instance if we create money for greening the economy, and the velocity stays constant then \( \delta P_t \) would be even greater. For instance, if we assume the velocity stays constant, if we keep increasing our money supply by approximately 5% per year, and if we assume pessimistic growth rates (Figure 8), then we may have an inflation rate close to 6%.

That remark leads to a huge political problem, if it holds true, then, if one wants to transition to a low carbon economy before 2050, according to Section 3, it must decrease its output. Unfortunately, it may leads to inflation if the money supply is kept constant or increase, but central banks would then fight that inflation by increasing its rate, which would slow down green investments. If rates are kept low, then it would be a huge problem for the private banking sector. On one hand they would be pushed by the public and the regulator to grant very low interest loans to finance green energy sources, but, on the other hand it would increase the inflation that destroys its profits margin. One possibility that seems to be highly discussed is monetary policy targeted towards green assets, see for example the European taxonomy. That would indirectly grant low interest loans to green projects.

6 Possible evolutions of our parameterization

As briefly discussed in Section 5 we would like to discuss the possible developments of our model. If we assume Equation (5) or Equation (6) are relevant models, then we have 2 choices, either we want to fight climate change at the expanse of real GDP growth, or we try to increase produced goods to lift a part of the
population out of poverty today at the expanse of future climate change. Which will permanently change the living condition of all living species. That change will have huge detrimental impacts for future generations. Also, we want to stress the fact that we considered the best case scenario of global cooperation, with no free-rider. Our model also lacks essential damages. First of all, as Stern (2015) states, the total factor productivity could itself be damaged by climate change with a function of the following form:

\[ A_c(t) = (1 - d_c)A_c(t - 1). \]

Here \( d_{c,c} \) is a coefficient that grows with the climate warming. Stern states that global warming might displace or even kill populations which would result in a loss of knowledge and thus TFP. Also, the capital value under the DICE model only depreciates at a given rate. That rate should depend on climate change. Mainly because climate change is a threat to a lot of infrastructure and it is very sensitive to the issue of the stranded assets. Thus, it would be of great interest to use the following definition:

\[ K(t) = (1 - \delta_K(T(t)))K(t - 1) + I(t). \]

With \( \delta_K(T(t)) \) a function that measures the impact of climate change on capital depreciation. That function should have several properties:

- \( \lim_{T(t) \to \infty} \delta_K(T(t)) = 1 \),
- \( \delta_K(0) = \delta_K \),
- The function \( \delta_K(T(t)) \) should be increasing with respect to the mean temperature increase.

Also, while we transition from an energy to another, which means fossil fuel use decreases we will probably see higher values of depreciation. A recent French investigation from Giraud (2021) estimated that the financial sector invested 3395 billion euros in the fossil fuel industry from 2016 to 2020, these investments are at high risk to devaluation from future climate policy. If we decide to ban coal power plants then every coal plants and mines would be valued at 0. Which in fact increases capital depreciation. To summarize it would be of great interest to model capital depreciation as a function of the transition path, in the DICE model, that path is model by the parameter \( \mu(t) \), in our proposed model it could be a function of the energy mix:

\[ K(t) = (1 - \delta_K(T(t), \mu(t)))K(t - 1) + I(t). \]

Our model also lacks interactions between climate change and human death toll. Some reports state that an increase of (Mora et al., 2017; Hajat et al., 2006) 4°C might have disastrous impacts on death rate and cause millions if not billions of deaths. Unfortunately such articles only answer the question of deaths caused by heat waves. Climate change might increase mortality through many other risks, flood, fire, starvation ... Thus we would like to adapt the aforementioned model to take into account death toll. With \( L \) being also a function of climate change. That function shares the same property as \( \delta_K(t) \). As we have some conditions on the value it should take. For example, \( (L(t) - L(T(t)))_t \) must be increasing with respect to temperature increase. If \( T(t) \) is zero, then \( (L(t) - L(T(t)))_t \) should also be zero. If \( (T(t))_t \) tends to infinity then \( (L(t) - L(T(t)))_t \) should go to \( L(t) \).

**Conclusion**

The main conclusion of this work is the importance of exergy variations to explain total factor productivity variations. In almost all Western economies, even through the Covid-19 and the 2008 financial crisis, our model seems to explain relatively well observed data. The exergy contribution to the TFP seems to be equivalent for all countries. We introduce a proxy for imports that improved our model. It underlines
its importance in modern western economies. A secondary contribution of this work is the improvement of forward looking scenarios used in real world exercises such as stress-testing (ACPR, 2020) or public pension funds policy (COR, 2021). Confidence intervals are crucially important for these exercises, and our methodology introduces a natural framework to compute them.

That work contributes to a larger discussion where economists try to introduce energy as a main factor of economic development Ayres et al. (2003); Santos et al. (2018, 2021); Bercegol and Benisty (2022). Our work also tries to help policy makers understand the effect of different energy policy. Directly integrating tax or grants in exergy scenarios seems to be a natural extension of this work. Further study of the methodology presented in Section 3.1.2 are crucial to improve the usefulness of our model. Also, building a disintegrated model with sectorized scenarios is a legitimate future research direction. With such development, integrating that model to stress-test exercises would be even easier. Santos et al. (2021) argued that the TFP valued are inflated by a poor measurement of capital and labour. They also argued that adjusting quality of capital and labour may improve the link between TFP and exergy.

Unfortunately the results of our work are drastically different from the conclusions of the DICE model (Nordhaus, 2018). Our conclusion are drastically different because we directly integrate the fundamental effect of natural resources (and more specifically energy) in economic process. In this regard our model may be closer to other IAM models such as the very famous World 3 model and its most recent update (Meadows et al., 2013). Depending on the trajectory of exergy our results tend to be closer to one or the other model. It may be a way to reconcile both school. However despite being able to draw conclusion similar to World 3, we chose to not include other natural resources such as iron, copper, oil, . . . We assumed these materials are substituable by other materials even though some may have very unique physical property. Energy on the other hand is a very basic physical notion for any economic process that can not be ignored and can not be substituted by anything else.

In this article, we mainly discussed the relationship between Growth Domestic Product and exergy. Unfortunately beside the green house gas problem, we still have a lot of ecological puzzle to solve and maybe, access to clean energy would result in an ecological catastrophe. Even with a clean energy, destroying forests to grow crops, over-fishing or dumping plastic into the ocean would still result in a biodiversity disaster. Considering other dimensions of the ecological problem is mandatory to correctly handle the problem.
Appendix

A  List of countries

<table>
<thead>
<tr>
<th>Country</th>
<th>color</th>
</tr>
</thead>
<tbody>
<tr>
<td>United States</td>
<td>Blue</td>
</tr>
<tr>
<td>Japan</td>
<td>Orange</td>
</tr>
<tr>
<td>Italy</td>
<td>Green</td>
</tr>
<tr>
<td>Germany</td>
<td>Red</td>
</tr>
<tr>
<td>United Kingdom</td>
<td>Purple</td>
</tr>
<tr>
<td>France</td>
<td>Brown</td>
</tr>
<tr>
<td>Canada</td>
<td>Pink</td>
</tr>
</tbody>
</table>

B  Illustration of the long term relationship between TFP and \textit{exergy}

Figure 10: Fitted values and observed TFP from model

C  Computation of MSE and \( R^2 \)

If we assume we observed a sample \((\Delta A_t)_{t \in [1,T]}\), and its mean, \(\overline{\Delta A}\). If we also are able to compute \(\left(\overline{\Delta A_t}\right)_{t \in [1,T]}\), which could be the mean of the posterior density described by (5) or (6), then:

\[
\overline{MSE} = \frac{1}{T} \sum_{t \in [1,T]} \left( \Delta A_t - \overline{\Delta A_t} \right)^2, \tag{11}
\]

\[
R^2 = 1 - \frac{\sum_t \left( \Delta A_t - \overline{\Delta A_t} \right)^2}{\sum_t \left( \Delta A_t - \overline{\Delta A} \right)^2}. \tag{12}
\]
D Densities assumptions for Equation (5)

\[\begin{align*}
\alpha_{0,c} & \sim \mathcal{N}(\mu_{\alpha_0}, \sigma_{\alpha_0}^2) \\
\alpha_{1,c} & \sim \mathcal{N}(\mu_{\alpha_1}, \sigma_{\alpha_1}^2) \\
\beta_{0,c} & \sim \mathcal{N}(\mu_{\beta_0}, \sigma_{\beta_0}^2) \\
\Delta a_{c,t} | (\alpha_{0,c}, \alpha_{1,c}, \beta_{0,c}, \beta_{1,c}, \sigma, \Delta a_{c,t-1}, \Delta e_c(t)) & \sim \mathcal{N} (\alpha_{0,c} + \alpha_{1,c} \Delta a_c(t-1) + \beta_{0,c} \Delta e_c(t), \sigma^2)
\end{align*}\]

E Densities assumptions for Equation (6)

\[\begin{align*}
\alpha_{0,c} & \sim \mathcal{N}(\mu_{\alpha_0}, \sigma_{\alpha_0}^2) \\
\alpha_{1,c} & \sim \mathcal{N}(\mu_{\alpha_1}, \sigma_{\alpha_1}^2) \\
\beta_{0,c} & \sim \mathcal{N}(\mu_{\beta_0}, \sigma_{\beta_0}^2) \\
\beta_{1,c} & \sim \mathcal{N}(\mu_{\beta_1}, \sigma_{\beta_1}^2) \\
\Delta a_{c,t} | (\alpha_{0,c}, \alpha_{1,c}, \beta_{0,c}, \beta_{1,c}, \sigma, \Delta a_{c,t-1}, \Delta e(t)) & \sim \mathcal{N} (\alpha_{0,c} + \alpha_{1,c} \Delta a_c(t-1) + \beta_{0,c} \Delta e_c(t) + \beta_{1,c} \Delta \left( \sum_c e_c(t) \right), \sigma^2)
\end{align*}\]

F TFP trajectories for countries under optimistic scenario

<table>
<thead>
<tr>
<th>Primary energy source</th>
<th>Evolution per year</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coal</td>
<td>-5%</td>
</tr>
<tr>
<td>Natural gas</td>
<td>-5%</td>
</tr>
<tr>
<td>Crude oil</td>
<td>-5%</td>
</tr>
<tr>
<td>Nuclear</td>
<td>1%</td>
</tr>
<tr>
<td>Biofuels and waste</td>
<td>1%</td>
</tr>
<tr>
<td>Wind, solar, etc.</td>
<td>5%</td>
</tr>
</tbody>
</table>

Table 9: Primary energy scenario considered, each value represents the variation of a given energy source from a year to an other.
Figure 11: TFP trajectory using model (5) for 30 years under scenario presented in Table 9 for several countries without world exergy variations (model), red line represents the mean posterior and the blue area represents the 0.1 and 0.9 quantiles of the posterior density.
Figure 12: TFP trajectory using model (6) for 30 years under scenario presented in Table 9 for several countries with world exergy variations (model), red line represents the mean posterior and the blue area represents the 0.1 and 0.9 quantiles of the posterior density.
G  TFP trajectories for countries under intermediate scenario

<table>
<thead>
<tr>
<th>Primary energy source</th>
<th>Evolution per year</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coal</td>
<td>−5%</td>
</tr>
<tr>
<td>Natural gas</td>
<td>−5%</td>
</tr>
<tr>
<td>Crude oil</td>
<td>−5%</td>
</tr>
<tr>
<td>Nuclear</td>
<td>−3%</td>
</tr>
<tr>
<td>Biofuels and waste</td>
<td>0%</td>
</tr>
<tr>
<td>Wind, solar, etc.</td>
<td>5%</td>
</tr>
</tbody>
</table>

Table 10: Primary energy scenario considered, each value represents the variation of a given energy source from a year to another.
Figure 13: TFP trajectory with model (5), under scenario presented in Table 10 for several countries without world exergy variations, red line represents the mean posterior and blue area 0.1 and 0.9 quantiles of the posterior density.
Figure 14: TFP trajectory with model (6), under scenario presented in Table 10 for several countries without world exergy variations, red line represents the mean posterior and blue area 0.1 and 0.9 quantiles of the posterior density.
References


C. A. Hall. Introduction to special issue on new studies in eroi (energy return on investment), 2011.


IPCC. The physical science basis. Contribution of working group I to the Fourth Assessment Report of the Intergovernmental Panel on Climate Change, 996, 2007.

IPCC. The sixth assessment report - the numbers behind the science. 2021.


